



**SEARCH AND THE FIRM'S CHOICE OF THE OPTIMAL LABOR
CONTRACT**

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Titolo: Search And The Firm's Choice Of The Optimal Labor Contract.

Search and the Firm's Choice of the Optimal Labor Contract

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Abstract

This article studies the behavior of a firm searching to fill a vacancy. The main assumption is that the firm can offer two different kinds of contracts to the workers, either a short-term contract or a long-term one. The short-term contract acts as a probationary stage in which the firm can learn the worker's type. After this stage, the firm can propose a long-term contract to the worker or it can decide to look for another worker. We show that, if the short-term wage is fixed endogenously, for the firms can be optimal to start a working relationship with a short-term contract, but that this policy has a negative impact on unemployment and welfare. On the contrary, if this wage is fixed exogenously, this policy could be optimal also from welfare point of view.

Keywords: Search, Temporary Employment, Short-Term Wage.

JEL Classification: J31, J41, J64.

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1 Introduction

In this paper, we study the behavior of a firm searching to fill a vacancy. The main assumption is that firms may offer two different kinds of contracts to workers, either a short-term contract (STC)¹ or a long-term one (LTC).

The empirical evidence shows that the share of temporary work in total employment has been increasing in Europe in recent years. At the end of the seventies, labor market regulations restricted temporary jobs to specific tasks, characterized by large variations in production. These regulations have changed somewhat since then and, in a number of European countries, it is now possible to hire workers on a temporary basis even for jobs which are not characterized by high output variability.

For instance, an OECD study shows that, while in 1983 only 4% of the employees in the EC held temporary jobs, this share has risen to 10% by 1991.² After 1991, the percentage has been increasing in some countries (e.g., Belgium and Italy) and decreasing in others (e.g., Spain).

Temporary contracts are often considered a measure of labor market flexibility. They offer an instrument to ensure that the returns to the entrepreneurs and the start-up and demise of firms are unconstrained by institutional rigidities such as employment restriction legislation and trade union activity. In periods of rapid technical change or high demand volatility, temporary contracts allow firms to hire workers as they wish. In addition, STCs can also be viewed as a screening device, which allows employers to observe the productivity of the job-worker pair. In this perspective, job matches can be interpreted as "experience good", in the tradition of Jovanovic (1979, 1984). From this viewpoint, firms may use a STC as a probationary period, allowing them to select the right worker for the job.

This second interpretation is consistent with the results of several empirical studies. For instance, in their study of the duration pattern of STCs, based

¹We will also refer to short-term contract as temporary contracts.

²See OECD (2002) for a detailed description of fixed-term contract regulation in the OECD countries.

on micro data from the Spanish Labour Force Survey, Güell and Petrongolo (2000) observe an important spike at duration around 1 year. This observation supports the idea that STCs are used as a screening device: successful workers obtain permanent renewals well before the legal limit of their contracts (3 years). The authors also observe another spike in the hazard at 3 years, suggesting that STCs also provide to some firms a cheap option for adjusting their employment levels.

Booth, Francesconi, and Frank (2000), using data from the British Household Panel Survey, find that temporary workers report lower levels of job satisfaction, receive less training, and are paid less than their counterparts in permanent jobs. Conversely, they find that experience on STCs may lead to high wage growth if the workers move to permanent full-time jobs. This is because workers who have such contracts enjoy high returns to "experience capital", once they acquire a permanent job.

Also relevant are the results of Bentolila and Dolado (1994) and Bentolila and Saint-Paul (1992). Using a panel of Spanish firms, they show that the introduction of STCs is equivalent to a reduction in firing costs and that its impact on unemployment is ambiguous.

In the more theoretical literature, Wasmer (1999) and Cahuc and Postel-Vinay (2002) have introduced temporary jobs in matching models based on the classic equilibrium models of the labor market, built on Diamond (1982), Mortensen (1982), and Pissarides (2000).

Wasmer (1999), in a model with exogenous job destruction, shows that, in periods of low growth, firms are more willing to use STCs and that this increases employment. Cahuc and Postel-Vinay (2002), in a model with endogenous job destruction, show that the combination of temporary jobs and firing restrictions may be both inefficient in terms of aggregate welfare and inadequate as a weapon to fight unemployment. Their result follows from the fact that the share of temporary jobs transformed into permanent jobs is decreasing in the level of firing costs. This could explain the dramatic growth in temporary jobs in France, Italy and Spain, countries characterized by high levels of employment protection.

On the contrary, in the United States and Britain, countries with relatively low levels of employment protection, the proportion of the workforce on fixed term contracts has been fairly stable.

The increase in the share of STCs has had many consequences. Notably, it has affected the bargaining position of workers, negatively for the employees with short-term contracts, and possibly positively for the employees with long-term contracts (see, Bentolila and Dolado, (1994)).

Here, we focus the analysis on the role of STCs as a screening device. That is, we assume that the only way to determine the quality of a particular match is "to form the match and experience it". When this is the case, firms may use STCs to select the right worker for the job.

Our model is related to Acemoglu (2001) and Marimon and Zilibotti (1999). The first author constructs a model where firms offer jobs of two different "qualities". He studies how the size of unemployment benefits and minimum wages affects the equilibrium composition of good *vs.* bad jobs. The second paper also studies the impact of unemployment benefits on the economy in a model with heterogeneous agents. The authors assume that both firms and workers are uniformly distributed along a circle and that a worker's productivity depends on the location of the firm, decreasing with the distance between worker and firm.

As in this last paper, we construct a model with heterogeneous workers, distributed on $[0, 1]$, but with homogeneous firms. Search costs are captured by the discount factor. Moreover, we assume that, in each period, workers have an exogenous probability to leave the market. In order to preserve the stationarity of the types' distribution in the labor market, we assume that there is an exogenous incoming flow in the search market of workers of the same quality. In our economy, the only way to determine the quality of a particular match is "to form and experience it". That is, the initial STC acts as a probationary stage to select the right workers.

In this framework, we study the firm's optimal policy and its welfare impact. When firms meet a worker for the first time, they don't know his type. A policy

is defined as the choice of the contract to offer to the heterogenous workers. The firm has two possibilities: Either it only offers long-term contracts or it offers a short-term contract to begin with, switching to a long-term one if it is satisfied about the productivity of the job-workers pair. We assume that firms post a long-term wage for the job and that workers either take it or leave it. Moreover, at first we assume that the short-term wage is fixed exogenously, independently of the worker's type. In the last section, we consider the case where the short-term wage is posted by the firms.

When the short-term wage is exogenous and sufficiently high, STCs can be optimal for both firms and social welfare. This is because a high short-term wage pushes every worker to accept a STC. As already mentioned, with this contract, firms will be able to screen workers only hiring with a long-term contract the most qualified. From a social welfare viewpoint, the screening has a negative impact on unemployment, but it can be compensated by a higher productive efficiency.

When the short-term wage is posted by the firms (and we allow firms to use contingent wages), at the equilibrium STC may be optimal for the firms. STCs are profitable only if the costs, due to the probability of being unmatched, are compensated by the surplus from the long-term matching. The higher the workers' unemployment benefits, the lower the surplus that it can be obtained by waiting to find a good worker. However, in this case STCs are never optimal from a welfare viewpoint.

Moreover, we show that the regime with fixed short-term wage dominates, in terms of welfare, the regime with posted short-term wage, provided that the short-term wage exceeds a threshold value.

We also establish that, when the short-term wage is exogenously given, the STC wage is higher than the LTC one, which is equal to the worker's reservation wage, given by the unemployment income. This is because unemployment benefits are the outside option of a workers faced with a STC. Similarly, the exogenous short-term wage is the outside option of a worker faced with a LTC.

The model is formally introduced in Section 2. Sections 3.2 and 4 present the

main results. Section 5 analyzes the welfare effects of the two kinds of contracts.

2 The Model

2.1 Workers and Firms

Workers are characterized by a real-value parameter defining the worker's type, x , distributed according to a continuous distribution function $F(x)$ with full support on $[0, 1]$. Its density denoted by $f(x)$.

Time is discrete and $t = 0, 1, \dots + \infty$. β is the probability that a worker leaves the market ("dies") when going from period t to period $(t + 1)$. We assume that, in each period, new workers enter the market. Their types are distributed so that the actual distribution of workers alive is time-invariant, i.e., a worker of type x enters the market if and only if a worker of type x dies. This assumption is strong, but it allows to keep the analysis sufficiently simple. The main results of the paper are independent of it.

In each period firms and workers meet. Firms offer a job-contract pair and workers can accept or reject the offer. If the offer is accepted, production takes place. At the end of the period, wages are paid and output is sold.

As in Albrecht and Axel (1984), we assume that workers can earn different levels of income when unemployed³, $b(x)$. However, while Albrecht and Axel (1984) assume that homogenous workers can have just two possible levels of unemployment incomes, here we assume that there is a continuum of unemployment income levels, depending on workers' types. More precisely, we assume that unemployment income is proportional to the worker's type, i.e., that $b(x) = \gamma x$, with $\gamma \in [0, 1]$.

We assume, there are $M > N$ homogenous firms. At each moment of time, a firm can have either a filled position or a vacancy.⁴ An active firm with a

³It can be interpreted as including the value of leisure and home production, net of search costs. This wide notion of unemployment income also justifies the assumption that benefits are related to the type.

⁴We will assume that firms have not cost to open a vacancy, so that we can avoid limited

filled position employs one worker and obtains a revenue selling its output.

If the position is filled, an exogenous layoff arrives at each period with probability β (this is because in each period t , matched worker dies with probability β). Moreover, we assume that, if a firm leaves the market, another firm enters the market. These stationarity assumptions are convenient, because they allow us to focus on the main point of the paper (the optimal choice of the contract), avoiding unnecessary complexities. The main results are robust to several alternative, and weaker, assumptions on this issue.

For simplicity, for each firm, the production function is:

$$Y = f(x) = x \tag{1}$$

where x is the type of the worker and y the technology of the homogenous firms. To simplify notation, we set $y = 1$.

If there is a vacancy, in each t , firms can create a position without cost. At each meeting in the search market, firms are not able to observe the type of the worker (hence, their unemployment income). We allow firms to offer a probationary contract (lasting one period) to the workers. During this period, firms learn their worker's type and decide whether or not to offer him a long-term contract.

All the agents (firms and workers) have a common discount factor, denoted $0 \leq \delta \leq 1$. Impatience also captures search costs.

2.2 Search and Matching

Total employment is $N_a = N(1 - u_a)$, where N is the labor force and u_a the unemployment rate. Moreover, let v_a denotes the vacancy rate. These values depend on a , the firm's choice of the contract.

Unemployed workers are matched to the recruiting firms according to a simple random matching technology, α , that is assumed to be independent of the number of participants in the search market. The matching function exhibits constant returns to scale.

liability issues.

The rate at which firms find an unemployed worker, denoted q , will depend on α , the matching technology, and on u_a and v_a (rate of unemployed and vacancies). We simply assume that q is a fixed proportion of the unemployment/vacancy ratio.⁵ Hence,

$$q = \alpha \frac{Nu}{Mv}.$$

2.3 Optimal firms' behavior

In determining the optimal firms' behavior, an important role is played by the short-term wage w_O . We will mostly focus on the more interesting and relevant case when w_O is exogenously given, for instance because it is fixed by law. The last section of the paper will summarize the result for the case when firms also post w_O . Hence, from here we treat the short term wage w_O as exogenous.

For a firm, a policy, a , is the choice of the contract to offer to the workers. We assume that it is impossible for a firm to fire a worker before the expiration of the contract (or that to fire a worker is infinitely costly).

The firm has two possible policies:

- (L) It supplies a long-term employment contract at a wage rate $w_L(x)$, contingent on the type ($a = L$).
- (SL) It supplies a short-term (one-period) contract to begin with, switching, possibly, in the following period, a long-term employment contract at a wage rate $w_{SL}(x)$, contingent on the type ($a = SL$).

The advantage of this last policy is that the firm will have full information about the worker's type when it offers him the long-term contract.

Contracts are chosen by the firm so as to maximize its profit subject to the participation of workers, corresponding to the expected utility of a worker not accepting the contract.

Let

$$V_a(x) = b(x) + \delta(1 - \beta) [\alpha W_a(x) + (1 - \alpha)V_a(x)] \quad (2)$$

⁵Given that $M > N$, evidently $v > u$.

be the expected utility of a type- x unemployed with $a \in \{L, SL\}$ and $x \in [0, 1]$.

An (L) policy is fully characterized by a wage rate w , meaning that a worker of type x will receive a wage $w_L(x)$. The expected utility for a worker is implicitly defined by

$$W_L(x) = w_L(x) + \delta(1 - \beta)W_L(x). \quad (3)$$

Given our simple matching technology (and the assumption $M > N$), where α is the probability of a workers receives a job offer, a worker accepts if and only if $W_L(x) \geq V(x)$. Let ρ_L be the (measurable) subset of workers accepting the contract.

An (SL) contract is fully characterized by a short-term wage w_0 , a long-term wage rate $w_{SL}(x)$ and a subset σ of workers for which the firm is willing to extend the contractual relation over the long-period. The expected utility of for a worker is implicitly defined by

$$W_{SL}(x) = \begin{cases} w_0 + \delta(1 - \beta)V_{SL}(x), & \text{if } x \notin \sigma, \\ w_0 + \delta(1 - \beta)W_L(x; w) & \text{if } x \in \sigma \end{cases}. \quad (4)$$

Clearly, a worker accepts if and only if $W_{SL}(x) \geq V_{SL}(x)$. Let ρ_{SL} be the (measurable) subset of workers accepting the contract.

Now, when a firm supplies a contract, workers in ρ_{SL} accept and worker in $\rho_{SL} \cap \sigma$ extend to the long-period one.

The Problem firms' policy choice Knowing w_0 , $w_{SL}(x)$, $w_L(x)$ σ and w_0 we can define the problem firms'policy choice.

Let $J(x)$ be the expected profit of firm with a long-term contract, that is,

$$J(x) = (x - w_a(x)) + \delta(1 - \beta)J(x)$$

where $a \in \{L, SL\}$ and $(1 - \beta)$ is the probability that the worker matched will survive to the next period and $w_a(x)$ is the long-term wage that, by hypothesis, is posted by firms and that workers either take it or leave it.

Moreover, let Π_L denote the expected discounted payoff of a firm searching to fill a vacancy with a policy L , With probability $(1 - q)$, it does not and it

will try again to match next period. If the firm decides to offer directly a LTC ($a = L$), it will engage, by hypothesis, every workers.

$$\Pi_L = q \int_{x \in \rho_L} J(x) f(x) dx + \delta(1 - q \int_{x \in \rho_L}) \Pi_L(w_0) \quad (5)$$

where ρ_L is he (measurable) subset of workers accepting the contract.

And, let Π_{SL} denote the expected discounted payoff of a firm searching to fill a vacancy. Given the short-term wage w_0 ,⁶ the expected profit of the firm is

$$\max_{\sigma} \Pi_{SL} = q \int_{x \in \rho_{SL}} (x - w_0) f(x) dx \quad (6)$$

$$+ q\delta(1 - \beta) \int_{x \in \sigma_{SL} \cap \rho_{SL}} J(x) f(x) dx \quad (7)$$

$$+ \delta \left(1 - q \int_{x \in \rho_{SL}} f(x) dx + q\delta(1 - \beta) \int_{x \notin \sigma_{SL} \cap \rho_{SL}} f(x) dx \right) \Pi_{SL}.$$

(notice that, by continuity of the distribution function, single points have zero probability of occurrence).

The first term is the one-period expected profit from a short-term job, as the contract will be accepted by workers of the subset ρ_{SL} ; the second term is the expected profit from an extension to a long-term contract, which occurs only for types in the subset σ_{SL} ; the third term collects all cases in which the firm supplies again a short-term contract, either because the the matching was unsuccessful, or because the short-term contract was rejected, or because the firm refuses to extend the contract to a long-run position.

If the firm chooses to start with a STC, $a = SL$, during the probationary stage, it will learn the worker's type. In this stage, the worker will receive a short-term wage: w_O . In period 2, the firm will propose a LTC only to the workers in the subset σ_{SL} . Given that the firm learns the worker's type, future wages will be contingent on the worker's type: $w_a = w_{SL}(x)$.

The firm maximizes its profits by choosing policy SL if $\Pi_{SL} \geq \Pi_L$, otherwise policy L will be optimal.

⁶We will assume this wage is exogenously given. The last section of the paper will summarize the result for the case when firms also post w_0 .

3 Optimal contract choice

We study the optimal contract choice in four steps:

A) we determine the subset of workers (ρ_L, ρ_{SL}) accepting the contract proposition. To this scope we need to determine the wages;

B) we determine the subset of workers (σ_{SL}) accepted by firm after a short-term contract;

C) we determine the unemployment rates;

D) we find the optimal contract choice.

3.1 Wage posting

The profit maximizing firm chooses the wage subject to $W_a(x) \geq V_a(x)$, with $a \in \{L, SL\}$. Since the firm has no incentive to offer to the worker anything over and above the minimum required to make him to accept its offer (see Diamond (1971)), this condition reduces to

$$W_a = V_a. \tag{8}$$

Let's first consider an (L) policy. In this simple wage setting set-up, the condition (8), where W_L is given by (3) and V_L is given by (2), implies that w_L is driven to the worker's reservation wage:

$$w_L(x) = b(x) = \gamma x. \tag{9}$$

Notice that w_L is type-contingent. This implies that subset of workers accepting the contract, ρ_L , is the full support $[0, 1]$. Also, observe that, even with type-contingent wages, firms would rather hire high productivity (and highly paid) workers because, per period, their profits are $(1 - \gamma)x$.

Let consider the an (SL) policy. The condition (8), where W_{SL} is given by (4) and V_L is given by (2), immediately implies that the long-term wage associated with SL is⁷

⁷Notice that this is only an hypothetical wage, as the firm will extend the contract only to workers in σ .

$$w_{SL}(x) = \frac{\gamma x + \alpha \delta (1 - \beta) w_O}{1 + \alpha \delta (1 - \beta)} \quad (10)$$

Proposition 1 *Under the maintained assumptions, given w_O , all the workers with $x \in (x^+, 1]$ always reject the STC. For all the workers with $x \in [0, x^+)$, $w_O \geq w_{SL}(x) \geq b(x)$, where $x^+ = \frac{w_O}{\gamma}$.*

Proof To establish the first claim, observe that workers with $x \in \sigma_{SL}$ will accept a SL if and only if $(w_O + \delta(1 - \beta)W_{SL}(x)) > W_{SL}(x)$ where $W_{SL}(x)$ and V_{SL} are implicitly given by (4) and (2). By (10), the previous inequality is satisfied if and only if $w_O > b(x) = \gamma x$. Hence, all the workers with $x \in (x^+, 1]$, with $x^+ = \frac{w_O}{\gamma}$, will always reject the short-term contract.

Given that, for all the workers accepting the SL , $w_O > b(x)$, the second claim follows immediately from (10).

This result is of some interest. Due to the assumption $b(x) = \gamma x$, when w_O is exogenous, high productivity workers will reject short-term contracts, unless w_O is sufficiently high. However, high levels of w_O also imply high levels of $w_{SL}(x)$. This trade-off may make STC less profitable than LTC. The result is coherent with the wage formation theories suggesting that temporary workers are paid more than workers in long-term contract to compensate them for the less advantageous characteristics of temporary jobs. Moreover, in our model, the long-term wage obtained after a short-term contract (i.e., after the firm observes the type of the worker) is not driven to the worker's reservation wage. This is because, for the workers, the STC is an outside option: If the long-term wage is equal to the reservation wage, the worker will reject the firm's proposal and wait for the next STC proposal.

3.2 Search equilibrium in the STC case

Knowing that only workers with $x \in [0, x^+)$ will always accept the short-term contract, the firm's strategy in the SL policy is to engage after the probationary stage only workers in the set σ_{SL} . Hence, for the stationary strategy profile

(σ_{SL}), we define the expected payoff of a firm as Π_{SL} . We focus on equilibria in undominated strategies, firm accepts a worker of type x if and only if $J^{SL}(x) > E\Pi_{SL}$.

Proposition 2 *The optimal $SL(x^-, x^+)$ is given by $\sigma_{SL} = (x^-, x^+]$, where*

$$x^- = \frac{q\delta^2(1-\beta) \left(\int_{x^-}^{x^+} xf(x)dx - \int_{x^-}^1 f(x)dx - \frac{D}{C} \right) + \frac{(1-\delta(1-\beta))}{C} \left[q\delta \left(\int_0^{x^+} xf(x)dx - w_0 \right) \right]}{[1-\delta+qE\delta-q\delta^2(1-\beta)]}, \text{ while } x^+ = \frac{w_O}{\gamma},$$

E is the probability to meet a worker in the set $[0, x^+)$, $C = \frac{1-\gamma+q\delta(1-\beta)}{1+q\delta(1-\beta)}$ and $D = \frac{q\delta(1-\beta)w_O}{1+q\delta(1-\beta)}$.

Proof. See Appendix 1. ■

Proposition 2 characterizes the search equilibrium in the case of STCs. In this equilibrium, firms partition workers who have accepted the short-term contract, into two subintervals, hiring forever only workers whose ability is above the threshold x^- .⁸

An increase in w_O has a double effect on the economy. First, given that $x^+ = \frac{w_O}{\gamma}$, it increases the proportion of workers accepting a STC (and this has a positive effect on E). Second, it pushes up the long-term wage w_{SL} , as we can see from (10).

To better understand this result, it is instructive to consider a simple example, where abilities are uniformly distributed on $[0, 1]$, $N = M$, $q = \alpha = 0.8$, $\gamma = 0.5$, $\delta = 0.8$, and $\beta = 0.05$. The next figure shows how the interval of workers accepting and accepted in the long-term job (i.e., with $x \in [(x^+) - (x^-)]$) varies with w_O .

⁸Remember, however, that all the workers with $x \in (\frac{w_O}{\gamma}, 1]$ will always reject the *STC*.

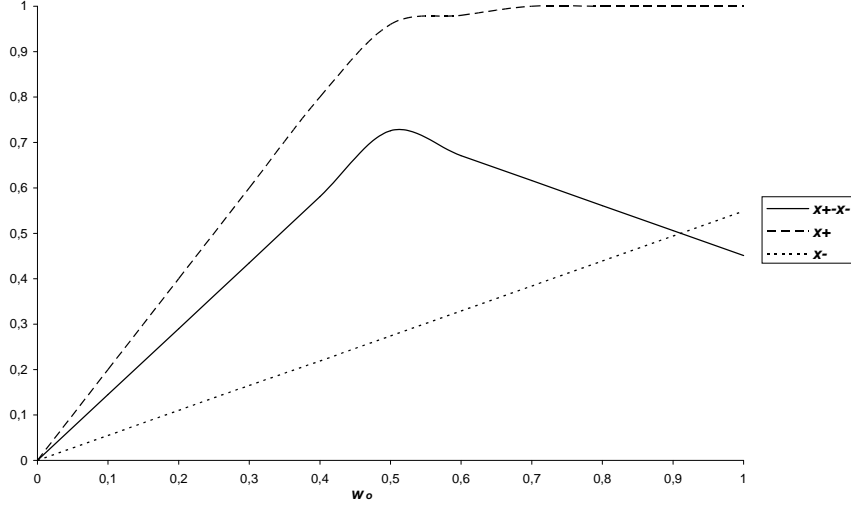


Figure 1: The interval of workers accepting/accepted in the long-term job

If w_O is low, the first one of the two effects discussed above dominate the second: only some workers accept a STC (x^+ is low). On the other hand, firms are not very selective in screening the workers (x^- is low). When w_O is higher, the opposite is true: x^+ takes a value near 1 (i.e., every worker accepts the STC), and x^- is also high (because the long-term wage is so high that firms only find profitable to engage highly qualified workers).

3.3 Unemployment

In order to determine the unemployment rate, under the SL policy, we shall consider unemployment rates for workers with $x \leq x^-$ (similarly for $u_{x^- < x < x^+}^t$ and $u_{x \geq x^+}^t$). Also, let's index with p_{SL}^t the proportion of workers engaged in a STC .

Given the policy SL , the expected value of $u_{x^- < x < x^+}^t$, evolves over time according to:

$$u_{x^- < x < x^+}^{t+1} = u_{x^- < x < x^+}^t (1 - \beta)(1 - \alpha) + \beta$$

$$p_{x^- < x < x^+}^t = u_{x^- < x < x^+}^{t-1} (1 - \beta)\alpha$$

because a fraction α of the unemployed who survive (with a probability $(1 - \beta)$) at t is expected to get a job, a fraction β of them is expected to die and to be replaced by a fraction β of the work force with $x \in [x^-, x^+]$ (these individuals are necessarily unemployed in the first period of their life, given the time structure of the model).

Hence, the unique stationary state occurs at

$$\bar{u}_{x^- < x < x^+} = \frac{\beta}{\alpha(1 - \beta) + \beta} \quad (11)$$

and

$$\bar{p}_{x^- < x < x^+} = \bar{u}_{x^- < x < x^+} (1 - \beta)\alpha. \quad (12)$$

Similarly, for workers with $x < x^-$, the dynamics is described by.

$$u_{x < x^-}^{t+1} = u_{x < x^-}^t (1 - \beta)(1 - \alpha) + p_{x < x^-}^t (1 - \beta)(1 - \alpha) + \beta$$

and

$$p_{x < x^-}^t = u_{x < x^-}^{t-1} (1 - \beta)\alpha$$

The difference with respect to the previous case reflects the share of people employed at t in a STC that, given that $x < x^-$, become unemployed (if still alive) at $(t + 1)$.

Hence, the unique stationary state is described by

$$\bar{u}_{x < x^-} = \frac{\beta}{\alpha^2(1 - \beta) + \beta} \quad (13)$$

$$\bar{p}_{x < x^-} = \bar{u}_{x < x^-} (1 - \beta)\alpha \quad (14)$$

Obviously, the unemployment rate of the workers with $x > x^+$, is always 1, because they never accept to work.

Total unemployment is:

$$\bar{u}_{SL} = F(x^+ - x^-)\bar{u}_{x^- < x < x^+} + F(x^-)\bar{u}_{x < x^-} + F(1 - x^+) \quad (15)$$

with a percentage $\bar{p} = F(x^+ - x^-)\bar{p}_{x^- < x < x^+} + F(x^-)\bar{p}_{x < x^-}$ of workers are engaged in a one period contract.

Substituting (11) and (13) in (15), we find the value of the total unemployment:

$$\bar{u}_{SL} = 1 - \frac{\alpha(1 - \beta)}{\alpha(1 - \beta) + \beta} \left[F(x^+) - \frac{\beta(1 - \alpha)}{\alpha^2(1 - \beta) + \beta} F(x^-) \right] \quad (16)$$

where the endogenous values x^+ and x^- are given respectively the Proposition 1 and by Proposition 2.

Looking again at the numerical example above (see Figure 2), the unemployment rate is decreasing in w_O if $w_O < \gamma$. After this point it starts increasing. When $w_O = \gamma$, $x^+ = 1$, and so all the agents accept the temporary employment proposition. After this point, the increase of the short and, consequently, of the long-term wages pushes the firms to hire permanently only the most productive agent to compensate the high cost of labor.

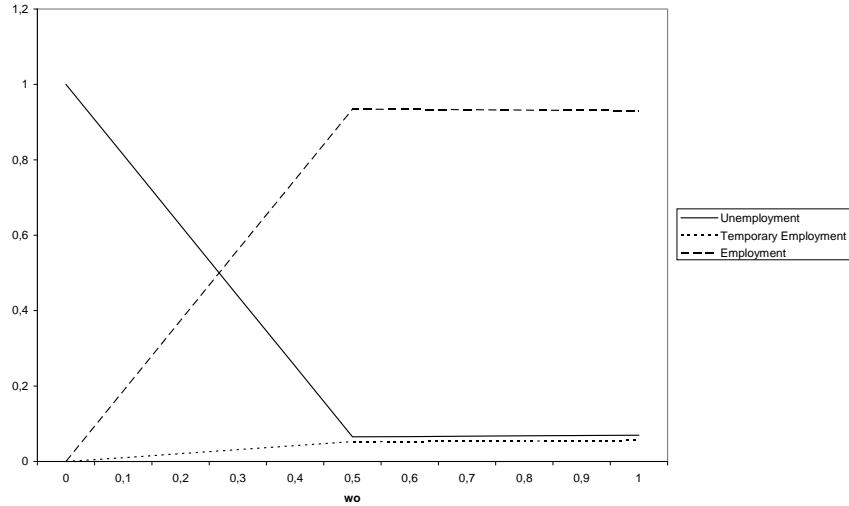


Figure 2: Unemployment, Employment and Temporary Employment

We now turn to the analysis of the unemployment rate with the strategy L . Given the strategy L , u_L^t is expected to evolve according to:

$$u_L^{t+1} = u_L^t(1 - \beta)(1 - \alpha) + \beta.$$

Hence, a stationary state occurs if and only if

$$\bar{u}_L = \frac{\beta}{(1 - \beta)\alpha + \beta} \quad (17)$$

It is easy to check that $\bar{u}_L \leq \bar{u}_{SL}$. This is because, in policy SL , firms screen workers and engage long-term term only workers in the interval (x^-, x^+) . On the contrary, if $a = L$, this interval is the full support $[0, 1]$.

4 Choice of the contract

Studying now the optimal contract choice: Policy SL is an optimal contract if $\Pi_{SL} > \Pi_L$, where $E\Pi^{SL}$ is defined in Appendix 1 by the equation (22), and, $E\Pi^L$ may be rewritten, from the equation (6) above, as:

$$\Pi_L = \frac{q\delta(1 - \beta)(1 - \gamma) \int_0^1 x f(x) dx}{[1 - (1 - \beta)\delta][1 - (1 - q)\delta]}$$

Looking again at the numerical example (see figure 3), $a = SL$ is the firms' optimal contract if and only if w_O takes a value near γ .

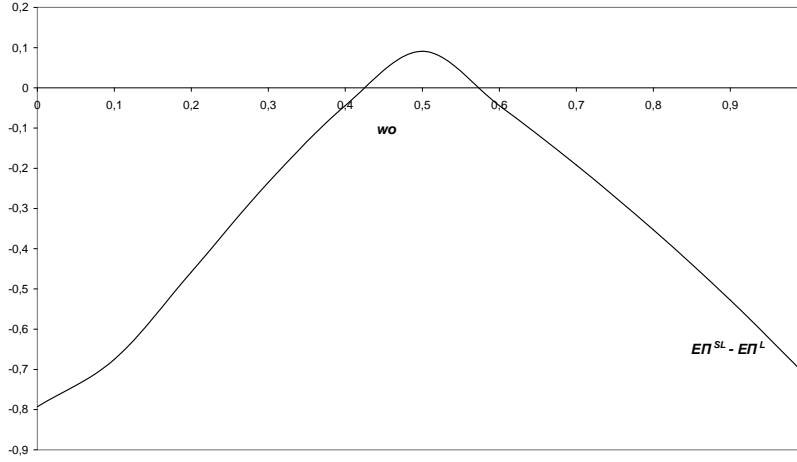


Figure 3: Firms' optimal contract choice

For w_O less than γ , the more qualified workers will refuse to work. For w_O near γ , all qualified workers accept a short-term contract. This value of w_O implies an high level of w_{SL} but the high wages are compensated because firms can screen the workers in the first period. For w_O near 1, the screening cannot compensate the high wages and STC is suboptimal.

5 Welfare

We define welfare in terms of the present discounted value of output. The social planner is not interested in wages, since wages determine only the distribution of output and, by assumption, distributional considerations are excluded from the social welfare function. In our model, the critical issue is the firms' policy choice. We need to check if and when a SL policy can be optimal for both the firms and the social planner. We define as Ω_a the welfare value with $a = SL, L$.

The value function of the planner, with $a = L$, satisfies the equation

$$\Omega_L = \sum_{t=0}^{+\infty} \delta^t N \left[(1 - \bar{u}_L) \int_0^1 x f(x) dx + \bar{u}_L(\gamma) \int_0^1 x f(x) dx \right] \quad (18)$$

where \bar{u}_L is given by (17).

With $a = SL$, it satisfies the equation

$$\begin{aligned} \Omega_{SL} = & \sum_{t=0}^{+\infty} \delta^t N [F(x^-)(1-\bar{u}_{x<x^-}) \int_0^{x^-} xf(x)dx + (F(x^+) - F(x^-))(1-\bar{u}_{x^- \leq x \leq x^+}) \int_{x^-}^{x^+} xf(x)dx + \\ & F(x^-)(\bar{u}_{x<x^-})\gamma \int_0^{x^-} xf(x)dx + (F(x^+) - F(x^-))(\bar{u}_{x^- \leq x \leq x^+})\gamma \int_{x^-}^{x^+} xf(x)dx \\ & + (1 - F(x^+))(\gamma) \int_{x^+}^1 xf(x)dx] \end{aligned} \quad (19)$$

where $x^- < x < x^+$ is given by (11), while $\bar{u}_{x<z}$ is given by (13) and (14). In each period, only workers with type $x^- < x < x^+$ will be hired on LTC. On the contrary, workers with $x < x^-$ will be hired exclusively on STC.

We need to check when

$$\Omega_{SL} \geq \Omega_L \quad (20)$$

Obviously the result depends upon the distribution $F(\cdot)$ and the values of the parameters.

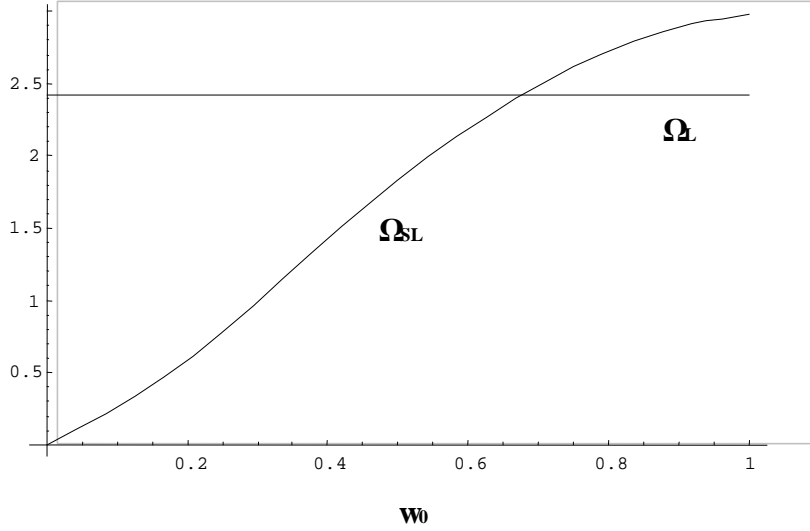


Figure 4: Welfare Comparison

In our numerical example (see figure 4), an *SL* policy is optimal when the short-term wage is fixed at a value sufficiently high, so that $x^+ = 1$, i.e., so that all the workers enter the market for temporary jobs. In this case, the higher unemployment rate coming from the firms' screening is compensated by the higher productive efficiency. Workers in the subset $[0, x^-)$ will be always rejected by firms after the short-term contract. These workers will have a higher unemployment rate, nevertheless in the short-term relation they will obtain a wage w_O larger than their productivity.

6 Posted short-term wage

Let's now assume that the firm can also post the wage of the *STC*. The associated long-term wage is given by equation (9). When firms meet a worker for the first time, they don't know the type. We assume that the firm can choose to offer a (short-term) two-part wage. This wage would be $w_O = [\tilde{w}(x) + (w(x) - \tilde{w}(x))]$, where $\tilde{w}(x)$ is the fixed component, while $w(x)$ is contingent upon the type of the worker (that the firm will discover just at the end of the period). Hence, the firm will pay a salary $w(\tilde{x})$ independent of the worker type, and, at the end of the period (after learning the type), it will pay a "bonus" equal to the difference between the worker's type and x , i.e., $[w(x) - \tilde{w}(x)]$. From the proof of Lemma 1, it follows that a worker will accept a *STC* only if the expected utility of accepting this proposal is larger than the one of rejecting it. Therefore, if $(w_O + \delta W_x^{SL}) > V_x^{SL}$.

Clearly, at the equilibrium, given that types are distributed on $[0, 1]$, the equilibrium value of $\tilde{w}(x)$ will be 0 and $w(x) = \gamma x$. Hence, wages on long-term contracts (signed after the trial stage) and short-term wage are both driven to the worker's reservation wage:

$$w_{SL} = w_O = b(x) \tag{21}$$

Notice that, with a two-part wage, all the workers (also the high productivity ones with $x \in (\frac{w_O}{\gamma}, 1]$) will enter the short-term market.

The main result for the case of endogenous short-term wage is summarized in the following proposition:

Proposition 3 *When w_O is endogenous, firms only hire workers in the interval $(z, 1]$, where $z = \frac{q\delta \left[\delta(1-\beta) \left(\int_z^1 (x-z)f(x)dx \right) + (1-\delta(1-\beta)) \left(\int_0^1 xf(x)dx \right) \right]}{1-\delta+q\delta-q\delta^2(1-\beta)}$*

Proof. See Appendix 2. ■

If w_O is endogenous, as we have seen $w_{SL} = w_O$ (see eq. (21) above). In this case, every worker accepts a short-term contract offer, i.e. $e = 1$. Moreover, firms will select only the agents in the subset $(z, 1]$ for the long term work relation.

Looking again at the numerical example above, we find that $\bar{u}_{sl} = 29\%$, $\bar{p} = 4,7\%$ and $z = 0,46$. Compared to the case of endogenous w_O , the rate unemployment is lower only when w_O is close to γ . Moreover, we find that $a = SL$ is always optimal.

For the case of uniform distribution on $[0, 1]$, it is straightforward to check that total welfare always attains a maximum with policy L . With a SL policy, firms screen workers. The screening process has a negative effect on unemployment and this generates a higher search cost. This effect could be compensated from the viewpoint of the workers on short-term contracts, because $w_O > w_L$. However, by definition, this is irrelevant from a social welfare viewpoint.

We can conclude that the introduction of a temporary market can be optimal only if the short-term wage is fixed exogenously at a value sufficiently high.

The assumption of posted short-term wage modifies social welfare only as far as Ω_{SL} , as the value Ω_L is independent of w_O . This simplifies the welfare comparison between the two regimes. In particular, according to our simulations (see figure 4), the regime with fixed short-term wage dominates, in terms of welfare, the regime with posted short-term wage, provided that w_0 exceeds a threshold value.

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Appendix 1: Search Equilibrium on the SL policy with w_O

exogenous

Proof of Proposition 2.

First, observe that the set X of workers that the firm accepts in SL contract is always an interval $(z, 1]$, some z . Indeed, by the definition of undominated equilibrium, $\frac{x}{1-\delta(1-\beta)} > \delta E \Pi^{SL}(x)$. Hence for any $x' > x$, $\frac{x'}{1-\delta(1-\beta)} \geq \delta E \Pi^{SL}$.

When w_O is fixed exogenously, by Proposition 1, all the workers with $x \in (\frac{w_O}{\gamma}, 1]$ will always reject the STC.

Thus, the search problem faced by the firm may be rewritten, from 6 above, as:

$$\max \Pi^{SL} = \frac{(1-\delta(1-\beta)) \left[q\delta(1-\beta) \left(\int_0^{x^-} xf(x)dx - w_0 \right) \right] + q\delta^2(1-\beta)^2 \left[\int_{x^-}^{x^+} xf(x)dx - Ew_{sl} \right]}{(1-\delta(1-\beta)) \left(1 - \delta(1-qE) - q\delta^2(1-\beta) \left(1 - \int_{x^+}^1 f(x)dx \right) \right)}$$

or

$$\max \Pi^{SL} = \frac{(1-\delta(1-\beta)) \left[q\delta(1-\beta) \left(\int_0^{x^-} xf(x)dx - w_0 \right) \right] + q\delta^2(1-\beta)^2 \left[C \left(\int_{x^-}^{x^+} xf(x)dx \right) - D \right]}{(1-\delta(1-\beta)) \left(1 - \delta + qE\delta - q\delta^2(1-\beta) \left(1 - \int_{x^+}^1 f(x)dx \right) \right)} \quad (22)$$

where $Ew_{sl} = \frac{\gamma \int_0^z xf(x)dx + \alpha\delta(1-\beta)w_O}{1+\alpha\delta(1-\beta)}$, $x^+ = \frac{w_O}{\gamma}$, E is the probability to meet a worker in the set $[0, x^+)$.

$$C = \frac{1-\gamma+\alpha\delta(1-\beta)}{1+\alpha\delta(1-\beta)} \quad \text{and} \quad D = \frac{\alpha\delta(1-\beta)w_O}{1+\alpha\delta(1-\beta)}.$$

The first-order conditions with respect to x^- are given by

$$\begin{aligned} & -x^- f(x^-) C q \delta^2 (1-\beta)^2 (1-\delta(1-\beta)) \left[\left(1 - \delta + qE\delta - q\delta^2(1-\beta) \left(1 - \int_{x^-}^1 f(x)dx \right) \right) \right] \\ & \quad + q\delta^2(1-\beta)f(z)(1-\delta(1-\beta)) \\ & \left[q\delta^2(1-\beta)^2 \left[C \left(\int_{x^-}^{x^+} xf(x)dx \right) - D \right] + (1-\delta(1-\beta)) \left[q\delta(1-\beta) \left(\int_0^{x^-} xf(x)dx - w_0 \right) \right] \right] = 0 \end{aligned}$$

or

$$x^- = \frac{q\delta^2(1-\beta) \left(\int_{x^-}^{x^+} xf(x)dx - \int_{x^-}^1 f(x)dx - \frac{D}{C} \right) + \frac{(1-\delta(1-\beta))}{C} \left[q\delta \left(\int_0^{x^+} xf(x)dx - w_0 \right) \right]}{[1-\delta+qE\delta-q\delta^2(1-\beta)]} \quad (23)$$

To check that the solution is unique, observe that the left-hand side of equation (23) is increasing in x^- , with range $(0,1)$. On the other hand, the right-hand side of equation (23) is decreasing in x^- , falling from

$$\frac{q\delta^2(1-\beta) \left(\int_{x^-}^{x^+} xf(x)dx - \int_{x^-}^1 f(x)dx - \frac{D}{C} \right) + \frac{(1-\delta(1-\beta))}{C} \left[q\delta \left(\int_0^{x^+} xf(x)dx - w_0 \right) \right]}{[1-\delta+qE\delta-q\delta^2(1-\beta)]}$$

to $\frac{q\delta^2(1-\beta)(-\frac{D}{C}) + \frac{(1-\delta(1-\beta))}{C} \left[q\delta \left(\int_0^{x^+} xf(x)dx - w_0 \right) \right]}{[1-\delta+qE\delta-q\delta^2(1-\beta)]}$. Hence there exists a unique solution to (23).

Appendix 2: Search Equilibrium on the SL policy with w_O

endogenous

Proof of Proposition 3.

As in the Proof of Proposition 1, the search problem faced by the firm may be rewritten as

$$\max \Pi^{SL} = \frac{(1 - \delta(1 - \beta)) \left[q\delta(1 - \beta)(1 - \gamma) \left(\int_0^1 x f(x) dx \right) \right] + q\delta^2(1 - \beta)^2 \left[(1 - \gamma) \left(\int_z^1 x f(x) dx \right) \right]}{(1 - \delta(1 - \beta)) \left(1 - \delta(1 - q) - q\delta^2(1 - \beta) \left(1 - \int_z^1 f(x) dx \right) \right)}$$

with $w_{SL} = w_O = \gamma x$.

The first-order conditions with respect to z are given by

$$\begin{aligned} & -zf(z)q\delta^2(1-\beta)^2(1-\gamma) \left[(1 - \delta(1 - \beta)) \left(1 - \delta + q\delta - q\delta^2(1 - \beta) \left(1 - \int_z^1 f(x) dx \right) \right) \right] \\ & \quad + q\delta^2(1 - \beta)f(z)(1 - \delta(1 - \beta)) \\ & \left[q\delta^2(1 - \beta)^2(1 - \gamma) \left[\left(\int_z^1 x f(x) dx \right) \right] + (1 - \delta(1 - \beta)) \left(q\delta(1 - \beta)(1 - \gamma) \left(\int_0^1 x f(x) dx \right) \right) \right] = 0 \end{aligned}$$

or

$$z = \frac{q\delta}{1 - \delta + q\delta - q\delta^2(1 - \beta)} \left[\delta(1 - \beta) \left(\int_z^1 (x - z) f(x) dx \right) + (1 - \delta(1 - \beta)) \left(\int_0^1 x f(x) dx \right) \right] \quad (24)$$

To check that the solution is unique, observe that the left-hand side of equation (24) is increasing in z , with range (0,1). On the other hand, the right-hand side of equation (24) is decreasing in z , falling from

$$\frac{q\delta}{1 - \delta + q\delta - q\delta^2(1 - \beta)} \left[\delta(1 - \beta) \left(\int_0^1 (x - z) f(x) dx \right) + (1 - \delta(1 - \beta)) \left(\int_0^1 x f(x) dx \right) \right]$$

to $\frac{q\delta}{1 - \delta + q\delta - q\delta^2(1 - \beta)} \left[(1 - \delta(1 - \beta)) \left(\int_0^1 x f(x) dx \right) \right]$. Hence there exists a unique solution to (24).

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