



SHAPE OF U.S. BUSINESS CYCLE AND LONG-RUN EFFECTS OF RESSIONS

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Titolo: Shape of U.S. business cycle and long-run effects of recessions

Shape of U.S. business cycle and long-run effects of recessions*

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Abstract

In this paper, we propose a generalised specification of a time varying transition probability Markov switching model for U.S. industrial production index. The model is specifically designed to investigate the presence of asymmetries in the shape of the cycle, given its relevance in the debate about long-run effects of recessions on output level. We can think about asymmetries in the shape of the cycle along two main dimensions. First, we can think about patterns of variation in growth rates over the course of expansions and recessions. Second, we can consider to which extent recessions are simply negative expansions. The model, estimated using Bayesian methods, generates posterior probabilities of being in recessions which correspond to the *NBER* dated recessions, provides support to the presence of a recovery early in expansions, consistent with what found in the literature, and estimates the shape of recessions to be linear, contrary to some previous parametric studies. When we investigate the ability of our specification to produce plausible business cycle features, where those features are a set of statistics proposed by Harding and Pagan (2002), we find that the model is able to capture all of them. Finally, the effects of recessions on long-run output level implied by our specification lie between those predicted by two important benchmark models of this literature.

JEL Classification: C22; E32.

Keywords: Business cycles; Non-linear time series models; Asymmetries.

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1 Introduction

The behavior of macroeconomic variables over the phases of business cycles has been object of interest since the early investigations of Mitchell (1927), Keynes (1936) and Burns and Mitchell (1946). A systematic approach to the analysis of business cycle characteristics started in the 1940s with a group of researchers at the National Bureau of Economic Research (Burns and Mitchell, 1946) who investigated the behavior of large set of economic time series and produced important evidence about timing and duration of business cycle. An important aspect of this old and large literature is about the symmetry or asymmetry of the cycles: to them recessions cannot be thought as just the mirror image of expansions. In the last decades, the literature has provided more formal support to many of their ideas. Hamilton in its 1989's seminal paper captures asymmetries in phases's duration allowing the mean growth rate of GNP to switch between the two states (negative trend growth and positive trend growth) according to a Markov Chain variable. More recently, Harding and Pagan (2002) after establishing a link between the turning point definition of the cycle in a series and the moments of the random variables assumed to represent that series, suggest some measures which are useful in capturing the nature of the cycles, such as duration and amplitude of the cycle and its phases, cumulative movements within phases. The last feature has received particular attention in the literature and it is intended to capture the shape of the phases, namely the pattern of variation in output growth rates over the course of expansions and recessions. Using this measure, Harding and Pagan conclude that US expansions tend to be concave, that is, output growth tends to be faster earlier in expansion and slower as expansion persists.

The purpose of this paper is to propose and estimate a generalized version of a Markov switching model and evaluate its ability to reproduce a group of selected features observed in actual cycles. The characteristics we consider are the group of non-parametric statistics identified by Harding and Pagan and our emphasis will be on the investigation of the shape of cycle phases. Indeed, the shape of the phases has important implications with regard the debate about the nature of recessions and their long-run effects on output level. For example, a concave shape of expansions, implying the presence of a recovery phase immediately after recessions end, supports models of fluctuations in which recessions are mostly transitory deviations from trend and not movements of the trend. The specification we estimate is a generalisation of the model proposed by Kim, Morley and Piger (2005), with the important difference being that we do not impose any symmetry in the shape of cycle across the phases. To clarify this point, we can think of the shape of output dynamic along two main dimensions. First, we can think of the pattern of variation in growth rates over the course of expansions and recessions. Second, we can consider the extent to which recessions are simply negative expansions. Kim et al. (2005) have focussed their attention on the first dimension, concluding that there are important asymmetries over the course of expansions. In doing so, they force similar pattern over recessionary phase. Our idea is to augment their model to estimate both dimensions separately. Indeed, forcing recessions

to be mirror images of expansions regarding their shape might imply an incorrect evaluation of long-run effects of recession on output level. Moreover, estimating separately the shape of output dynamic across cycle phases will result in a better understanding about potential theoretical model behind business cycle dynamic. Contrary to Kim et al., in our specification, we take into account the possibility of structural break in output volatility allowing the variance of the disturbance term to switch between two states according to a Markov chain process, independent from the one governing the mean¹. Moreover, we estimate the model using a Bayesian approach and the Gibbs sampler². The main problem associated to the estimation of Markov switching model using Maximum likelihood is that the state space for the unobservable Markov Chain variable grows with sample size³. On the contrary, using the Gibbs Sampler, we can obtain marginal posterior distribution for the parameters of interest by sampling from conditional distributions, easily derived given the nature of our model.

We find evidence to support the presence of a recovery early in expansion, as obtained by Kim et al., and we find that recessions have linear shape, contrary to what is implied by Kim et al.'s model. When we investigate the ability of our specification to produce plausible business cycle features, where those features are the statistics proposed by Harding and Pagan, we find that the model is able to capture most of them. Finally, we find that the effects of recessions on long-run output level implied by our specification are smaller than what implied by Hamilton's model, but greater than what predicted by Kim et al.'s model. The latter finding is strictly related to the alternative shape of cycle phases implied by the three models, suggesting that our specification, estimating separately the shape of output across cycle phases, is the most appropriate to derive conclusion about long-run effects of recessions on output.

The structure of the paper is the following. Section 2 relates the paper to some important contributions in the literature and describes a set of stylized facts for US business cycles. Section 3 describes both the model and the econometric methodology, and presents the parameter estimates. In section 4, we compare our model with alternative specifications proposed in the literature performing two main exercises. First, we investigate the ability of our specification to produce plausible business cycle features. Second, we evaluate what the model implies regarding the effects of recessions on long-run output level. Final section concludes.

2 Background

Historically, the approach of BM to the investigation of business cycles has had great influence, confirmed also by the fact that measurements of cycle phases' duration continue to be made

¹There is an extensive literature documenting structural break in output volatility; see, among others, McConnell and Perez-Quiros (2000), and Kim and Nelson (1999b).

²Kim et al. (2005) estimate their specification using Maximum likelihood.

³The estimation is still possible but inference is complicated by the fact that asymptotic properties of test statistics are difficult to establish.

by the *NBER* to this day, following their framework. Nevertheless, their methodology has been subject to important criticism for its lacking sound statistical foundations. In the last decades, the literature has provided more formal support to many of their ideas, for example proposing models that treat data generating processes and business cycle phases together, consistent with the BM's idea of duration and expected duration as crucial for business cycle dynamic. More recently, Harding and Pagan have established a link between the turning point definition of the cycle in a series and the moments of the random variables assumed to represent that series. Harding and Pagan also suggest some measures which are useful in capturing the nature of the cycles, such as duration and amplitude of the cycle and its phases, any asymmetric behavior of the phases, cumulative movements within phases. The last feature has received particular attention in the literature since it is intended to capture the shape of the phases, namely the pattern of variation in output growth rates over the course of expansions and recessions. Table 1 shows the features suggested by Harding and Pagan for monthly and quarterly frequency data of US economic activity in the last four decades.

	US	IPI	GDP*
Mean duration (months/quarters)			
Recessions		12	3
Expansions		58	17.8
Mean amplitude (%)			
Recessions		-8	-2.5
Expansions		26	20.2
Mean excess (%)			
Recessions		0.4	-0.1
Expansions		1.5	1.1

Table 1: Business cycle characteristics for US. * The result for GDP are from Harding and Pagan (2002).

We identify the turning points for *IPI* series using the Bry and Boschan algorithm (1971), appropriately set out for monthly observations⁴, while we use the algorithm proposed by Harding and Pagan for dating the *GDP* series⁵. Duration statistics are measured in months (second column) and in quarters (third column), while the other two characteristics are measured in terms of percentage changes. The average cycle length, around 65 months, results from an asymmetric duration between the two phases; indeed the expansions duration is on average more than 4 times longer than contractions duration. The amplitude in expansions is around

⁴For a detailed explanation of the Bry and Boschan algorithm see Bry and Boschan (1971). We use the GAUSS implementation of the Bry and Boschan algorithm written by Denson and Watson for monthly observations.

⁵The turning points identified by the dating algorithms coincide almost perfectly with the *NBER* dates.

three times larger (in absolute value) than the amplitude in recessions. The “excess measure” indicates the deviation of output from a triangle approximation and is intended to capture the shape of the phases. The evidence that contractions are on average followed by relatively short period of very high growth is supported by the positive “excess measure”. While output tends to have a concave-shaped expansions, there is not evidence of similar shape during recessions.

Fig 1 shows the average growth rates of *IPI* in the first three semesters after a contraction phase. There is a tendency of growth rates to be higher, on average, in the first months of expansions, and then to decrease as expansions persist.

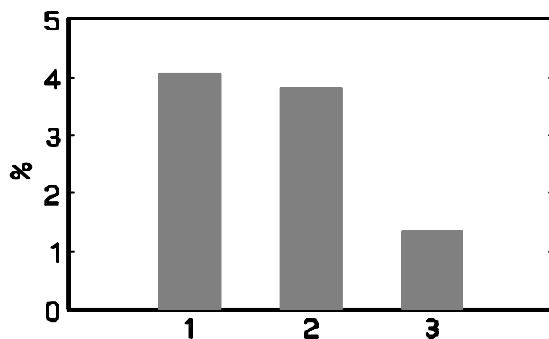


Figure 1: Average growth rates in the first, second and third semester after US recessions (IPI).

Fig 2 shows the average growth rates of *IPI* over the course of contractions. Specifically, the three histograms show the average growth rate in the first quarter, in the second quarter and later on in the recession phase, respectively. Overall, growth rates seem to be constant, consistent with the previous non parametric evidence of linear shape of output over the course of contractions.

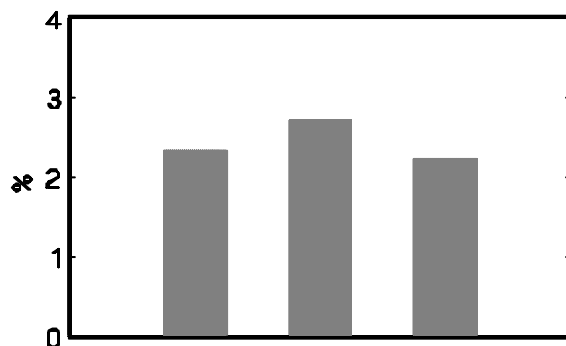


Figure 2: Average growth rates in the first quarter, second quarter and later on in the US recessions (IPI).

The idea that US contractions are typically followed by a short period of fast growth has found more formal support in the work by Sichel (1994) such that he suggests that “ postwar

fluctuations in real output in the United States should be thought of as having three phases rather than two- contractions, high-growth recoveries, and moderate-growth periods following recoveries”. An approach to modelling the high recovery phase has been to add an additional regime to Hamilton’s model. Hamilton’s specification , by construction, implies that recessions have long-run effects on output level since its two-phases business cycle implies that following the trough, output switches back to the expansion regime, never regaining what lost during recessions. Clements and Krolzig (1998) propose and estimate a three-state heteroscedastic model with a switching intercept. One implication of this approach is that recoveries are independent of the preceding recession, while some empirical evidence seem to suggest the opposite. Beaudry and Koop (1993) augment a standard ARMA model of output growth with a “current-depth-of-recession” dummy variable that is supposed to capture the fall of output below its previous historical maximum. They find that this variable is useful for predicting changes in output. In their paper, Kim et al. (2005) propose a model that takes into account most of these issues. They augment a standard Hamilton’s model in a way that allow them to estimate the magnitude of the recovery early in expansion. Rather than introducing an additional regime, they include a “bounce-back” term directly in the output process. This term is related to the length of each recession (and thus to the endogenously estimated unobservable state variable), and can potentially capture the particular shape of US expansions. Moreover, it does not place particular constraints on the effects of recession on long-run output level. They find a large “bounce-back” effect with the implication that recessions have small permanent effects on output. As we will show in the next section, although Kim et al. refer to their additional term as a “bounce-back” term, this term is capturing more complex dynamic. Indeed, since it starts to operate during recessions, it is implicitly forcing recessions to be mirror images of expansions regarding their shape, a fact that might lead to underestimate the long-run effects of recession on output level.

3 The model

The model we propose is intended to capture a selected set of characteristics of US *classical cycles*, which include duration, amplitude and shape of the cycle and its phases. Our focus will be in particular on the shape of the phases, namely the pattern of variation in output growth rates over the course of expansions and recessions given its relevance regarding the debate about the nature of US recessions and their long-run effects on output level. The specification we consider and estimate is a generalisation of Kim et al.’s (2005) model. The main limitation of their model is that, by construction, business cycle are symmetric regarding the shape which might lead to an incorrect evaluation of long-run effects of recession on output level. To clarify this point, we can think of the shape of output dynamic along two main dimensions. First, we can think of the pattern of variation in growth rates over the course of expansions and recessions. Second, we can consider the extent to which recessions are simply

negative expansions. Kim et al. find important evidence supporting the first dimension, with the implicit conclusion being that there are important asymmetries in the shape of output dynamic within phases. In deriving this conclusion, they are *de facto* imposing that recessions are mirror images of expansions (regarding the shape). Moreover, forcing rather than testing a “recovery” during recessions might imply an underestimation of long-run effects of recession on output level. We propose to augment their model in order to account for both dimensions of the cycle shape, and thus without imposing *a priori* any particular relation in the shape of output dynamic across cycle phases.

The output process Let Δy_t denote the output growth in period t , and suppose that S_t is an unobserved first-order Markov-switching state variable that takes on discrete values of 0 and 1, corresponding to a period of low and high growth, respectively. We model the deviation of output growth from its mean as the following stationary *AR* process:

$$\phi(L)(\Delta y_t - \alpha_0 - a_1 S_t - \alpha_2 S_t \sum_{j=1}^m (1 - S_{t-j}) - \alpha_3 (1 - S_t) \sum_{j=1}^m (1 - S_{t-j})) = \varepsilon_t \quad (3.1)$$

$$\varepsilon_t \sim N(0, \sigma_t^2)$$

where $\phi(L)$ is the lag operator with roots outside the unit circle, and the phases are identified by restricting α_1 to be positive. The Markov chain variable S_t switches from one state to the other according to transition probabilities $p = P(S_t = 1 | S_{t-1} = 1)$, the probability of remaining in expansion, and $q = P(S_t = 0 | S_{t-1} = 0)$, the probability of remaining in recession. Kim et al. introduce the idea of estimating the shape of the cycle phases including the summation term $\sum (1 - S_{t-j})$ in the output process. This term is related to the length of each recession and thus to the endogenously estimated unobservable state variable. Figure 3 shows output dynamic (solid line), the evolution of the summation term $\sum_{j=1}^m (1 - S_{t-j})$ over the course of recessions and expansions (the trapezium at the bottom of the graph), and the contractionary regime (shaded area). To see how the summation term works, let us consider first the switches of S_t from 1 to 0 which causes an impact fall in output up to $(E[\Delta y_t | I_t] = \alpha_0 < 0)$. As recession persists the summation term tends to increase reaching eventually its maximum at the beginning of expansion. Meanwhile, for $\alpha_3 > 0$, output starts to recover during the recessionary phase since the expected growth rate moves from the negative value α_0 to an higher (but still negative) value $\alpha_0 + \alpha_3 \sum_{j=1}^m (1 - S_{t-j})$, as it is evident by the changing slope of the solid line during contraction. After the recession ends, namely S_t switches to 1, the summation term reaches its maximum and for $\alpha_2 > 0$, output growth rate rises substantially up to $\alpha_0 + a_1 + \alpha_2 \sum_{j=1}^m (1 - S_{t-j})$. This rapid recovery is temporary since as expansions persist the summation term goes to zero and expected growth rate approaches its medium/long term growth rate in expansion, $\alpha_0 + a_1$.

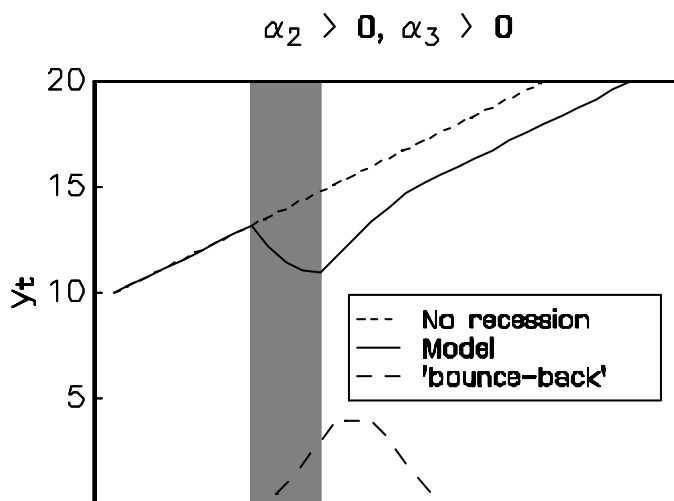


Figure 3: Output dynamic and cycle phases.

Notice also that, although the recovery in expansion is temporary, its effect on long-run output level is permanent⁶. Finally, we can think about the overall effects of recession on output level as the distance between the dashed and the solid lines.

Parameters α_2 and α_3 are therefore intended to capture variation in growth rates over the course of expansions and recessions. Kim et al. estimate a model in which they constraint $\alpha_2 = \alpha_3$, namely they constraint recessions to be mirror images of expansions regarding the shape. In figure 4, we show output dynamic under alternative signs for the parameter α_3 , conditional on a positive value for α_2 .

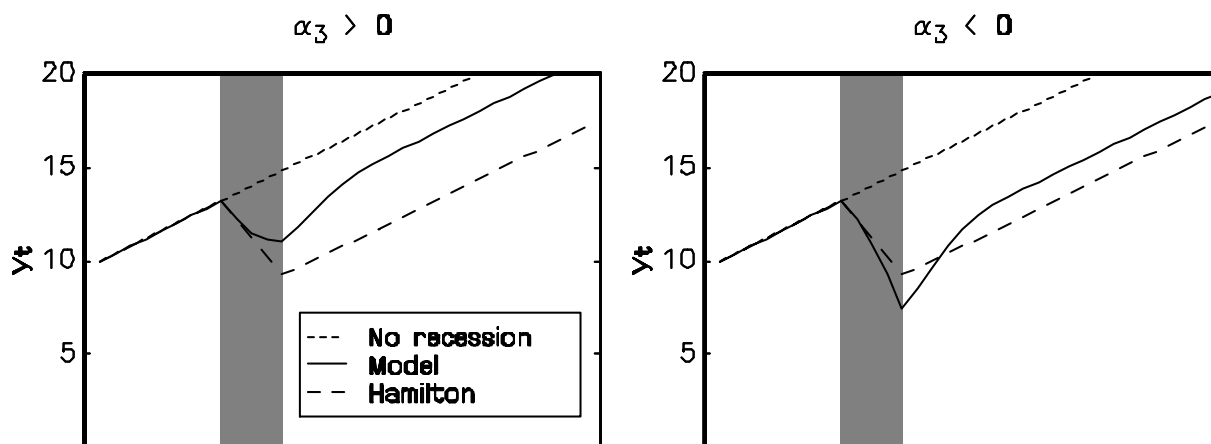


Figure 4: Output dynamic under alternative shape of recessions.

⁶Similarly, also the pattern of growth rate during recession has effects on long-run output level.

Specifically, on the left hand side, we compare output dynamic after a recessionary episode implied by our model when both α_2 and α_3 are positive (solid line) with the one implied by standard Hamilton's model, namely $\alpha_2 = \alpha_3 = 0$, (dashed line). On the right hand side, we consider the case in which α_3 is negative and α_2 is kept fixed at a positive value. It is clear that, for a given positive value of α_2 , alternative values of α_3 imply different long-run output level (compare solid line in the LHS and in the RHS). Specifically a positive α_3 implies a smaller drop of output as recession persists and hence a lower output cost of recession, where the cost of recession can be thought as the distance between the dotted and solid lines.

Regarding expansions, a positive value of α_2 implies a concave shape of booms, namely a temporary period of rapid recovery immediately after recessions which will merge in a period of slower and positive growth rate (as it is evident from the pattern of the slope of the solid lines). It is then clear that the overall effect of recessions on long-run output level, captured by the distance between dotted and solid lines, is the result of different magnitude and sign of the parameters α_2 and α_3 . Notice that Kim et al.'s specification is a restricted version of our generalized specification for $\alpha_2 = \alpha_3$, while for $\alpha_2 = \alpha_3 = 0$ we have the standard Hamilton's model⁷.

Transition equations for the unobserved states Following Filardo and Gordon (1998), we consider a specification with time-varying transition probabilities, such that the evolution of the unobserved state depends on the information contained in leading indicator data. The implication of this inclusion is that the conditional expected duration of a phase is no longer constant, but can vary across time. A latent variable version of the probit model is used to describe the transition probabilities:

$$S_t^* = \gamma_0 + \gamma'_z z_t + \gamma_s s_{t-1} + u_t \quad (3.2)$$

where z_t is a vector of information variables that affect the transition probabilities of the business cycle phases, γ'_z is a vector of state-dependent slope parameters capturing the information of leading indicators about the probability the economy persists or exits from a business cycle phase, and u_t is assumed to be normally distributed. At any point in time, the probability to be in expansion is equal to the probability that S_t^* is positive,

$$P(S_t = 1) = P(S_t^* > 0)$$

while the transition probabilities p and q are given by:

$$p_t \equiv P(S_t = 1 | s_{t-1} = 1) = 1 - \Phi_{U|Z}(-\gamma_0 - \gamma'_z z_t - \gamma_s) \quad (3.3)$$

$$q_t \equiv P(S_t = 0 | s_{t-1} = 0) = \Phi_{U|Z}(-\gamma_0 - \gamma'_z z_t) \quad (3.4)$$

⁷Since Kim et al. find that $\alpha_2 \equiv \alpha_3 > 0$, output dynamic implied by their model is represented by the LHS picture.

where $\Phi_{U|Z}$ is a $N(0, 1)$ conditional cumulative density function. The transition probabilities p and q are a measure of the persistence of booms and recessions, while $1 - p$ and $1 - q$ are the probabilities of switching from boom to recession and from recession to boom respectively.

Moreover, following the seminal works of McConnell and Perez-Quiros (2000) and Kim and Nelson (1999b), we allow the possibility that the variance of ε_t , σ_t^2 , is subject to structural break⁸. Specifically, we assume that:

$$\sigma_t^2 = \sigma_0^2(1 + kV_t)$$

where we model V_t as a first order two state Markov chain variable, independent of S_t , with transition probabilities given by $p_t^v = P(V_t = 1|V_{t-1} = 1)$ and $q_t^v = P(V_t = 0|V_{t-1} = 0)$. We identify the high variance regime $\sigma_1^2 \equiv \sigma_0^2(1 + k)$ with $V_t = 1$, by restricting $\sigma_1^2 > \sigma_0^2$, namely $k > 0$. Again, we employ a latent variable version of the probit model to describe the transition probabilities:

$$V_t^* = \delta_0 + \delta_v v_{t-1} + \xi_t \tag{3.5}$$

where the probability to be in the high volatility regime is equal to the probability that V_t^* is positive,

$$P(V_t = 1) = P(V_t^* > 0)$$

while the transition probabilities are given by:

$$p^v \equiv P(V_t = 1|v_{t-1} = 1) = 1 - \Phi_{U|Z}(-\delta_0 - \delta_v) \tag{3.6}$$

$$q^v \equiv P(S_t = 0|s_{t-1} = 0) = \Phi_{U|Z}(-\gamma_0) \tag{3.7}$$

where $\Phi_{U|Z}$ is a $N(0, 1)$ conditional cumulative density function.

3.1 Estimating the model: a Bayesian approach

We estimate the model described by equations 3.1, 3.2 and 3.5. In order to avoid some problems associated to the classical inference, Bayesian method is used to calculate posterior probability⁹. Within this framework, the parameters of the model are treated as random variables having

⁸A more general specification than the one we propose would have been allowing the conditional mean of recessions and expansions to evolve according to structural break in output volatility, namely $\alpha(S_t, V_t)$. It is indeed the specification we firstly tried to estimate. The difficulty is that singularity problems are likely to arise for many draws of the posterior parameters. The reason is that the regressors of the autoregressive process, under some posterior draws of the parameters, tend to be close to be linearly dependent and the inversion of the matrix becomes unfeasible.

⁹The problems associated with the classical inferences are mainly due to the fact that for a given sample size of T observations, we need to consider 2^T possible cases. Although it is still possible to estimate the model (Hamilton, 1990), the asymptotic properties of the test statistics for many hypothesis are difficult to derive (see also Hansen, (1996)). Kim et al. estimate their specification using Maximum likelihood.

known probability distribution. For Bayesian inference, marginal posterior distribution may be obtained from the joint posterior distribution $P(\theta|Y)$. Nevertheless, the nature of the model is such that direct estimation of the joint posterior distribution is unnecessary. Indeed, the model allows us to employ Gibbs sampling techniques for obtaining marginal posterior distribution by sampling from conditional distributions, easily derived in the model¹⁰. Under Bayesian methodology, also the unobserved elements of the model are treated as additional parameters to be estimated, such that the parameters of interest are:

$$\theta = \left\{ \alpha, \phi, \sigma_0^2, \sigma_1^2, \gamma, \delta, S^T, \{s_t^*\}_1^T, \{p_t\}_1^T, \{q_t\}_1^T, V^T, \{v_t^*\}_1^T, p^v, q^v \right\} \quad (3.8)$$

where S^T and V^T denote $\{s_t\}_1^T$ and $\{v_t\}_1^T$, respectively. Let $Y^T = [y_1 \ y_t]'$ and $z^T = [z_1 \ z_t]'$, the following steps describes the application of Gibbs sampling technique to our model:

- (i) generate σ_0^2 from $p(\sigma_0^2|Y^T, \alpha, \phi, \sigma_1^2, S^T, V^T)$, where knowledge of the parameters $\gamma, \{p_t\}_1^T, \{q_t\}_1^T$ and $\{s_t^*\}_1^T$ is redundant conditional on S^T and knowledge of $\delta, p^v, q^v, \{v_t^*\}_1^T$ is redundant conditional on V^T .
- (ii) generate σ_1^2 from $p(\sigma_1^2|Y^T, \alpha, \phi, \sigma_0^2, S^T, V^T)$, where knowledge of the parameters $\gamma, \{p_t\}_1^T, \{q_t\}_1^T$ and $\{s_t^*\}_1^T$ is redundant conditional on S^T and knowledge of $\delta, p^v, q^v, \{v_t^*\}_1^T$ is redundant conditional on V^T .
- (iii) generate ϕ from $p(\phi|Y^T, \alpha, \sigma_0^2, \sigma_1^2, S^T, V^T)$, where knowledge of the parameters $\gamma, \{p_t\}_1^T, \{q_t\}_1^T$ and $\{s_t^*\}_1^T$ is redundant conditional on S^T and knowledge of $\delta, p^v, q^v, \{v_t^*\}_1^T$ is redundant conditional on V^T .
- (iv) generate α from $p(\alpha|Y^T, \phi, \sigma_0^2, \sigma_1^2, S^T, V^T)$, where knowledge of the parameters $\gamma, \{p_t\}_1^T, \{q_t\}_1^T$ and $\{s_t^*\}_1^T$ is redundant conditional on S^T and knowledge of $\delta, p^v, q^v, \{v_t^*\}_1^T$ is redundant conditional on V^T .
- (v) generate S^T from $p(S^T|Y^T, z^T, \alpha, \phi, \sigma_0^2, \sigma_1^2, V^T, \gamma, \{p_t\}_1^T, \{q_t\}_1^T)$ where knowledge of $\{s_t^*\}_1^T$ is irrelevant and knowledge of $\delta, p^v, q^v, \{v_t^*\}_1^T$ is redundant conditional on V^T .
- (vi) generate $\{s_t^*\}_1^T$ from $p(\{s_t^*\}_1^T | S^T, z^T, \gamma)$, where conditional on those, $\{s_t^*\}_1^T$ is independent of all the other parameters.
- (vii) generate γ from $p(\gamma | \{s_t^*\}_1^T, S^T, z^T)$, where conditional on those, γ is independent of all the other parameters.
- (viii) generate V^T from $p(V^T|Y^T, \alpha, \phi, \sigma_0^2, \sigma_1^2, S^T, \delta, p^v, q^v)$, where knowledge of the parameters $\gamma, \{p_t\}_1^T, \{q_t\}_1^T$ and $\{s_t^*\}_1^T$ is redundant conditional on S^T .
- (ix) generate $\{v_t^*\}_1^T$ from $p(\{v_t^*\}_1^T | V^T, \delta)$, where conditional on those, $\{v_t^*\}_1^T$ is independent from all the other parameters.

¹⁰On the contrary, direct estimation of the joint distribution would have been more cumbersome.

(x) generate δ from $p(\delta | \{v_t^*\}_1^T, V^T)$, where conditional on those, γ is independent of all the other parameters.

The above procedure is similar to the ones firstly proposed by Albert and Chib (1993), and used also in Filardo and Gordon (1998), and Kim and Nelson (1999b)¹¹.

3.1.1 Priors Specifications and Posterior Distributions

Although the parameters to estimate include 6 time series, the time invariant parameters $\{\alpha, \phi, \sigma_0^2, \sigma_1^2, \gamma, \delta\}$ are sufficient to define the posterior distribution for all the parameters of interest. Indeed, for each draw of the parameters γ and δ , using equations (3.2), (3.5), (3.3), (3.4), (3.6) and (3.7), and given the data on leading indicators, we can recover the 6 time series of interest. We consider diffuse priors for the parameters of autoregressive process, considering very flat prior distributions. Specifically, we assume that σ_i^2 are inverse gamma distributions with single degree of freedom and small scale parameter,

$$\sigma_i^2 \sim IG\left(\frac{\hat{v}_i}{2}, \frac{\hat{v}_i \hat{\sigma}_i^2}{2}\right) \sim IG\left(\frac{1}{2}, \frac{0.001^2}{2}\right)$$

explicitly designed to put most weight on sample information. The parameter vector ϕ is assumed to have truncated normal prior distribution,

$$\phi \sim N(\hat{\phi}, \hat{A}_\phi) I_{|\phi| < 1}$$

where $I_{|\phi| < 1}$ is an indicator function for stationarity and prior variance is set at high value given by $\hat{A}_\phi = 10000 \cdot I_4$. We also assume conjugate prior distribution for parameter vector α such that

$$\alpha \sim N(\hat{\alpha}, \hat{A}_\alpha) I_{\alpha_1 > 0}$$

where the restriction on the support of α allows us the identification of the two regimes, and the prior variance is assumed to be $\hat{A}_\alpha = 10000 \cdot I_4$.

We employ natural conjugate Normal priors also for the parameters γ and δ , governing the transition of Markov chain variables S_t and V_t , respectively. Specifically, we assume that prior distributions are given by:

$$\gamma \sim N(\hat{\gamma}, \hat{A}_\gamma)$$

$$\delta \sim N(\hat{\delta}, \hat{A}_\delta)$$

Regarding the process for S_t , although the inclusion of leading indicators in itself might lead to a good identification of business cycle phases, we use relatively tight informative prior

¹¹See the appendix for a more detailed explanation of the Gibbs sampler steps.

in order to have a more precise estimate of the business cycle transition ¹². Specifically, the moments for γ are given by:

$$\hat{\gamma} = \begin{bmatrix} \hat{\gamma}_0 \\ \hat{\gamma}_{zc} \\ \hat{\gamma}_{ze} \\ \hat{\gamma}_s \end{bmatrix} = \begin{bmatrix} -1.3 \\ 0 \\ 0 \\ 3.2 \end{bmatrix} \quad \hat{A}_\gamma = \text{diag} \left[\begin{array}{cccc} 0.1 & 2 & 2 & 0.1 \end{array} \right]$$

Those values are chosen to imply transition probabilities that generate expected whole cycles phase durations similar to the average duration of cycles identified by the *NBER*; indeed, under fixed transition probabilities, the average duration of a regime is given by $(1 - \rho)^{-1}$, where ρ is the probability to persist in a given regime. The parameters γ_{zc} and γ_{ze} are state dependent slope and describes the marginal contributions of the leading indicators for the transition from one phase to the other and viceversa and it is intended to capture the variation of the cycle durations around their averages. We center their prior distribution for γ_{zc} and γ_{ze} at zero but allowing relatively high variance. The general idea behind the choice of the prior distribution for γ is of being able to derive precise dates of US business cycles. Indeed, the objective of the paper is not only to show that the model is able to capture the *NBER* dated recessions, something already shown in previous papers (including the seminal Hamilton's 1989 paper). The main motivation is to shed some lights on the pattern of growth rates variation over the cycle phases. Moreover, since there are some evidence that a particular output dynamic might take place early at the beginning of the phases, it is then clear that a precise identification of turning points plays a crucial role.

The priors parameters for δ are given by:

$$\hat{\delta} = \begin{bmatrix} \hat{\delta}_0 \\ \hat{\delta}_v \end{bmatrix} = \begin{bmatrix} -2.3 \\ 4.6 \end{bmatrix} = \quad \hat{A}_\delta = \text{diag} \left[\begin{array}{cc} 0.1 & 0.1 \end{array} \right]$$

They are chosen such that they imply a probability to persist in the same regime around 0.99, and hence a relatively high average duration both in the low and high volatility regime¹³.

Given the assumption about the prior distribution, we can now be more explicit regarding the distributions from which posterior values for the parameters of interest are drawn at each iteration of the Gibbs sampler¹⁴.

¹²In the fixed transition probability models of Albert and Chib (1993), the use of tight priors about the transition probabilities parameters p and q , is also necessary to obtain their results.

¹³The implied average duration in the same regime is around 78 months.

¹⁴See the appendix for a more detailed explanation of the Gibbs sampler steps.

Table 2 list the Gibbs sampler steps (first column), the corresponding parameter to be drawn (second column) and the sampling distribution (third column).

Step of Gibbs Sampler	Parameter	Distribution
(i)	σ_0^2	Inverted Gamma
(ii)	σ_1^2	Inverted Gamma
(iii)	ϕ	Normal
(iv)	α	Truncated Normal
(v)	S^T	Bernoulli
(vi)	$\{s_t^*\}_1^T$	Truncated Normal
(vii)	γ	Normal
(viii)	V^T	Bernoulli
(ix)	$\{v_t^*\}_1^T$	Truncated Normal
(x)	δ	Normal

Table 2: Gibbs Sampler and distributions.

3.2 Parameter estimates

The data for y_t is the log of US seasonally adjusted Total Industrial Production series (1992=100) from the Federal Reserve Board. The series for the information variable z is the seasonally adjusted Composite Index of Eleven Leading Indicators (1987=100), now published by the Conference Board. The monthly sample run from 1959/10 to 2003/03. Following Hamilton, the lag order of the AR process is 4¹⁵. We normalize the Composite Index of leading Indicators by their 12 months moving average in order to have stationary series for z . The estimates are based on 12,000 passes of the Gibbs sampler. Moreover, starting values $\alpha^{(0)}$, $\phi^{(0)}$, $\sigma_1^{2(0)}$, $\gamma^{(0)}$, $\delta^{(0)}$, $\{S_t^{(0)}\}_1^T$, $\{V_t^{(0)}\}_1^T$ are required for the simulation; the starting values for the sequence $\{s_t\}_1^T$ are the *NBER* dates while for the sequence $\{v_t\}_1^T$ are built according to the evidence of structural break in output volatility occurring in mid 80's (see McConnell and Perez-Quiros (2000), and Kim and Nelson (1999b)). Notice that the posterior estimates are robust to the choice of different starting values. Moreover, in order to attenuate the effect of those starting values and let the Gibbs-sampler converge, the first 2000 observations were discarded leaving the 10000 observations to calculate the posterior moments¹⁶. The Gibbs sampler estimates seem to settle down fast, such that very small variations in the estimated parameters and probabilities do occur after few hundred draws. We have computed the recursive mean and graphically checked for the convergence (CUMSUM statistic). For the summation term, we choose $m = 17$, in

¹⁵We have checked the robustness of the results to alternative number of lags.

¹⁶In order to eliminate the autocorrelation existing among draws, we use one draw every 5 to compute the posterior estimates.

order to capture the length of all dated postwar US recession¹⁷. Table 3 shows the first and the second moments for the parameters of interest of equation (3.1).

Parameter	Estimate	St.error
α_0	-0.82	0.24
α_1	1.14	0.23
α_2	0.039	0.018
α_3	0.044	0.028
ϕ_1	0.09	0.049
ϕ_2	0.10	0.045
ϕ_3	0.11	0.046
ϕ_4	0.10	0.047

Table 3: Gibbs Sampling estimates under our specification.

The parameters for recession (α_0) and expansion ($\alpha_0 + \alpha_1$) are negative and positive respectively, and are both statistically significant. This implies that $S_t = 1$ corresponds to an “expansionary” regime. The estimate of α_2 is positive and statistically significant providing support to the presence of a recovery early in expansions. Specifically, the estimate we find implies that, after a recession of say 10 months, the quarterly output growth rate early in expansion is around 1% above the long-run expansion growth rate. It is implicit in this formulation that the longer the recession is, the higher the output response will be, but this high response will have small persistence. The extreme case is when the duration of the preceding recession is exactly of m months or more; the summation term reaches its maximum value, m , in the first period of expansion phase, but then output growth tends to decrease rapidly from its maximum. Focusing on first moments, the estimate of α_3 seems to confirm Kim et al.’s finding that “the leveling off of output during a prolonged recession appears to be an important aspect”. It turns out that this effect is not statistically different from zero. Therefore, contrary to what is implied by Kim et al.’s model, we cannot reject the null hypothesis that recessions are linear. Linear contractions for US is a common view in the literature, and is consistent with the *excess* measure we have found and with formal findings of Harding and Pagan, Galvão (2002) and Balke and Wynne (1995). There are some problems for the significance of the autoregressive parameter ϕ_1 , while the other autoregressive parameters are statistically significant. Table 4 shows the parameters estimates for the transition equation of S_t and V_t , and for both σ_0^2 and σ_1^2 , the low and high volatilities values respectively. The posterior estimates for γ_0 and γ_s imply an average cycle duration close to what we observe in the data, but this is also induced by the tight priors we assume. The positive value of γ_{ze} , the coefficient that captures the time-varying profile of transition probabilities during expansions, is consistent with the idea that an increase in the value of leading indicator increases the value for the latent variable S_t^* and therefore

¹⁷The results are robust to the choice of similar number of lags.

increases the probability of being in expansion. On the contrary, the marginal contribution of leading indicators in contractions, captured by γ_{zc} , is not statistically significant. The values we find for the volatilities estimates clearly confirm a large structural break in output volatilities, being the volatility in the high regime 3 times larger than the volatility in the low regime.

Parameter	Estimate	St.error
γ_0	-1.20	0.08
γ_{zc}	0.03	0.06
γ_{ze}	0.18	0.09
γ_s	3.1	0.08
δ_0	-2.1	0.19
δ_v	4.3	0.06
σ_0^2	0.18	0.02
σ_1^2	0.56	0.06

Table 4: Gibbs Sampling estimates under our specification.

Figure 5 shows the recessions identified by the *NBER* (shaded bars), the posterior probability of being in the low mean regime (solid line), the posterior probability of being in the low volatility regime (dotted line). Overall, there is a good correlation between the posterior probability of being in recession and the *NBER* dates. Moreover the reduction in volatility is estimated to occur in the second quarter of 1984, consistent with the findings of McConnell and Perez-Quiros and Kim and Nelson.

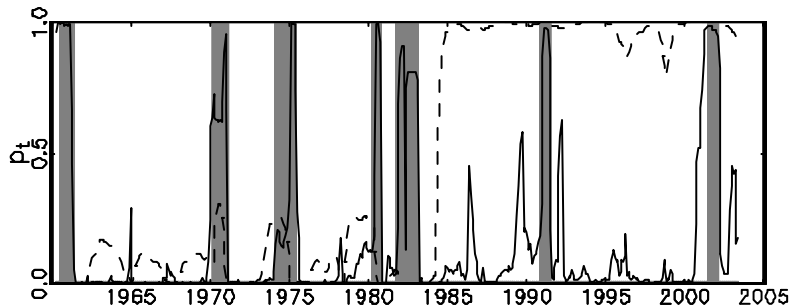


Figure 5: Posterior probabilities of being in recessions (solid line), of being in low volatility regime (dotted line) and NBER recessions (shaded bars).

Thinking about the two main dimensions for the shape of output dynamic, we can summarize our results as follow. First, we find asymmetry of the shape over the course of expansions. Large output growth rates tend to be located at the beginning of expansions and then, as expansions persist, output growth tends to converge to lower positive long-run value. Although there is some evidence that the patterns of variation of growth rates are similar, in absolute value,

between expansions and recessions, we cannot reject the null that recessions have linear shape. We have estimated again the model under the assumption that $\alpha_3 = 0$, namely that recessions are linear. The results are shown in the following table.

Parameter	Estimate	St.error
α_0	-0.934	0.156
α_1	1.238	0.160
α_2	0.027	0.012
ϕ_1	0.085	0.047
ϕ_2	0.096	0.045
ϕ_3	0.117	0.044
ϕ_4	0.119	0.047

Table 5: Gibbs Sampling estimates under our specification.

The estimated value for α_0 is smaller, in absolute value, than what found before, although the comparison is not straightforward since in the previous specification output growth during recessions was not constant. We find that long-run output growth during expansions, $\alpha_0 + a_1$, is very similar to what found before, while the magnitude of the recovery phase is slightly smaller. The estimated values for the other parameters are almost unchanged.

4 Evaluating the model

In this section, we compare the performance of our model with alternative specifications proposed in the literature by performing two main exercises. First, we investigate the ability of our specification to produce plausible business cycle features. Second, we attempt to evaluate what the model implies regarding the effects of recessions on long-run output level.

4.1 Business cycle characteristics

We investigate the ability of our specification to produce the business cycle features proposed by Harding and Pagan and shown in section 2. By treating these non-parametric statistics as additional functions of interest, we can obtain their posterior distributions from the Gibbs Sampling routine. In particular, for each posterior draw of model's parameters, we simulate the model (conditional on actual initial output values), we date the artificial cycles using the Bry and Boschan algorithm (see Bry and Boschan (1971)) and compute business cycle characteristics. Indeed under the Gibbs sampler, given a general function h , a parameter vector α and the vector of data y , the following relation holds:

$$E(h(\alpha)) = \frac{1}{J} \sum_j h(\alpha^{jL})$$

where α^{jL} is the $j*L$ -th draw after skipping an initial number of iterations. For comparative purpose, we have done the same exercise also for the specifications proposed by Hamilton (1989) and Kim et al.(2005). Table 6 shows the mean estimates and the 68.3% confidence interval (the values inside the square brackets are the lower and upper bounds) for the features of interest under the three alternative specifications. Specifically, the symbols D , A and E stand for duration, amplitude and “excess measure” respectively, while tp (trough-to-peak) and pt (peak-to-trough) denote expansions and recessions, respectively.

Several points seem to emerge. Focusing on first moments’ estimates, all the three models are able to reproduce a good asymmetry between phases duration, being the number we find very close to what we observe in the data. This was partly expected, given the known ability of Markov switching model to identify *NBER* dates.

	Model	Kim et al.	Hamilton	Data
D_{tp}	61 _[45 76]	65 _[47 79]	63 _[49 78]	58
D_{pt}	13 _[10 15]	14 _[11 16]	11 _[9 14]	12
A_{tp} (%)	35 _[24 44]	38 _[29 46]	36 _[26 42]	26
A_{pt} (%)	-9 _[-7 -11]	-8 _[-6 -9]	-8 _[-6 -9]	-8
E_{tp} (%)	0.9 _[0.2 1.5]	0.95 _[0.3 1.5]	0.01 _[-0.02 0.03]	1.5
E_{pt} (%)	0.2 _[-0.05 0.4]	-0.3 _[-0.6 -0.01]	-0.005 _[-0.01 0.03]	0.4

Table 6: Business cycle characteristics.

An high degree of asymmetry seems to emerge also for the amplitude (which is partly related to the asymmetry in phases duration), although amplitude in expansion for artificial cycles seems to be excessive¹⁸. The measure of *excess* we find for expansions under our model supports the idea of concave expansions, consistent with what emerges from actual cycles. Kim et al.’s specification implies very similar dynamic for expansion while the standard Hamilton model is unable to reproduce this feature (as already noted by Harding and Pagan (2002) and Galvão (2002), among others). The measure of *excess* for contractions under our specification implies that US recessions are linear, consistent with what we observe for actual cycles. On the contrary, the specification proposed by Kim et al. implies that recessions tend to have similar shape of expansions and hence the negative “excess measure”. To clarify this point, if output growth rate is large at the beginning of expansions and decreasing as expansion persists, the deviation of output from a triangular approximation is positive and so it is the “excess measure”. If recessions have similar shape, namely relatively large (in absolute value)

¹⁸Harding and Pagan (2002), after investigating the ability of linear and non linear specification to reproduce those business cycle features, note that non linear process, including Markov Switching models, “... seem to go too far, producing cycles that are too extreme”.

growth rates at the beginning of the phase, the deviation from a triangular approximation will be negative. Regarding second moments, things are not so neat since the empirical distributions of some business cycle characteristics tend to be very skewed, with long tails. For example, the lower and the upper bound of A_{tp} are 24% and 44% respectively. Considering this interval for making inference, from one side, we cannot reject that the model is able to capture the value for actual cycles 26%. From the other, the width of the confidence interval is quite large and the actual value lies relatively close to the lower extreme. Similar conclusions can be drawn for the *excess* measure statistics. Indeed, under our model, the confidence interval for E_{tp} and E_{pt} goes from 0.2% to 1.5%, and from -0.05% to 0.4% respectively. Finally, the model tends to perform remarkably well regarding the precision of the estimates for duration of expansion and contractions, and amplitude in contractions. These results are only in part unexpected. At the root, there is the difficulty to make inference having few observations, namely recession and expansion episodes, and thus few business cycle characteristics¹⁹. More related to our approach is the fact that we are taking into account parameter uncertainty, typically neglected in most works²⁰. Indeed, fixing the parameters to their posterior mean, the Monte Carlo distributions of the HP statistics would have been much less skewed, with a dramatically decrease of the uncertainty²¹.

4.2 Long-run effects of US recessions

There has been a large debate in the literature about the nature of US recessions, with a particular emphasis on the effects of recessions on long-run output level. There is not a general agreement about whether recessions are mostly transitory deviations from trend or, on the contrary, are movements of the trend. Hamilton' model implies, by construction that output never regains what lost in recessions and output level is therefore permanently lower. In order to capture the consequences of recessions on the output level, Hamilton (1989) proposes to consider the expected difference in the long-run output level if at date t economy is in recession rather than in boom, given by:

$$\Xi = \lim_{j \rightarrow \infty} \{E_t [y_{t+j}|S_t = 1, \psi_{t-1}] - E_t [y_{t+j}|S_t = 0, \psi_{t-1}]\} \quad (4.9)$$

In the basic Hamilton model, this expression has the following form:

$$\Xi = \lim_{j \rightarrow \infty} \{E_t [y_{t+j}|S_t = 1] - E_t [y_{t+j}|S_t = 0]\} = \frac{\alpha_1(-1 + p + q)}{(2 - p - q)} \quad (4.10)$$

where α_1 is the parameter attached to the state variable S_t while p and q are the probabilities of remaining in booms and recessions respectively. This expression is computed holding the

¹⁹We have not stressed the fact that the characteristics of actual cycles we refer to hold on average. Therefore, an important issue is related to the uncertainty around these first moments.

²⁰For example, Kim et al. perform a similar exercise but simulating their model conditional on their Maximum likelihood parameters estimates.

²¹The results are available from the author on request.

current level of output constant, that is conditions on y_t and ψ_{t-1} . If we want to consider the effect on future and *present* level of output of a shift from $S_t = 1$ to $S_t = 0$, with the history of ε' s and past values of s_{t-j} constant, we add α_1 to the previous equation and we obtain the dynamic multiplier Λ , given by:

$$\Lambda = \frac{\alpha_1}{(2 - p - q)} \quad (4.11)$$

Kim et al. find a closed-form expression for the dynamic multiplier in their “bounce-back” specification, that is given by:

$$\Lambda_1 = \frac{\alpha_1 + m\alpha_2}{(2 - p - q)} \quad (4.12)$$

where m is the number of lags in the “bounce-back” summation and α_2 is the parameter attached to the “bounce-back” term. Our model has no closed-form solution for the expression Λ . Therefore we compute it *via* simulation. Specifically, we simulate the model computing output levels conditional on starting value for S being equal to 1 and 0 respectively, and then we take the difference between the average long-run levels of simulated output series. We compute Λ for the three different specification inside the Gibbs sampling routine, so taking into account also parameter uncertainty²². The following table shows the values for Λ implied by Hamilton’s model, Kim et al.’s model and our specification.

	Hamilton	Kim et al.	Model
$\Lambda(\%)$	9.8	5.3	6.4
SE	3.3	2.6	2.9

Table 7: Effects of recessions on long run output level (in absolute value)

The consequences on long-run output level of being in recession rather than in expansion are large in all the models, being the output drop between 9.8% and 5.7%²³. Those value are much larger than the values that Hamilton and Kim el al. found in their estimation. It is important to remember that they use real *GDP* series rather than *IPI* series. Not surprising, under Hamilton’s model recessions tend to have the largest (negative) effects on long-run output level. In particular, Hamilton’s model estimates imply long-run output drop 1/3 larger than the one implied by our model, and 70% larger than what implied by Kim et al.’ model. Fig 6 provides a graphical support by showing output dynamic after a recession under the three specification.

In our model the permanent drop of output is greater than the one implied by Kim et al.’s model for two main reasons. The first reason is that the parameter estimates under Kim et al.’s model implies a larger recovery early in expansion, graphically given by the steeper

²²Regarding our model, we consider the specification which implies that recessions are linear and expansions are concave.

²³For the first two models, these values can be computed analytically.

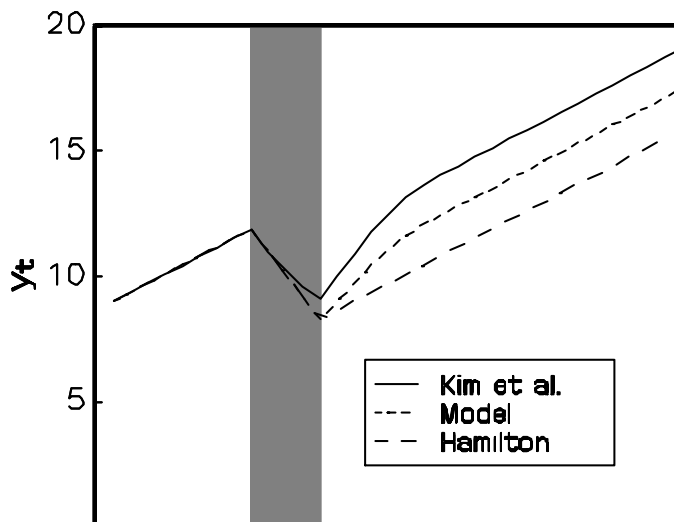


Figure 6: Long-run output level after a recession.

slope of the solid line (Kim et al.) *versus* the dashed line (our specification) at the beginning of expansion. Second, and most important, under their model there is a clear leveling off of output during a prolonged recession, which turns out statistically insignificant in our specification, and which implies that output drop during a given recession is lower in their specification (as it is evident from the solid line being above the dashed line at the end of recession). Leveling off of output during a prolonged recession implies contractions which are mirror images of expansions regarding the shape. Similarly to what we have found with business cycles features, things are not so neat when we consider second moments. Taking into account parameter uncertainty, something neglected in most of the works, strongly affects the standard error for the statistics of interest and makes the confidence interval relatively large.

5 Conclusions

In this paper, we estimate a generalized specification of a time varying transition probability Markov switching model for US industrial production index. The model, estimated using Bayesian methods and the Gibbs sampler, is able to capture important asymmetries. It generates posterior probabilities of being in recessions which correspond to the *NBER* dated recessions, consistent with duration asymmetries between cycle phases. It is also able to capture the presence of asymmetries in shape of the cycle. In particular, we can think about the shape of the cycle along two main dimensions. First, we can think about patterns of variation in growth rates over the course of expansions and recessions. Second, we can consider to which extent recessions are simply negative expansions. We find evidence to support the presence of a recovery early in expansions, as already found by Kim, Morley and Piger (2005). Although there

is some evidence that variations of growth rates have similar pattern in recessions, we cannot reject the null that recessions have linear shape, contrary to what is implied by Kim et al.'s (2005) model. When we investigate the ability of our specification to produce plausible business cycle features, where those features are the statistics proposed by Harding and Pagan (2002), we find that the model is able to capture most of them on average. However, the dispersion of their posterior distributions is relatively large. Finally, the effects of recessions on long-run output level implied by our specification are smaller than what Hamilton's model (1989) would imply, but greater than what Kim et al.'s (2005) model would predict.

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A Bayesian methods and the Gibbs sampler

Starting values $\left\{ \alpha^{(0)}, \phi^{(0)}, \sigma_1^{2(0)}, \gamma^{(0)}, \delta^{(0)}, \left\{ S_t^{(0)} \right\}_1^T, \left\{ V_t^{(0)} \right\}_1^T \right\}$ are required for initializing the simulation. The starting values for the sequence $\left\{ S_t^{(0)} \right\}$ are the *NBER* dates, while the starting values for $\left\{ V_t^{(0)} \right\}_1^T$ are derived according to the conclusions by McConnell and Perez-Quiros (2000) and Kim and Nelson (1999b) which document a structural break in US output volatility occurring in the mid 80's. A single iteration of the Gibbs sampler involves the following steps.

A.1 step (i): generating σ_0^2

Given values for α , S^T , let us define the new variable $\tilde{y}_t = \Delta y_t - \alpha/x_t$, where $x_t = [1 - s_t s_t \sum_{j=1}^m (1 - s_{t-j}) (1 - s_t) \sum_{j=1}^m (1 - s_{t-j})]'$ such that we can express equation (3.1) as:

$$\tilde{y}_t = \phi_1 \tilde{y}_{t-1} + \phi_2 \tilde{y}_{t-2} + \dots \phi_r \tilde{y}_{t-r} + \varepsilon_t, \quad t = \{1 + r, \dots, T\} \quad (\text{A.1})$$

Given values for ϕ , V^T and k , we generate σ_0^2 from the following inverse-gamma distribution:

$$\sigma_0^2 \sim IG\left(\frac{\bar{v}_0}{2}, \frac{\bar{v}_0 \bar{\sigma}_0^2}{2}\right) \quad (\text{A.2})$$

$$\bar{v}_0 = \hat{v}_0 + T - r \quad (\text{A.3})$$

$$\bar{\sigma}_0^2 = \bar{v}_0^{-1} \left[\hat{v}_0 \hat{\sigma}_0^2 + \sum_{t=r+1}^T \left[\frac{\tilde{y}_t - \phi_1 \tilde{y}_{t-1} - \dots \phi_r \tilde{y}_{t-r}}{(1 + kv_t)^{0.5}} \right]^2 \right] \quad (\text{A.4})$$

where \hat{v}_0 and $\hat{\sigma}_0^2$ are respectively the degrees of freedom and the scale parameter of the inverse gamma prior distribution.

A.2 step (ii): generating σ_1^2

Given equation (A.1), and conditional on the realisation of σ_0^2 , we draw $(1+k)$ from the following inverse-gamma distribution

$$(1+k) \sim IG\left(\frac{\bar{v}_k}{2}, \frac{\bar{v}_k ssr}{2}\right) I_{(k>0)} \quad (\text{A.5})$$

$$\bar{v}_k = \hat{v}_k + T_k \quad (\text{A.6})$$

$$ssr = \bar{v}_k^{-1} \left[\hat{v}_k \hat{\sigma}_k^2 + \sum_{t=r+1}^{T_k} v_t \left[\frac{\tilde{y}_t - \phi_1 \tilde{y}_{t-1} - \dots \phi_r \tilde{y}_{t-r}}{\sigma_0} \right]^2 \right] \quad (\text{A.7})$$

where $I_{(k>0)}$ is an indicator function which allow us to identify the high variance regime, and T_k is the number of elements in T for which $v_t = 1$, \hat{v}_k and $\hat{\sigma}_k^2$ are respectively the degrees of freedom and the scale parameter of the inverse gamma prior distribution. The high variance state is then given by:

$$\sigma_1^2 = \sigma_0^2(1+k) \quad (\text{A.8})$$

A.3 step (iii): generating ϕ

Conditional on realisations for σ_0^2 , and $(1+k)$, and given the variable \tilde{y}_t defined in (A.1), we define the new variable

$$\hat{y}_t = \frac{\tilde{y}_t}{\sigma_0(1+kv_t)^{0.5}} \quad (\text{A.9})$$

such that equation (3.1) takes the following form:

$$\hat{y}_t = \phi_1 \hat{y}_{t-1} + \dots + \phi_r \hat{y}_{t-r} + \varepsilon_t, \quad t = \{1+r, \dots, T\} \quad (\text{A.10})$$

Define \hat{Y} to be the matrix of the right hand side variables, and \hat{y} to be the vector of the left hand side; we draw ϕ from the following normal distribution:

$$\phi \sim N(\bar{\phi}, \bar{A}_\phi) I_{|\phi| < 1} \quad (\text{A.11})$$

$$\bar{A}_\phi = (\hat{A}_\phi^{-1} + \hat{Y}'\hat{Y})^{-1} \quad (\text{A.12})$$

$$\bar{\phi} = \bar{A}_\phi(\hat{A}_\phi^{-1}\hat{\phi} + \hat{Y}'\hat{y}) \quad (\text{A.13})$$

where $I_{|\phi| < 1}$ is an indicator function ensuring stationarity. \hat{A}_ϕ and $\hat{\phi}$ are respectively the variance and the mean of the prior normal distribution.

A.4 step (iv): generating α

Conditional on realisations for σ_0^2 , $(1+k)$ and ϕ , and given values for S^T and V^T , we can define the following new variables:

$$\ddot{y}_t = \frac{\Delta y_t - \phi_1 \Delta y_{t-1} \dots - \phi_r \Delta y_{t-r}}{\sigma_0(1+kv_t)^{0.5}} \quad (\text{A.14})$$

$$\ddot{x}_{it} = \frac{x_{it} - \phi_1 x_{it-1} \dots - \phi_r x_{it-r}}{\sigma_0(1+kv_t)^{0.5}} \text{ for } i = 0, 1, 2, 3 \quad (\text{A.15})$$

where x_{it} is the i^{th} regressor of (3.1), such that we can express equation (3.1) in the following form:

$$\ddot{y}_t = \alpha_0 \ddot{x}_{0t} + \alpha_1 \ddot{x}_{1t} + \alpha_2 \ddot{x}_{2t} + \alpha_3 \ddot{x}_{3t} + \varepsilon_t \quad (\text{A.16})$$

We define \ddot{X} to be the matrix of the right hand side variables, then we draw the vector α from the following normal distribution:

$$\alpha \sim N(\bar{\alpha}, \bar{A}_\alpha) I_{\alpha_1 > 0} \quad (\text{A.17})$$

$$\bar{A}_\alpha = (\hat{A}_\alpha^{-1} + \ddot{X}'\ddot{X})^{-1} \quad (\text{A.18})$$

$$\bar{\alpha} = \bar{A}_\alpha(\hat{A}_\alpha^{-1}\hat{\alpha} + \ddot{X}'\ddot{y}) \quad (\text{A.19})$$

where $I_{\alpha_1 > 0}$ is an indicator function which allows the identification of the two regimes. \hat{A}_α and $\hat{\alpha}$ are respectively the variance and the mean of the prior normal distribution.

A.5 step (v): generating S^T

Following Albert and Chib (1993) and Filardo and Gordon(1998), we draw $\{s_t\}_1^T$ from a multinomial Bernoulli distribution. Let $Y^n = \{\Delta y_1, \dots, \Delta y_n\}$, $S^n = \{s_1, \dots, s_n\}$ and $S_{-t}^T = \{s_0, \dots, s_{t-1}, s_{t+1}, \dots, s_T\}$ the full conditional distribution for $\{s_t\}_1^T$ is given by:

$$P(s_t|Y^T, Z^T, S_{-t}^T) \propto P(s_t|s_{t-1}, z_t)P(s_{t+1}|s_t, z_{t+1})$$

$$\times P(\Delta y_t, \dots, \Delta y_r | Y^{t-1}, S^r) \prod_{j=r+1}^{t+r} f(\Delta y_j | Y^{j-1}, S^j), \quad t \leq r \quad (\text{A.20})$$

$$P(s_t|Y^T, Z^T, S_{-t}^T) \propto P(s_t|s_{t-1}, z_t)P(s_{t+1}|s_t, z_{t+1})$$

$$\times \prod_{j=t}^{t+r} f(\Delta y_j | Y^{j-1}, S^j), \quad r+1 \leq t \leq T-r+1 \quad (\text{A.21})$$

$$P(s_t|Y^T, Z^T, S_{-t}^T) \propto P(s_t|s_{t-1}, z_t)P(s_{t+1}|s_t, z_{t+1})$$

$$\times \prod_{j=t}^T f(\Delta y_j | Y^{j-1}, S^j), \quad T-r \leq t \leq T \quad (\text{A.22})$$

Draws for values of s_t can be made backwards from $t = T$ to $t = 1$ from a series of Bernoulli distributions, using the probabilities generated by equations (A.20), (A.21), (A.22).

A.6 step (vi) and (vii): generating $\{s_t^*\}_1^T$ and γ

Conditional on the realisations of s_t , equation.(3.2) determining the latent variable s_t^* becomes a probit model. Therefore, given the vector γ , values of s_t^* can be drawn by the following truncated normal distribution:

$$s_t^*|s_{t=1} \sim I_{s_t^* \geq 0} N(\gamma_0 + \gamma'_z z_t + \gamma_s s_{t-1}, 1) \quad (\text{A.23})$$

$$s_t^*|s_{t=0} \sim I_{s_t^* < 0} N(\gamma_0 + \gamma'_z z_t + \gamma_s s_{t-1}, 1) \quad (\text{A.24})$$

where $I_{s_t^*}$ is an indicator function to ensure that the condition $P(s_t^* > 0) = P(s_t = 1)$ holds, namely to ensure that s_t^* is non negative when $s_t = 1$, and negative when $s_t = 0$. Conditional on the simulated values for s_t^* , equation (3.2) becomes a linear regression model with unit

variance. Let us define W to be the matrix of the right hand side variables and given the prior distribution, the posterior distribution of γ has the following normal form:

$$\gamma \sim N(\bar{\gamma}, \bar{A}_\gamma) \quad (\text{A.25})$$

$$\bar{A}_\gamma = (\hat{A}_\gamma^{-1} + W'W)^{-1} \quad (\text{A.26})$$

$$\bar{\gamma} = \bar{A}_\gamma(\hat{A}_\gamma^{-1}\hat{\gamma} + W's^*) \quad (\text{A.27})$$

where \hat{A}_γ and $\hat{\gamma}$ are the prior variance and prior mean, respectively.

A.7 step (viii), step (ix) and step (x): generating V^T , $\{v_t^*\}_1^T$ and δ

Following steps (v), (vi) and v(ii), with appropriate changes on the parameters we condition on, it is possible to draw the time series V^T , $\{v_t^*\}_1^T$, and the vector δ , respectively. Given the normal prior distribution for δ , the posterior has the following normal distribution:

$$\delta \sim N(\bar{\delta}, \bar{A}_\delta) \quad (\text{A.28})$$

$$\bar{A}_\delta = (\hat{A}_\delta^{-1} + W'_v W_v)^{-1} \quad (\text{A.29})$$

$$\bar{\gamma} = \bar{A}_\delta(\hat{A}_\delta^{-1}\hat{\delta} + W'_v v^*) \quad (\text{A.30})$$

where \hat{A}_δ and $\hat{\delta}$ are the prior variance and prior mean, respectively, W'_v is the matrix of right hand side variables and v^* is the vector of left hand side variable of the following equation:

$$v_t^* = \delta_0 + \delta_v v_{t-1} + \xi_t \quad (\text{A.31})$$

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