



**A REALISTIC MODEL FOR OFFICIAL INTEREST RATES  
MOVEMENTS AND THEIR CONSEQUENCES**

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**Titolo: A REALISTIC MODEL FOR OFFICIAL INTEREST RATES. MOVEMENTS AND THEIR CONSEQUENCES**

# A Realistic Model for Official Interest Rates Movements and Their Consequences\*

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## Abstract

This paper extends the VAR methodology to examine the consequences of monetary policy decisions by considering two types of nonlinearities in the determination of official interest rates: 1) the asymmetry related to the different nature of the discrete and infrequent positive and negative interest rate movements determined by central bankers; and 2) the convexity in the transmission of policy shocks induced by the nonnegativity constraint in interest rates. For the UK, we find evidence of both types of asymmetries. Moreover, the operational independence granted to the Bank of England involved drastic changes on the interpretation of the reaction function of the monetary authority and the consequences of monetary shocks. In the US, responses to unexpected interest rate shocks are far more symmetric. Results highlight the importance of considering all types of asymmetries when studying monetary transmission.

*Keywords:* Monetary shocks, impulse-response functions, monetary policy, Markov Switching models.

*JEL Codes:* C22, C32, E52.

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# 1 Introduction

This paper proposes a new methodology for the empirical analysis of the manner in which official interest rates are determined. In particular, the framework presented here allows for the separate study of factors affecting the magnitude of interest rate changes (both positive and negative) as well as their probabilities. Recently, empirical literature has shown an increasing interest in modelling the discrete and infrequent changes of central bank rates. Here, we extend previous research allowing for a different characterization of the probability of positive and negative movements in a single model<sup>1</sup>.

A traditional methodology for modelling the dynamics of official interest rate series is the use of conventional ordered logit and probit models; Eichengreen et al (1985) and Davutyan and Parke (1995) are two relevant examples. In these models, the magnitude of interest rate changes is conditioned to a set of fundamental economic variables. Hamilton and Jorda (2002) propose the so-called autoregressive conditional hazard (ACH) models to analyze separately two different decisions by policy makers: 1) whether or not to change interest rates; and 2) by how much. They specify a hazard rate associated to an interest rate change for a given interval as a function of a set of economic variables and of the length of time between two previous interest rate changes. For the analysis of factors affecting the magnitude of change they use an ordered probit model.

In our particular context, two main limitations of this approach can be mentioned. Firstly, the ACH model does not differentiate between the probability laws governing positive and negative changes. As we will explain latter, this can be especially relevant when modelling interest rate movements in countries where the probability of negative and positive interest rate changes have been historically different. A second limitation concerns the fact that the two questions of magnitude and probability of change are considered in the context of two different models that are specified and estimated separately.

Here, we propose a model that explains interest rate changes as a function of three fundamental economic variables: inflation, exchange rate and output gap. However, parameters of the function are allowed to change with a latent three state variable that drives negative, no movement and positive interest rate interventions. In turn, the probability of being in each state is modelled to depend on interest rate decisions in the previous period and on a set of economic variables.

We denote this framework as the General Probabilistic and Magnitude (GPM henceforth) model for interest rate changes. The GPM model allows for the analysis of the different factors affecting four fundamental decisions

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<sup>1</sup>Gauss codes used in this analysis can be obtained from the authors upon request.

about interest rate movements: the magnitude of (1) positive and (2) negative interest rate changes; and the probability of (3) positive and (4) negative changes.

The specification and estimation of the GPM model is particularly simple as it shares some similar features with the Markov Switching model (MS hereafter) introduced by Hamilton (1989) and later extended by Filardo (1994) and Diebold et al. (1994) to the case where the transition probability matrix can change along time.

A separate contribution of the paper is the use of GPM models to define 4 different types of monetary shocks. The first two types relate to the situation where the monetary authority increases and decreases official interest rates when no change was expected. The third and fourth shocks arise when central bankers do not change official interest rates even though a positive or negative change had been expected. We compute numerically the impact of these shocks on inflation for the US and UK and compare our results with the standard VAR approach.

In order to obtain a more realistic analysis of the effect of interest rate shocks, we add a new type of asymmetry in the simulation that comes from the fact that interest rate series are censored at zero. We do this by following the Fisher Black's interpretation of interest rates as an option, in the sense that when interest rates are negative, economic agents prefer to keep the currency instead of lending it. This means that observed interest rates can be considered as an option on the shadow interest rate that would have existed in the absence of this option. This restriction is important even if interest rates are not zero because it affects the way in which economic agents form expectations about future values of the monetary instrument.

The lower bound restriction on interest rates and its importance in the transmission of monetary policy has been previously considered in the economic literature. Ruge-Murcia (2002) analyzes the consequences of the non-negativity constraint for short-run interest rates on the transmission of monetary policy to long-run interest rates. Ruge-Murcia considers rational expectations in the simulation process and shows that the non-negativity restriction generates an asymmetry in the transmission of monetary policy. He shows that a decrease in interest rate has a lower economic impact than an increase. Also, he finds that this asymmetry becomes more important as the interest rate approaches the neighbourhood of zero. Orphanides and Wieland (1998) present a small structural model with rational expectations in order to investigate the consequences of the lower bound restriction on interest rate finding that the effects of the restriction are nonlinear with respect to the inflation target and produces an important deterioration of the economy when the inflation target is located between zero and one per cent.

A fundamental difference between this paper and the literature above is

that we integrate in a single analysis two types of asymmetries related to the different sign of the infrequent and discrete interest rate movements 3 and the asymmetry generated by the lower bound for interest rates at zero.

Our results indicate that all types of asymmetries are important to understand the impact of interest rate shocks. More specifically, in the US the probability and magnitude of positive and negative interest rate movements are fairly similar, and the zero lower bound does not affect significantly the transmission of interest rate shocks to inflation. Results for the UK indicate that, in general, unexpected negative interest rate movements have a bigger impact on inflation than positive ones for high levels of interest rates. However, when the interest rate approaches zero, the consequences of positive and negative shocks are smaller and more symmetric.

The remainder of this paper is organized as follows. In the next section, we present a simple model to explain the type of asymmetric discrete movements found for official interest rates. In section 3 we present the GPM model, comparing it with different approaches to estimate the determinants of changes in the official interest rates set by central bankers, within a reaction function context. Section 4 explains how to form expectations about future interest rates when they are censored at zero and the shadow interest rate is generated by a GPM model. This discussion is shown to be useful to understand the types of nonlinearities in the formation of interest rate expectations induced by each of the different features of the model. We explain in Section 5 how to integrate a GPM model in the VAR approach in order to simulate the effect of different types of interest rate shocks. Section 6 analyzes the importance of the model's different assumptions in order to explain the impact of monetary shocks on inflation in the US and the UK. Some concluding remarks follow in Section 7.

## 2 A Simple Theoretical Model

We present a simple one-period model to motivate the discrete interest rate movements by the monetary authority. Our framework consists of a stylized Phillips curve and aggregate demand equations given respectively by expressions (1) and (2)

$$\pi = \pi^e + \theta y + e, \tag{1}$$

$$y = \rho y_0 - \beta \Delta i + u \tag{2}$$

where  $\pi$  is the current inflation,  $\pi^e$  is the expected inflation,  $y$  and  $y_0$  are respectively the current and lagged deviation of output from potential,  $\Delta i$  denotes interest rate movements and  $e$  and  $u$  are unexpected shocks to inflation and aggregate demand respectively. The parameters of the model fulfil the following restrictions:  $\theta > 0$ ,  $0 < \rho < 1$  and  $\beta > 0$ . These constraints reflect the trade off between inflation and economic cycle inherent

in the Phillips curve and ensures that the aggregate demand equation is stationary and is negatively affected by positive interest rate movements.

Notice that we assume in expression (2) that what enters the aggregate demand equation is not the nominal interest rate, but its changes. We do this to be consistent with the empirical evidence that indicates that nominal and real interest rates are generated by  $I(1)$  processes, while output gap is stationary by definition.

The monetary authority can be assumed to set the changes in the nominal interest rate in order to minimize his loss function given by:

$$L = \begin{cases} (\pi - \bar{\pi})^2 + \delta y^2 + F & \text{if } \Delta i \neq 0 \\ (\pi - \bar{\pi})^2 + \delta y^2 & \text{if } \Delta i = 0 \end{cases} \quad (3)$$

where  $\bar{\pi}$  is an intermediate target for inflation,  $\delta$  is a positive parameter referring to the importance of output gap for the policymaker and  $F$  denotes the utility cost associated with interest rate changes.

An important point to notice is that expression (3) indicates that the policymaker dislikes changing interest rates and not necessarily in proportion to the magnitude of change. In fact, it is plausible to assume that there is a fixed cost associated even to marginal interest rate changes that comes, for example, from the effort of explaining policy actions to commercial banks and other economic agents. This is an important difference between our model and some previous papers in the literature that characterize interest rate smoothing by including the variability of the interest rate in the objective function of the monetary authority; see, for example, Woodford (2003) and Goodhart (1997) among others. By doing this, they explain the degree of persistence in interest rates levels while here we provide an intuitive explanation on why interest rates changes occur in discrete steps.

The model also includes an equation describing the cost of moving interest rate,  $F$ , as a (not necessarily linear) function of interest rate movements in the previous period,  $\Delta i_0$ . We denote this function by  $F = f(\Delta i_0)$ . The intuition behind this is that two consecutive interest rate movements can be difficult to be justified by the monetary authority, especially if they have the opposite sign.

For the sake of building intuition in the model, we consider first the case in which  $\delta = 0$ , inflation target is zero,  $\bar{\pi} = 0$  and central banks cannot observe contemporaneously shocks to inflation and aggregate demand. The monetary authority set  $i$  in order to minimize his loss function subject to (1) and (2). There are two possible actions to consider: (1) moving, (2) not moving interest rates. When  $\Delta i \neq 0$ , the loss function is minimized by setting  $\pi = 0$ . If the central bank does not alter interest rates, its expected loss function would be  $L = (\pi^e + \theta \rho y_0)^2$ . Therefore, the Taylor rule takes the form

$$\Delta i = \begin{cases} 0 & \text{if } (\pi^e + \theta \rho y_0)^2 < f(\Delta i_0) \\ \frac{\pi^e + \theta \rho y_0}{\theta \beta} & \text{if } (\pi^e + \theta \rho y_0)^2 \geq f(\Delta i_0). \end{cases} \quad (4)$$

Equation (4) indicates that the monetary authority only moves interest rates when the utility reported by this action is higher than the cost,  $f(\Delta i_0)$ . However, if the policymaker decides to change interest rates, s/he will set the expected inflation to the level that minimizes his loss function. Two important points must be mentioned about the Taylor rule described above. First, expression (4) indicates the conditions that determine the decisions to move interest rates (or not) and the magnitude of the movements. Interestingly, both decisions are affected by economic variables, however the functional form in which fundamentals affect the two actions are not identical. This suggests the use of an econometric model that characterizes the decision to move interest rates or not, as well as the magnitude of the change, as a function of economic variables. The second point relates to the type of policy suggested by expression (4). The relationship between expected inflation and interest rate changes exhibited in Figure 1 indicates the existence of an inaction range when the policymaker prefers not to alter his monetary instrument in response to economic conditions. In fact, interest rates do not change when expected inflation is in the range  $(-\sqrt{f(\Delta i_0)} - \theta \rho y_0, +\sqrt{f(\Delta i_0)} - \theta \rho y_0)$ . Notice that this simple model describes a type of behaviour that is similar to the so called *opportunistic approach* for monetary policy. Proponents of this view hold that when inflation is moderate but relatively close to the objective, the monetary authority will not undertake any policy action to reduce inflation but will wait for external circumstances to accomplish the reduction. In this sense, this model shares some similarities with the framework proposed by Orphanides and Wilcox (2002). They find that when the loss function of the monetary authority depends on the absolute value of the output gap, it results in a Taylor rule. This Taylor rule is a discontinuous function which indicates that active policies are only undertaken when inflation surpasses a certain range of values. An important difference with the model in this section lies in the specification of the utility function of the monetary authority, as we assume that this function is affected by the cost of moving interest rates instead of the absolute value of the output gap. A second difference with the paper by Orphanides and Wilcox (2002) is in the empirical emphasis of this research.

[INSERT FIGURE 1]

It is also straightforward to formalize policy actions when the monetary authority cares about both inflation and output gap,  $\delta \neq 0$ , inflation target has a positive value,  $\bar{\pi} \neq 0$ , and central banks can observe contemporaneously shocks to inflation and output,  $e$  and  $u$ . In this case, interest rate movements can be described by the following function:



$$\Delta i = \begin{cases} 0 & \text{if } L_{\Delta i=0} < L_{\Delta i \neq 0} \\ \frac{\theta(\pi^e + e - \bar{\pi}) + (\theta^2 + \delta)(\rho y_0 + u)}{(\theta^2 + \delta)\beta} & \text{if } L_{\Delta i=0} \geq L_{\Delta i \neq 0}, \end{cases} \quad (5)$$

where  $L_{\Delta i \neq 0}$  and  $L_{\Delta i=0}$  are the loss function of the policymaker associated with moving and not moving interest rate respectively with the following specifications:

$$L_{\Delta i \neq 0} = \delta^2(\pi^e + e - \bar{\pi})^2 + \delta \left( \frac{\theta(\pi^e + e - \bar{\pi})}{\theta^2 + \delta} \right)^2 + f(\Delta i_0) \quad (6)$$

$$L_{\Delta i=0} = ((\pi^e + \theta \rho y_0 + \theta u) + e - \bar{\pi})^2 + \delta(\rho y_0 + u)^2 \quad (7)$$

Using this framework, it is also possible to discuss some of the potential explanations for the asymmetric interest rate movements found in the literature. For example, some authors suggest that monetary policies have a smaller impact on output in an expansionary cycle than in a downturn cycle as the latter can activate financial constraints in the credit markets; see Kiyotaki and Moore (1997) and Bernanke and Gertler (1989) for some relevant examples. This could be incorporated to our model by giving a different value to parameter  $\beta$  in (2) for expansionary and tight policies. In this case, it is straightforward to notice that the monetary authority will behave more actively when raising rather than decreasing interest rates.

Another relevant explanation for asymmetric interest rate movements could come from the fact that the fixed cost associated with a reduction in interest rate is smaller than the cost of raising it. This is a plausible assumption because although commercial banks dislike changes in interest rates once they have set their contracts with customers, they would accept more easily a reduction than an increase in the price of money. If, in our model,  $F$  has a higher value when the policymaker increases rather than decreasing interest rates, expansionary monetary policies will be undertaken more often than contractionary ones.

### 3 A GPM Model for Interest Rate Decisions

In the empirical analysis, modelling the discrete and infrequent changes in bank rates has been a recurrent topic in the empirical literature. A typical example is based on the use of logit and probit models in which the dependent variable is referred to the magnitude of change of official interest rate (instead of interest rate levels) set by the monetary authority and assumes that all the relevant conditioning variables are included in the study; see for example Eichengreen et al. (1985) and Davutyan and Parke

(1995). A clear advantage of this approach is that it is well suited for the nature of changes in interest rates set by central bankers. However, as pointed out by Hamilton and Jorda (2002), an important drawback of this methodology is the presence of a potentially significant serial correlation in the latent residuals. A solution to this problem can be found in the autoregressive conditional duration (ACD henceforth) model of Engle and Russel (1997) and Engle (2000). In the ACD model, the length of time elapsed between previous events is taken to forecast future durations.

Starting from this consideration, Hamilton and Jorda (2002) proposed their autoregressive conditional hazard (ACH) model. In this case, their interest is not on the length of time between events but on the probability of an interest change tomorrow given the information available today. In their framework they study separately two types of decisions by central bankers: 1) the decision on whether or not to change interest rate; and 2) the magnitude of the change. For the first one, they specify and estimate an ACH model in which the hazard rate for interest rate change in a given period depends on a vector of fundamental variables. The second decision, on the other hand, is evaluated using an ordered probit model for the magnitude of change. However, the ACH approach to the analysis of interest rate changes in the UK encounters an important problem in the fact that, while the observation of changes seems to indicate that the probability of positive is clearly different from that of negative movements, ACH models make no distinction between the probability law governing positive and negative interest rate movements.

Here, we propose a simple model in which the probabilities of positive and negative interest rate movements are treated differently. Moreover, in contrast to the ACH model that studies the probability and magnitude of change in two separate models, the GPM model integrates these two decisions into a single framework.

We require a framework that explains the different probability of positive and negative interest rate changes, and also their different magnitudes, as a function of inflation and output gap. In order to do this, we define a latent variable  $s_t^*$  whose values 1, 2 and 3 are associated to negative, no movement and positive interest rate changes respectively. Notice that, although we can observe the variation of official interest rate at every period  $t$ , variable  $s_t^*$  can be thought as an unobservable indicator on how prone central bankers are to perform positive or negative interest rate changes given the available information. Also, it can be used to evaluate the probability of positive, nil and negative interest rate changes at period  $t$  given the information at  $t - 1$ .

Then, the GPM model can be defined as:

$$i_t = \begin{cases} i_{t-1} + c_1 + \beta_1^1 \pi_t + \beta_2^1 y_t + \beta_3^1 \Delta e_t + \sigma_1 \varepsilon_t & \text{if } s_t^* = 1 \\ i_{t-1} & \text{if } s_t^* = 2 \\ i_{t-1} + c_2 + \beta_1^3 \pi_t + \beta_2^3 y_t + \beta_3^3 \Delta e_t + \sigma_3 \varepsilon_t & \text{if } s_t^* = 3 \end{cases}, \quad (8)$$

where  $i_t$  is the interest rate controlled by the monetary authority;  $\pi_t$  is the rate of inflation;  $y_t$  is the cyclical output;  $e_t$  denotes the foreign exchange rate included to account for open market considerations in the interest rate rule,  $\varepsilon_t$  is a standard Normal disturbance<sup>2</sup>.

Two points must be mentioned at this stage. First, model (8) assumes that the monetary authority knows at time  $t$  all the simultaneous information on inflation, output gap and exchange rates. Second, the model can be easily generalized to allow for lagged explanatory variables. These two points will be more specifically outlined in the following section.

To define the probability of being in one of the three states, let  $\{p_{ij}\}_{i,j=1,2,3}$  the transition probability  $P(s_t^* = j/s_{t-1}^* = i)$ . We assume that  $s_t^*$  follows a three-state Markov change with transition probability matrix:

$$P = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix} \quad (9)$$

where  $p_{i3} = 1 - \sum_{j=1}^2 p_{ij}$ .

The described framework in (8) is similar to a standard MS model where the only peculiarity is the absence of an error term in the equation for state  $s_t^* = 2$ . MS models were initially introduced by Hamilton (1989) and later extended by Filardo (1994) and Diebold et al. (1994) to the case where the transition probability matrix can change along the time, depending on some observed variables,  $\mathbf{z}_t$ . In this case, we need some specification for the probabilities  $p_{ij,t}$ . Filardo (1994) uses the following logistic functions for the 2-states case:

$$p_{ii,t} = \frac{\exp(\phi_i + \mathbf{z}_t \vartheta_i)}{1 + \exp(\phi_i + \mathbf{z}_t \vartheta_i)} \quad (10)$$

where  $\phi_i$  and  $\vartheta_i$  ( $i = 1, 2$ ) are unknown parameters. Of course, the choice of  $\mathbf{z}_t$  is crucial and implies computational efforts and complications in the likelihood function. Filardo (1998) indicates the conditions to select  $\mathbf{z}_t$  to avoid estimation problems; in general a sufficient condition to justify the

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<sup>2</sup>The intuition of this model is analogous to the model with changes in regime presented by Hamilton (1994), Chapter 22. He observes a different evolution of the volume of dollar-denominated accounts held in Mexican banks before and after 1982. Then, he argues that although it is possible to estimate a different model for these two periods, the change in regime should not be considered as the outcome of a perfectly foreseeable deterministic event but a random variable.

use of the Hamilton filter (Hamilton, 1990) in a time varying transition probability Markov Switching (TVTP-MS hereafter) model to develop the maximum likelihood estimation, is that the elements in  $\mathbf{z}_t$  are conditionally uncorrelated with  $s_t$ . This is a plausible assumption according to our empirical analysis in the following section.

Here, we also extend the specification of Filardo to a 3-states case, using a multinomial logit:

$$p_{ij,t} = \frac{\exp(\phi_{ij} + \mathbf{z}_t \vartheta_{ij})}{1 + \sum_{h=1}^2 \exp(\phi_{ih} + \mathbf{z}_t \vartheta_{ih})} \quad (11)$$

Details about estimation of the model can be found in Tena and Otranto (2006). As said, the only difference with respect to a classical MS model is the presence of the second equation in (8), which is deterministic. To provide maximum likelihood estimates we have introduced a small approximation, which ensures to assign only observations equal to 0 to state 2. We have supposed that, when  $s_t^* = 2$ , the variable  $(i_t - i_{t-1})$  is Normal with mean 0 and variance  $10^{-9}$ ; in this way we can have a stochastic equation to estimate the transition probabilities relative to state 2, but with a variance practically equal to zero.

In the model described above, the magnitude of interest rate movements is asymmetrically affected by economic fundamentals. Moreover, our approach also characterizes the probabilities of positive, negative and no interest rate movements as a function of previous policy actions and economic variables.

Based on model (8), interest rate expectations one period ahead given that we are in the  $i - th$  state are formed as

$$E_t i_{t+1} = i_t + E_t p_{i1,t+1} (\beta_1^1 E_t \pi_{t+1} + \beta_2^1 E_t y_{t+1} + \beta_3^1 E_t \Delta e_{t+1}) + E_t p_{i3,t+1} (\beta_1^3 E_t \pi_{t+1} + \beta_2^3 E_t y_{t+1} + \beta_3^3 E_t \Delta e_{t+1}). \quad (12)$$

Notice that the approach proposed here also clearly differs from other papers that have used limited dependent models, like the multinomial logit/probit framework, to analyse monetary policy decisions; see, for example Chevapatrakul et al. (2001) and Bhattacharjee and Holly (2006). While a multinomial probit/logit model simply describes a model for the probability that interest rates will change within some ranges of values, the model here not only estimates these probabilities, but also the magnitude of changes associated with them and how these decisions are differently affected by economic variables.

Moreover, as we will show later, the number of different states for the range of interest rate variations in our model is not a subjective decision, as typically occurs in multinomial probit/logit models, but it is a decision

to be taken on the basis of suitable econometric tests. The second, and most important difference, between this literature and our approach, is that we extend the VAR approach to define new types of monetary shocks and estimate their impact on economic variables.

## 4 Including the Nonnegative Constraint in the GPM Model

We now extend the previous GPM models by including the lower bound at zero in interest rates. We do this by following the Fischer Black's interpretation of currency and interest rates as options, in the sense that, when interest rates are negative, agents can hold currency instead. Now, the observed interest rate at time  $t$ ,  $i_t$ , can be considered as an option on a shadow interest rate,  $i_t^*$ , with a strike price at zero. This variable can be interpreted as the interest rate available in the absence of the currency option. The observed and shadow interest rates are related by

$$i_t = \max(i_t^*, 0) \quad (13)$$

We can also express (13) as

$$i_t = \begin{cases} i_t^*, & \text{if } i_t^* > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

Notice that this formulation corresponds to a standard limited-dependent variable censored at zero, with  $i_t^*$  the associated latent variable following a GPM model. Thus, we can formulate expressions (8) as

$$i_t^* = \begin{cases} i_{t-1} + \beta_1^1 \pi_t + \beta_2^1 y_t + \beta_3^1 \Delta e_t + \sigma_1 \varepsilon_t & \text{if } s_t^* = 1 \\ i_{t-1} & \text{if } s_t^* = 2 \\ i_{t-1} + \beta_1^3 \pi_t + \beta_2^3 y_t + \beta_3^3 \Delta e_t + \sigma_3 \varepsilon_t. & \text{if } s_t^* = 3. \end{cases} \quad (15)$$

Given that  $i_t$  is a censored variable, this model cannot be used in order to form expectations about future interest rate movements because  $E_t \varepsilon_{1t+1} \neq 0$ . Therefore, we follow Ruge-Murcia (2002) and define

$$C_{t+1} = \frac{-E_t(i_{t+1}^*)}{E_t p_{1j,t+1} \sigma_1} = \frac{-[(i_t + E_t p_{1j,t+1} (\beta_1^1 E_t \pi_{t+1} + \beta_2^1 E_t y_{t+1} + \beta_3^1 E_t \Delta e_{t+1})) + E_t p_{3j,t+1} (\beta_1^3 E_t \pi_{t+1} + \beta_2^3 E_t y_{t+1} + \beta_3^3 E_t \Delta e_{t+1})]}{E_t p_{1j,t+1} \sigma_1}, \quad (16)$$

to write (14) as<sup>3</sup>

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<sup>3</sup>Notice that we use the fact that only negative interest rate movements, associated with  $s_t^* = 1$ , are affected by the lower bound constraint.

$$i_{t+1} = \begin{cases} i_{t+1}^* & \text{if } \varepsilon_{t+1} > C_{t+1} \\ 0 & \text{otherwise} \end{cases} . \quad (17)$$

Then, interest rate expectations at  $t + 1$  are

$$E_t i_{t+1} = E_t (i_{t+1}/\varepsilon_{t+1} > C_{t+1}) \Pr(\varepsilon_{t+1} > C_{t+1}) + E_t (i_{t+1}/\varepsilon_{t+1} \leq C_{t+1}) \Pr(\varepsilon_{t+1} \leq C_{t+1}). \quad (18)$$

Note that  $E_t (i_{t+1}/\varepsilon_{t+1} \leq C_{t+1}) = 0$ . Also,

$$E_t (i_{t+1}/\Omega_t, \varepsilon_{t+1} > C_{t+1}) = i_t + E_t p_{1j,t+1} (\beta_1^1 E_t \pi_{t+1} + \beta_2^1 E_t y_{t+1} + \beta_3^1 E_t \Delta e_{t+1}) + E_t p_{3j,t+1} (\beta_1^3 E_t \pi_{t+1} + \beta_2^3 E_t y_{t+1} + \beta_3^3 E_t \Delta e_{t+1}) + E (\varepsilon_{t+1}/\xi_{t+1} > C_{t+1}). \quad (19)$$

Finally, if we define the cumulative and density functions of a standard normal variable by  $\Phi(\bullet)$  and  $\phi(\bullet)$ , we have  $E (\varepsilon_{t+1}/\varepsilon_t > C_{t+1}) = \frac{\sigma_1 \phi(C_{t+1})}{[1 - \Phi(C_{t+1})]}$  and  $\Pr(\varepsilon_{t+1} > C_{t+1}) = [1 - \Phi(C_{t+1})]$ .

Then, we can write expression (18) as:

$$E_t i_{t+1} = [1 - \Phi(C_{t+1})] (i_t + E_t p_{1j,t+1} (\beta_1^1 E_t \pi_{t+1} + \beta_2^1 E_t y_{t+1} + \beta_3^1 E_t \Delta e_{t+1}) + E_t p_{3j,t+1} (\beta_1^3 E_t \pi_{t+1} + \beta_2^3 E_t y_{t+1} + \beta_3^3 E_t \Delta e_{t+1})) + \sigma_1 \phi(C_{t+1}). \quad (20)$$

In order to get insight on the importance of the non-negativity restriction on interest rates, notice that for high values of  $i_t$ ,  $C_t$  decreases and  $E_t(\varepsilon_t/\varepsilon_t > C_t)$  approaches to  $E_t \varepsilon_t$ . Therefore, the type of convexity induced by the non-negativity constraint becomes more important when interest rates is in the neighbourhood of zero.

Notice that for a GPM model with three states, there are two potential sources of asymmetries in the formation of interest rate expectations: 1) the different probabilistic laws that generate positive and negative interest rate movements; and 2) the convexity of expected negative interest rate movements induced by the non-negativity constraint. However, although the first type of asymmetry is, in principle, independent of the interest rate level, the second type becomes more important as interest rates approach zero.

## 5 Monetary Policy Shocks in a GPM Model

An interest rate shock can be defined as:

$$\varepsilon_{i,t} = i_t - E(i_t/\Omega_t), \quad (21)$$

where  $\varepsilon_{i,t}$  is the fundamental shock related to the equation for interest rate and  $\Omega_t$  refers to all the available information to the monetary authority up to period  $t$ .

The shock in expression (21) can be written as:

$$\varepsilon_{i,t} = i_t - i_{t-1} - (E(i_t/\Omega_t) - i_{t-1}). \quad (22)$$

According to expression (22), Hamilton and Jorda (2002) indicate that there are two types of monetary shocks that can be distinguished in an ACH model but not in a standard VAR model. The first one relates to the situation where the monetary authority changes official interest rates ( $i_t - i_{t-1} \neq 0$ ) when no change was expected ( $E(i_t/\Omega_t) - i_{t-1} = 0$ ). The second one is when central bankers fail to change official interest rates ( $i_t - i_{t-1} = 0$ ) even though a change ( $E(i_t/\Omega_t) - i_{t-1} \neq 0$ ) was expected.

A key difference between the ACH and the GPM framework is that the GPM model is entirely asymmetric and treats differently the probability of expansionary and contractionary monetary policy. Then, we can extend the previous definition allowing for the differentiation of 4 different types of monetary shocks in the following way:

$$\varepsilon_{i,t} = \underbrace{(i_t - i_{t-1})^+}_{\text{Shock 1}} + \underbrace{(i_t - i_{t-1})^-}_{\text{Shock 2}} - \underbrace{(E(i_t/\Omega_t) - i_{t-1})^+}_{\text{Shock 3}} - \underbrace{(E(i_t/\Omega_t) - i_{t-1})^-}_{\text{Shock 4}}, \quad (23)$$

where  $(x_t)^+$  is a function that takes value  $x_t$  if  $x_t > 0$  and zero otherwise; similarly  $(x_t)^-$  takes value  $x_t$  when  $x_t < 0$  and zero otherwise.

Hence, using the GPM model, we estimate the effect of four different types of monetary shocks. The first two shocks represent respectively an increase and a decrease in the official interest rates when no change was expected. Shocks 3 and 4 represent no change in interest rate when a positive and a negative movement were expected respectively. In GPM models, these four events do not necessarily have the same effect. An explanation of how to estimate the effect of these shocks follows.

Suppose that we are in period  $T$  and we want to estimate the effect on the economy of a certain value of  $i_T$  on a set of fundamental variables, denoted by  $\mathbf{Y}_t$ , at different time horizons. The elements in  $\mathbf{Y}_t$  can be split into two different groups: those that react with one lag delay to  $i_T$ , denoted by  $\mathbf{Y}_{1,t}$ ; and those that react at the same time as  $i_t$ , denoted by  $\mathbf{Y}_{2,t}$ . The first thing to do is to estimate the value of the variables that react simultaneously with  $i_T$ ,  $\hat{\mathbf{Y}}_{2,T}(i_T)$ . We do this by using linear equations. Then, we also use linear equations to predict the value of  $\mathbf{Y}_{1,T+1}(i_T, \Omega_T)$ . The one-step-ahead forecast of  $\hat{i}_{T+1}$  is then obtained from the GPM models as explained in the previous two sections.<sup>4</sup>

<sup>4</sup>A more detailed description of this process for the case when interest rate is not censored at zero can be found in Tena and Otranto (2006).

Iterating in this manner, we can compute

$$\widehat{\mathbf{Y}}_{T+k}(i_T) = \left( \widehat{\mathbf{Y}}_{1,T+k}(i_T, \Omega_{T+k}), \widehat{i}_{T+k}(i_T), \widehat{\mathbf{Y}}_{2,T+k}(i_T, \Omega_{T+k}) \right)'. \quad (24)$$

Then, we can easily extend the procedure in Hamilton and Jorda (2002) to answer the questions: what difference does it make if the Central Bank raises (decreases) the official interest rate by 25 basis points during month  $T$  compared to the case in which it keeps it constant? This can be done by computing the following expressions:

$$(0.25)^{-1} \left[ \widehat{\mathbf{Y}}_{T+k}(i_T) \Big|_{i_T=i_{T-1}+0.25} - \widehat{\mathbf{Y}}_{T+k}(i_T) \Big|_{i_T=i_{T-1}} \right], \quad (25)$$

$$(0.25)^{-1} \left[ \widehat{\mathbf{Y}}_{T+k}(i_T) \Big|_{i_T=i_{T-1}-0.25} - \widehat{\mathbf{Y}}_{T+k}(i_T) \Big|_{i_T=i_{T-1}} \right]. \quad (26)$$

We can also estimate what would happen if we predicted a positive and negative change in official interest rates, but no change in fact occurred, from the following expression:

$$(w_T)^{-1} \left[ \widehat{y}_{T+k/T}(i_T) \Big|_{i_T=i_{T-1}} - \widehat{y}_{T+k/T}(i_T) \Big|_{i_T=(\widehat{i}_{T/T-1})^+} \right], \quad (27)$$

$$(w_T)^{-1} \left[ \widehat{y}_{T+k/T}(i_T) \Big|_{i_T=i_{T-1}} - \widehat{y}_{T+k/T}(i_T) \Big|_{i_T=(\widehat{i}_{T/T-1})^-} \right], \quad (28)$$

where

$$w_T = \begin{cases} \left( i_{T-1} - \widehat{i}_{T/T-1} \right)^{-1} & \text{if } \left| i_{T-1} - \widehat{i}_{T/T-1} \right| > 0.05 \\ 0 & \text{otherwise} \end{cases}.$$

As suggested in Hamilton and Jorda (2002), the effect of the weight  $w_T$  in expressions (27) and (28) is to ignore observations for which no change was expected and to rescale forecast errors into units comparable to (25) and (26).

Notice that, departing from (22), it is also possible to define 6 different types of shocks (instead of 4): positive, negative and no movements when positive, negative and no movement was expected. However, we prefer to consider the decomposition in (23) for mainly two reasons: 1) having 6 shocks does not change the main conclusion of the analysis about the different effect of negative and positive interest rate movements; and 2) the cases where interest rates change in reverse direction to what it was expected are very rare in practice.



## 6 Empirical Results

This section appraises empirically the importance of the two types of asymmetries considered in the previous two sections. It does so by comparing inflation reactions to monetary shocks in a system that assumes that official interest rates are generated by a GPM model bounded below by zero with those obtained under two alternative benchmark specifications: 1) a standard VAR linear system; and 2) a VAR system that includes a GPM model to characterize interest rates without assuming they are censored at zero.

We start with the simplest case, the specification and estimation of two standard VAR linear models for the US and UK. The endogenous variables of interest for each of the two VAR models are (in this order): the annual difference of the logged consumer price index,  $\pi_t$ ; the output gap computed as the difference between the logged IPI and its HP trend,  $y_t$ ; the logged US/UK foreign exchange rate in first differences,  $\Delta e_t$ ; a measure of long run interest rates movements that are obtained from the 30-year Conventional Mortgage Rate for the US and 20-year interest rate for the UK,  $\Delta r_t$ ; and official interest rate movements by the central bank the last day of month  $t$ ,  $\Delta i_t$ , that for the UK was obtained from the Official Bank Rate history provided by the Bank of England whereas we used the Federal Fund Target for the US. The variables are on a monthly basis and cover the period from January 1971 to May 2006 and they were obtained from the UK Office for National Statistics, the Bank of England and the Federal Reserve Bank of St. Louis. The two structural VAR systems are identified by imposing the recursiveness assumption. This scheme is chosen because it is simple and its widespread use makes results comparable to previous studies.

We estimate the linear VAR system with 36 lags and focus on the inflation response to interest rate shocks. We do this for the sake of brevity and because central bankers explicitly admit that their main target when they move nominal interest rates is to keep inflation stable. However, based on experiments not reported here, the types of asymmetries shown in our paper also remains relevant when we study responses to other variables to policy shocks. Figures 2 and 3 show the cumulative inflation responses to negative interest rate shocks for the UK and the US respectively. Consistently with the economic theory, the figure shows that an expansionary monetary policy shock generates an increase in inflation in the long run for both countries. The figures also indicate that inflation reactions tend to be positive in the short run. We deal with this issue by 1) adding other variables in the system such as the commodity price index and different measures of monetary aggregates; and 2) trying different order for the variables under the recursiveness assumption. However, results were very similar in all cases. We deal with this issue by 1) adding other variables in the system such as the commodity price index and different measures of monetary aggregates; and 2) trying different order for the variables under the recursiveness assumption. However,

results were very similar in all cases. The perverse response is not new in the VAR literature; Christiano et al. (1999) and Sims (1992) document cases of responses that are not consistent with the economic literature in the short run. The most likely explanation for this is that the monetary authority sets policy by using private information that is not shared by the rest of the economy and cannot be captured in a VAR model.

[INSERT FIGURE 2]

[INSERT FIGURE 3]

For our second benchmark, we consider two similar VAR systems in which, for each of the two specifications, all the equations are linear except the official interest rate set by the central bank that was replaced by a GPM specification. One important decision in this context is the determination of the number of different states in the GPM equation for the US and the UK. Monthly Official interest rate series for the UK and the US are exhibited in Figure 2. Although the period of analysis includes different monetary regimes, two important stylized facts can be observed in practically all periods. First, in the UK, the number of negative interest rate movements, 108, almost double the number of positive movements, 57. However, the average magnitude of negative movements, 0.59, is much lower than the average magnitude of positive movements, 1.06.

For the US, the patterns of movements is far more symmetric. In practice, for the period of analysis, there were 101 (132) negative (positive) interest rate changes with an average magnitude of 0.69 (0.54). The intuition is that a GPM model with three states that treats differently the probabilistic law of positive and negative interest rate changes should fit well for the UK. However, interest rates in the US could be generated by a two-states GPM model that only considers the decisions of moving interest rates or not.

This intuition can be supported by the nonparametric Bayesian procedure of Otranto and Gallo (2002), which provides an empirical probability distribution for the number of states, via Gibbs sampling, analyzing directly the data and supposing that they are generated by a mixture of Normal distribution. We simply need to fix a parameter (it is called  $A$  in the Otranto and Gallo, 2002, paper) which provides the a priori distribution of the number of states. Fixing  $A = 0.4$  we obtain a prior with two modes in correspondence of 2 and 3 states. The posterior distribution of the number of states is explained in Table 1. There is a strong evidence for two states in the USA case, whereas the UK has a mode in correspondence of 3 states, with a certain probability also for the case of 4 states. Anyway we have tried to use several different priors, which yield the same result. For this reason we have estimated a two stage GPM model (that only distinguishes between the actions of moving interest rate or not) for the US and a three states GPM model (that distinguish positive, negative and no movement) for UK.

[INSERT TABLE 1]

For the estimation of the models we have acted in the following way. We have started from the models with all the explicative variables at different lags and then we have excluded the non significant variables; in particular for the USA the only significant variable is  $y_t$ , whereas for the UK  $y_t$  and  $\Delta e_t$ . To estimate the probabilities for the different states, we have used as  $\mathbf{z}_t$  the variables  $\pi_t$ ,  $y_t$  and  $\Delta e_t$  at different lags; for both the models we have chosen  $\pi_{t-1}$ , obtaining a specification similar to the GPM model of Tena and Otranto (2006). Moreover, in the UK case,  $\pi_{t-1}$  is significant only in the logit models for the probabilities  $p_{12,t}$  and  $p_{32,t}$ ; as a matter of fact the probabilities  $p_{21,t}$ ,  $p_{22,t}$  and  $p_{23,t}$  are constant (we erase the index  $t$  in Table 2). In Table 2 we show the estimation results.

[INSERT TABLE 2]

In the model specification, we assume that the monetary authority reacts simultaneously to movements in the different macroeconomic variables. This assumption is justified because it is reasonable to think that central bankers have real time monthly information on exchange rate, inflation and output gap; see, for example, Bernanke and Blinder (1992) and Wright (2002). This assumption is even less restrictive in our case because we collect the values of  $i_t$  the last day of each month. However, in order to have a stronger justification for the specification in equation (8), we test the null hypothesis of exogeneity with a Hausman (1978) test accepting the null hypothesis for all the explanatory variables at the conventional significance levels.

In our second benchmark specification two linear VAR systems are estimated but we replace the estimation of the interest equation for the relevant GPM model and then simulate inflation reactions to monetary shocks using the procedure described in the previous section. Figures 2 and 3 show the estimation of inflation responses to interest rate shocks in the US and UK respectively. As discussed above, reactions to shocks of different signs in the US are symmetric and therefore we only show inflation reactions to negative interest rate shocks. Two important points must be highlighted about the inflation reactions in the US. First, unexpected interest rate movements have a more significant effect on US inflation than no rate movement when a change was expected. This result is consistent with previous analysis by Hamilton and Jorda (2002). The second important point is that reactions to unexpected shocks under the GPM model for the interest rate equation are qualitatively similar to those obtained from the linear VAR model.

UK inflation responses show that the sign of the shock have an asymmetric influence on inflation. The asymmetric impact of monetary shocks is not a new result in the literature; see, for example, Atanasova (2003) and Giovannoni and Tena (2008). Two plausible explanations for this are the presence of frictions in the credit market and/or a convex Phillips curve. However, the main difference between our research and the aforementioned papers is that we consider a VAR system in which the only source of nonlin-

earity comes from the interest rate equation. In our framework, this asymmetric effect can be explained as, according to our estimations, negative interest rate movements increase the expectation about subsequent interest rate movements in the near future, however this effect is not so strong for positive movements.

Now, we turn to the analysis of the effect of the lower bound at zero in interest rates. Given the negligible effect of not changing interest rate when this change is expected, we focus on the effect of unexpected interest rate movements. Because inflation responses are potentially dependent on the interest rate level, we show these inflation responses in the US for three relevant ranges of official interest rate levels. The reactions are shown in Figure 4. It can be observed that they are symmetric but of smaller magnitude than the reactions obtained without imposing the lower restriction at zero for interest rates. Another relevant point to notice is the fact that the effects of monetary shocks are fairly similar for the two interest rate ranges [5, 15] and [2.5, 5]. This result is consistent with Orphanides and Wieland (1998) who show that the consequences of the zero bound are negligible in the US for all the relevant interest rate ranges.

[INSERT FIGURE 4]

Inflation reactions to monetary shocks in the UK are exhibited in Figure 5 for the same range of interest rate levels. It can be observed that the zero restriction matters for this country. More specifically, a decrease in the short rate produces a stronger inflation response than an increase of the same magnitude for high interest rate levels. However, as the interest rates approach zero, the nonnegative constraint introduces a nonlinear and convex relation between interest rates and inflation that offsets the type of asymmetry induced by the GPM specification in the second benchmark. Thus, when interest rates are between 1 and 2.5%, positive and negative interest rate movements have a similar effect.

[INSERT FIGURE 5]

Another important aspect to highlight from Figure 5 is the fact that inflation response to a change in interest rate is smaller at the neighborhood of zero regardless of whether it is a positive or a negative interest rate shock. This point can be clearly observed in Figure 6. The intuition for this result is that when interest rates approaches to zero, central bankers loose the possibility of affecting expectations on interest rate changes as economic agents will expect future interest rate increases regardless of the decision taken by the monetary authority. This last result is also consistent with the findings of Ruge-Murcia (2002, 2006).

[INSERT FIGURE 6]

We followed the recommendation in Nelson (2000) and tested for the presence of breaks in the reaction function of the monetary authority. Practically, for the US we tested for the possibility of a break in October 1982

to account for the new operating procedure when the FOMC abandoned the procedure of targeting the federal funds rate in favour of targeting on non borrowed reserves. However, there are not significant differences in our analysis when we estimate a GPM model with the whole sample compared to the case that only uses information after 1982. For the UK, we tested for the presence of the following breaks: 1) the premonetary targeting period at June 1976; 2) the "hard" exchange rate mechanism (from October 1990 to September 1992); and 3) the operational independence of the Bank of England in May 1997. We only found significant evidence of a structural break in May 1997. Moreover, economic tests strongly suggest that a two-states GPM model (with constant transition probabilities) is the best option to describe the pattern of interest rate movements after 1997, whereas a three-states TVTP model, as in (15), fits the previous period.<sup>5</sup> Therefore, we specify and estimate the following model for the shadow interest rate after May 1997 (standard errors between brackets):

$$i_t^* = \begin{cases} i_{t-1} - 0.006 + \frac{0.88}{(0.037)} \Delta i_{t-1} + \frac{6.43}{(4.970)} y_t + \frac{0.044}{(0.011)} \varepsilon_t & \text{if } s_t^* = 1 \\ i_{t-1} & \text{if } s_t^* = 2 \end{cases} \quad (29)$$

$$p_{11} = 0.415, \quad p_{22} = 0.750$$

$$(0.089) \quad (0.050)$$

Figure 7 shows the estimation of inflation responses to interest rate shocks using the reaction function in (29). In this case, the only source of asymmetry comes from the lower bound of interest rate at zero that makes the impact of positive shocks more important compared to negative ones. Given these results, a relevant question is whether to use reactions in Figure 5 or Figure 7 to understand the impact of unexpected interest rate shocks in the UK. However, there is not a clear answer to this question. On one hand, equation (29) can be deemed as a more realistic representation of the current reaction function of the Bank of England but, on the other hand, the last estimation uses very few observations of interest rate movements and, more importantly, the range of interest rate values used in the estimation is very narrow compared with that range in the historical sample from 1971. This is important under the plausible assumption that the observed pattern of interest rate fluctuations is different when interest rate levels are high compared to when they are close to zero. . Therefore, a reasonable strategy may be to use the policy rule in (29) but testing how consistent with the two frameworks future observations (in different ranges of interest rate values) will be.

[INSERT FIGURE 7]

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<sup>5</sup>The estimates of the first span are very similar to those shown in Table 2, relative to the full sample.

## 7 Concluding Remarks

This paper extends the VAR methodology by studying the consequences of the asymmetry of the discrete and infrequent positive and negative official interest rate movements and the zero lower bound on interest rates in the US and the UK. The econometric analysis suggests that a two-state GPM model is an accurate device to explain interest rate movements in the US. However, historical UK official interest rate decisions are generated by a three-state GPM model that differentiates the probabilistic law of positive and negative interest rate changes. When the zero restriction on interest rates is not considered, the GPM model for the US implies symmetric effects on inflation of unexpected interest rate movements of different signs. However, for the UK a decrease in interest rates has a stronger impact on inflation than an increase. This is because after a negative interest rate change, there is a relatively high probability of subsequent negative changes compared with the probability of positive changes after an interest rate increase. However, an important structural break is detected in 1997 when the Bank of England became independent.

Some important results are found once we include considerations for the non-negative restriction on interest rates. First, in general, the non-negativity constraint reduces the interest rate effect for both positive and negative rate movements. This is because the lower bound reduces policy makers' ability to affect agents' expectations on future interest rates. The second result is that in the UK the non-negativity constraint makes the effect of monetary policy shocks dependent on the level of the interest rate, in the sense that negative interest rate movements become relatively more important than positive ones as interest rates approach zero. In the US, however, inflation reactions are symmetric for all the relevant ranges of interest rates.

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Table 1: Empirical posterior distribution of the number of states using the Otranto and Gallo (2002) procedure.

	Number of states					
	1	2	3	4	5	6
UK	0.00	0.09	0.42	0.30	0.16	0.03
USA	0.01	0.80	0.18	0.01	0.00	0.00

Table 2: Estimates of the parameters of the GPM models for USA and UK interest rates (standard errors in parentheses).

USA							
$c$	$\sigma^2$	$\beta_2$	$\phi_1$	$\vartheta_1$	$\phi_2$	$\vartheta_2$	
0.002	1.310	6.615	0.035	19.905	1.953	-33.094	
(0.089)	(0.122)	(3.054)	(0.176)	(4.182)	(0.355)	(9.019)	
UK							
$c_1$	$c_3$	$\sigma_1^2$	$\sigma_3^2$	$\beta_2^1$	$\beta_2^3$	$\beta_3^1$	$\beta_3^3$
-0.386	0.115	0.015	1.288	2.353	13.427	-1.720	-14.584
(0.019)	(0.117)	(0.003)	(0.183)	(1.108)	(5.322)	(0.763)	(4.129)
$\phi_{11}$	$\phi_{12}$	$\vartheta_{12}$	$p_{21}$	$p_{22}$	$\phi_{31}$	$\phi_{32}$	$\vartheta_{32}$
1.014	2.030	-20.438	0.080	0.682	-1.104	1.112	-7.438
(0.445)	(0.625)	(9.568)	(0.022)	(0.029)	(0.446)	(0.401)	(4.156)

Figure 1: Expected Inflation as a Function of Interest Rate Changes

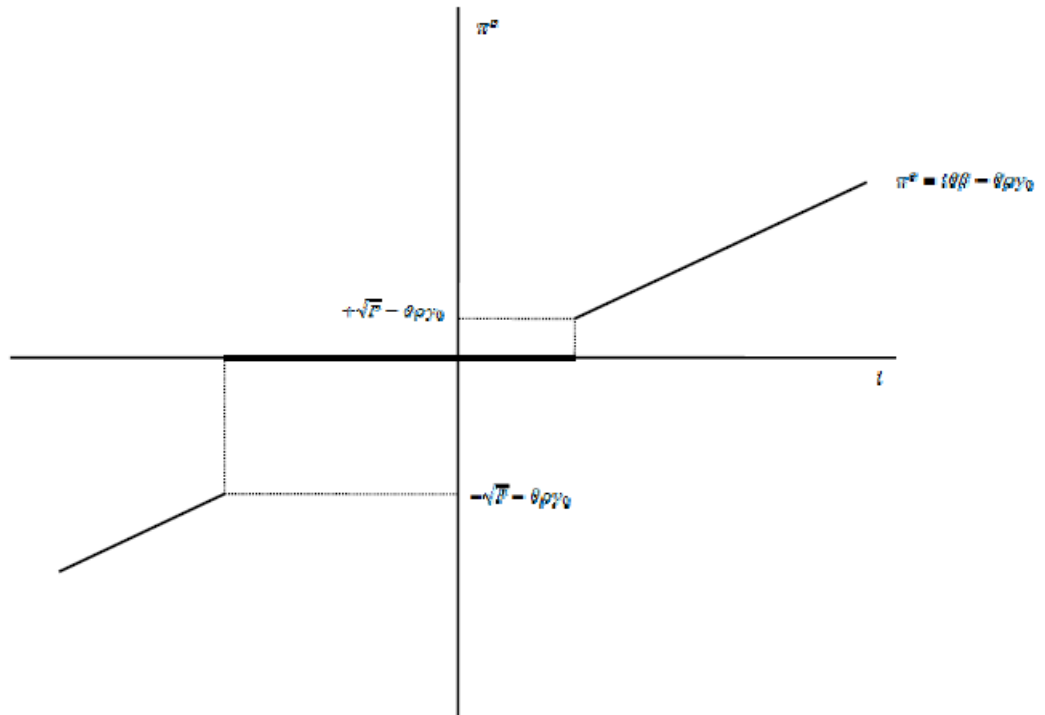
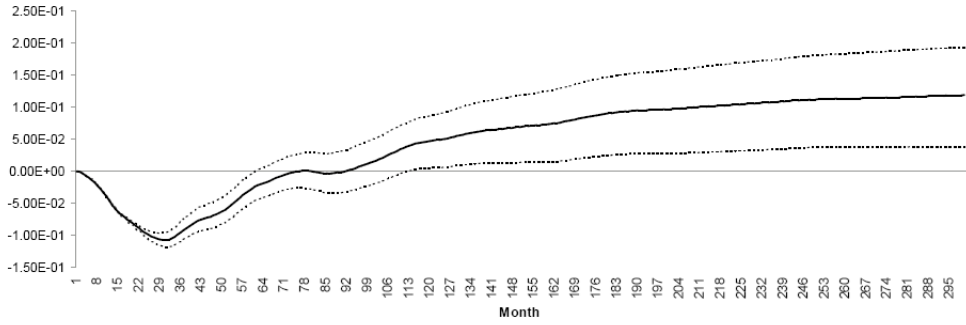
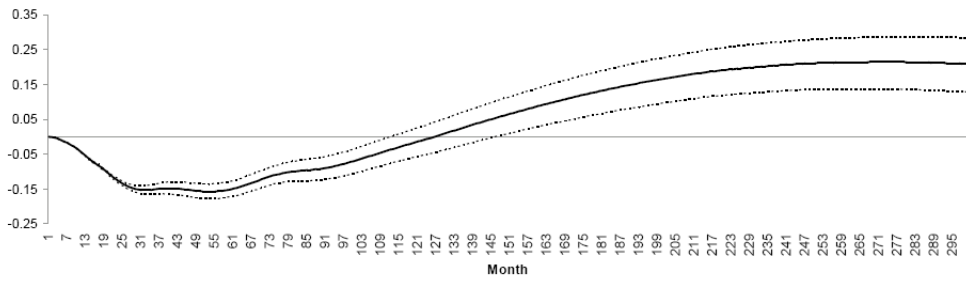


Figure 2: Responses to Interest Rate Shocks in the US.

**Cumulative reaction of US inflation to a negative 1std interest rate shock in a linear VAR model**



**Cumulative reaction of US inflation to an unexpected decrease of interest Rate in 25 basis points generated by a GPM model.**



**Cumulative reaction of US inflation when the fed fails to move interest rate generated by a GPM model.**

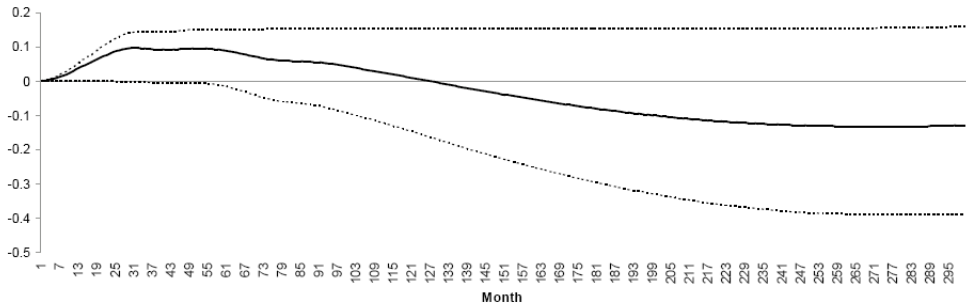
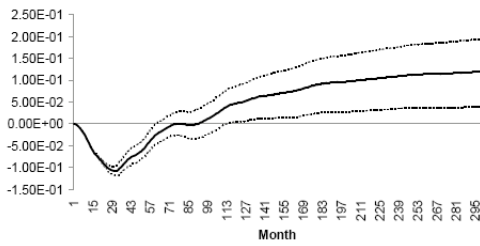
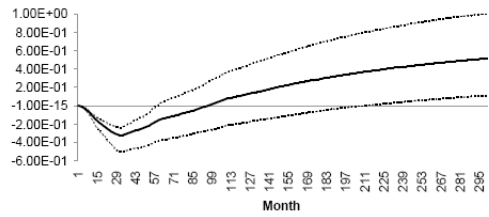


Figure 3: Responses to Interest Rate Shocks in the UK.

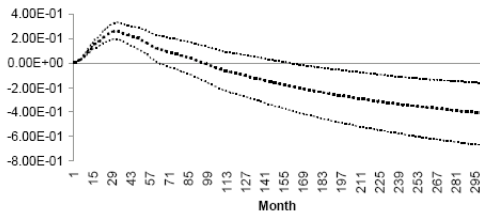
**Cumulative reaction of UK inflation to a negative 1std interest rate shock in a linear VAR model**



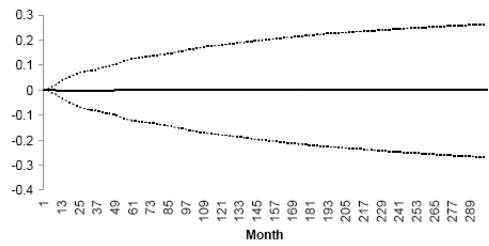
**Cumulative reaction of UK Inflation to a Decrease of Interest Rate in 25 basis points when no Movement was Expected**



**Cumulative reaction of UK Inflation to a positive interest rate movement in 25 basis points when no Movement was Expected**



**Cumulative reaction of UK inflation when the Bank of England fails to decrease interest rates**



**Cumulative reaction of UK inflation when the fed fails to increase interest rates**

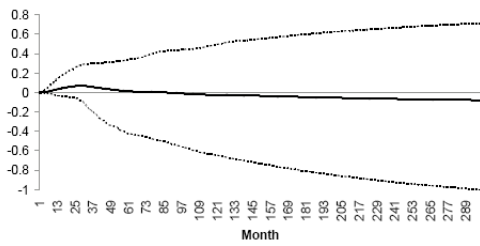
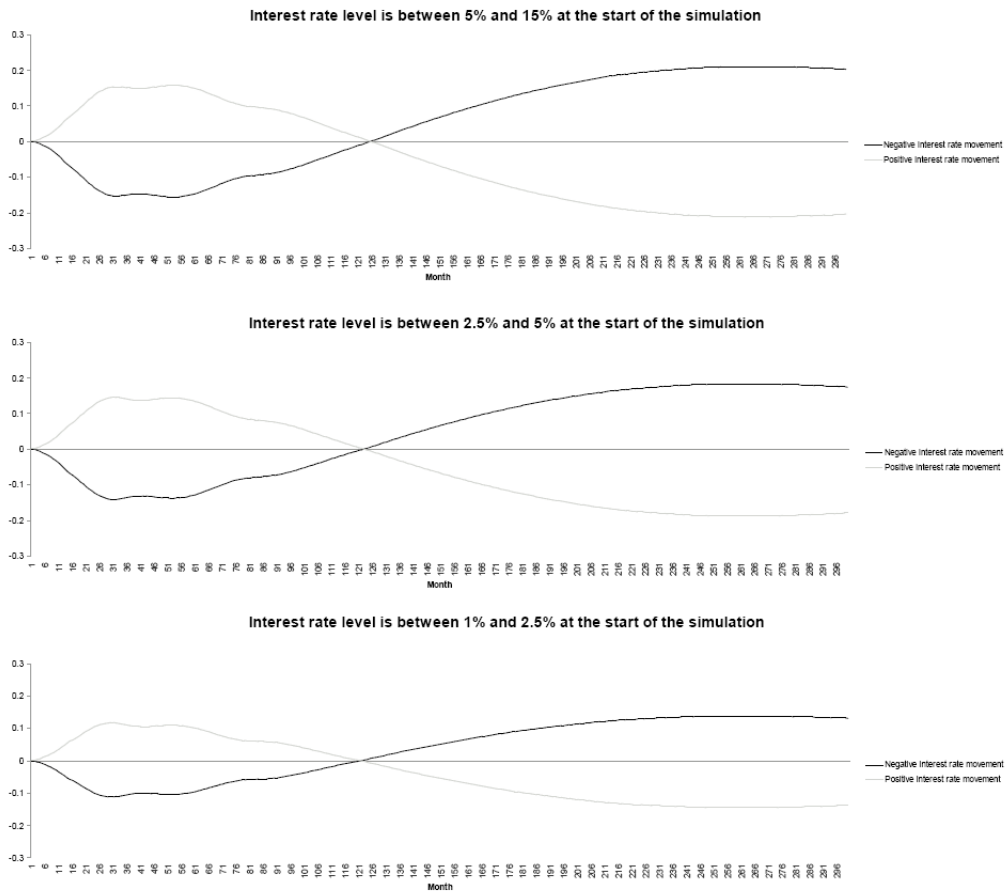
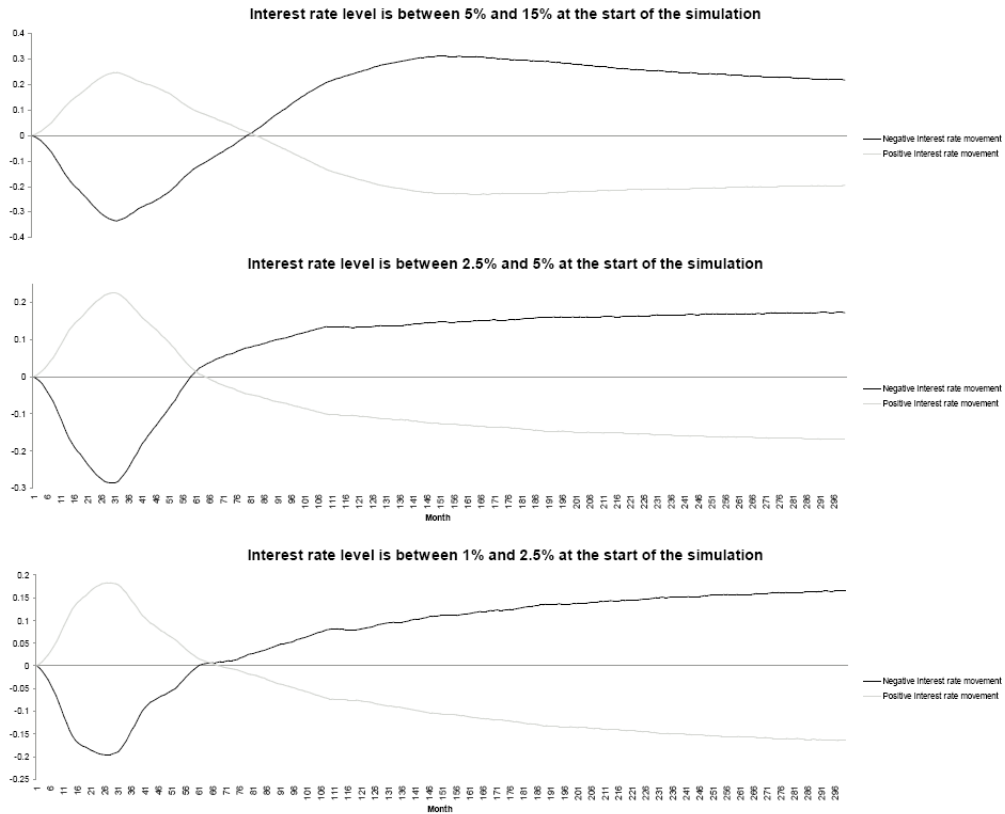


Figure 4: Responses to Interest Rate Shocks in the US when Interest Rates are Censored at Zero.<sup>(\*)</sup>



(\*) Cumulative reaction of US inflation to a positive or negative interest rate movement in 25 basis points when no movement is expected. Interest rates are censored at zero and shadow interest rates are generated by a GPM model.

Figure 5: Responses to Interest Rate Shocks in the UK when Interest Rates are Censored at Zero.<sup>(\*)</sup>



(\*) Cumulative reaction of UK inflation to a positive or negative interest rate movement in 25 basis points when no movement is expected. Interest rates are censored at zero and shadow interest rates are generated by a GPM model.

Figure 6: Responses of the UK inflation to an unexpected decrease of interest rates by 25 basis points when interest rates are censored at zero.

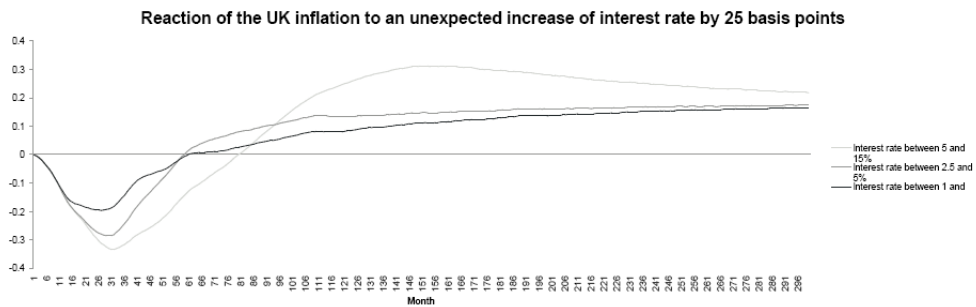
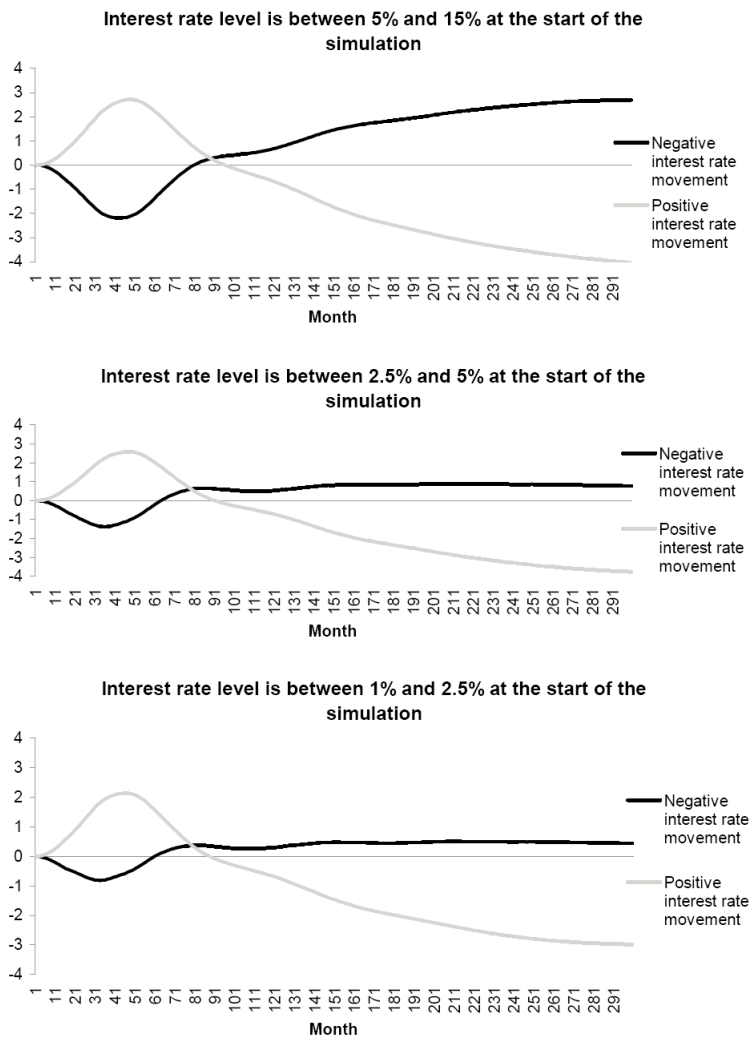


Figure 7: Comparable Reactions to no Change in Interest Rate when Positive and Negative Movements are Expected.<sup>(\*)</sup>



(\*) Reactions to no movement in interest rates when a positive movement is expected is multiplied by  $-1$ .

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