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BRAIN GAIN VS. BRAIN DRAIN. THE EFFECTS OF UNIVERSITIES' MOBILE STUDENTS ON TERRITORIAL INEQUALITIES

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Brain gain vs. brain drain. The effects of universities' mobile students on territorial inequalities

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Abstract

Our study examines the relationship between university student mobility and local economic dynamics. Universities are pivotal in shaping societies and economies as hubs of knowledge creation, innovation, and cultural exchange. While recent research underscores the significant impact of university students on local development, there is a notable gap in understanding the distinct effects of mobile versus resident students on the local economy. Using data from 90 NUTS3 provinces in Italy between 2013 and 2019, we investigate the spatial inequalities generated by student mobility. Our focus is on second-level university students, who are closer to entering the labor market and thus have a more immediate impact on the local economy. Employing a standard fixed effects growth model, our findings reveal that incoming students significantly boost the economic growth of the destination province, particularly in the Center-North regions (brain gain). Conversely, the southern provinces suffer reduced growth due to the loss of talented students (brain drain). Thus, student mobility exacerbates the enduring spatial disparities in Italy contributing to uneven economic development across regions.

Keywords and phrases: mobile university students, brain drain, spatial disparities, growth model
Jel Classification: I25, J61, O47, R11, C23

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1 Introduction

Both market power and financial friction negatively affect the output economy. Financial frictions, such as limited access to capital and liquidity constraints, can severely impact firm performance, especially in environments where the threat of bankruptcy looms large. Bankruptcy leads to the inefficient liquidation of assets, loss of organizational capital, and disruption of supplier and customer relationships, resulting in substantial economic losses. Key studies highlighting the adverse effects of financial frictions include Bernanke, Gertler, and Gilchrist (1999), who emphasize the impact of credit market imperfections on business cycles, and Kiyotaki and Moore(1997), who illustrate how credit constraints can amplify economic fluctuations. In environments where firms face significant financial frictions, the risk of bankruptcy becomes more pronounced, leading to resource losses that can propagate through the economy. For instance, a study by Bernstein, Colonnelli, Giroud and Iverson (2019) shows that bankruptcy can lead to significant job losses and decreased wages for employees, which can have a ripple effect on local economies.

At the same time, lower competition may produce a negative welfare effect. Recent empirical evidence suggests that firms have increased their market power in recent years. De Loecker et al. (2020) analyzed firm-level data to estimate the evolution of the aggregate markup from 1955 to 2016. Their results show that, while the aggregate markup was relatively constant from 1955 to 1980, it increased from 1980 (21% of the marginal cost) to 2016 (61%). This trend is consistent with the distribution of markups among companies and their productivity, thus providing evidence that market power has increased. Similarly, Diez et al. (2018) obtained comparable results using the same approach but analyzed both listed and private firms. They find an inverse U-shaped relationship between investment and market power and a monotonic negative relationship between market power and labor share. Further evidence of the negative welfare effects of increased market power comes from Autor et al. (2020), who provide comprehensive data showing that as market power increases, labor's share of income declines. This trend indicates that increased market power allows firms to capture a larger portion of economic gains at the expense of workers. Gutiérrez and Philippon (2017) also highlight that rising market power is associated with reduced business dynamism and lower rates of investment, which stifles innovation and long-term economic growth. Moreover, Covarrubias, Gutiérrez, and Philippon (2019) demonstrate that higher market concentration and increased market power have led to significant reductions in new firm entry rates and overall economic dynamism. These trends suggest that dominant firms can stifle competition by creating barriers to entry, reducing the overall level of competitive pressure in the market.

Recent empirical evidence suggests that common ownership, where institutional investors hold shares in multiple competing firms, has had negative effects on competition. Azar, Schmalz, and Tecu (2018) provide a foundational study showing that common ownership in the airline industry leads to higher ticket prices, indicating reduced competitive pressures. Their analysis highlights how common ownership can result in coordinated behavior among competing firms, leading to anticompetitive outcomes such as higher prices and reduced output[1].

[1] See Schmalz(2018,2021) for more details.

Both financial frictions and market power individually have negative effects on economic performance, yet their interplay remains ambiguous. Firms with higher levels of market power are generally more profitable and thus less likely to face bankruptcy. This suggests that market power could serve as a preventive mechanism against bankruptcies, potentially reducing their occurrence and yielding a positive effect for society as a whole. The aim of this paper is to investigate whether there exists an social optimal level of market power in the presence of financial frictions. By examining the dynamics between market power due to common ownership and financial frictions, this paper seeks to understand the conditions under which aggregate welfare can be maximized. This study examines how firms' market power, stemming from a common ownership structure, can affect general equilibrium in imperfect markets.

To achieve this, we introduce uncertainty into the model proposed by Azar and Vives (2021) by incorporating identical and uniformly distributed idiosyncratic shocks to consumer preferences. In our setup, firms determine their production plans and workers decide on their labor supply before the shocks' realization, with consumption decisions occurring afterward. Furthermore, we incorporate financial frictions by implementing the costly state verification framework as detailed in Bolton and Scharfstein (Chapter 5). Our findings suggest that under certain conditions, common ownership within sectors can have a positive effect. Specifically, when the percentage of credit recovered in bankruptcy is low, a more concentrated common ownership structure lowers credit costs, outweighing the surplus losses caused by market power, and resulting in positive effects. Conversely, when the percentage of credit recovered in bankruptcy is high, higher levels of common ownership reduce the likelihood of financial distress, but this benefit is offset by a decrease in output and workers' wages, leading to an overall negative effect. By parametrizing, we find that this optimal level of common ownership, which maximizes social welfare, does indeed exist. However, the level of common ownership observed in reality is higher than this optimal level. This discrepancy helps explain why previous literature has predominantly found negative effects associated with common ownership structures.

Our study makes significant contributions to the literature by exploring the interaction between financial frictions and market power (Jungherr and Stauss, 2017; Gale, 2019; Casares et al., 2023) as well as the macroeconomic effects of common ownership. The closest related work is by Casares et al. (2023), who also examine the interaction between market power and financial frictions, specifically considering scenarios where both non-verifiability of expected returns and moral hazard are present. However, our study diverges significantly in its approach. While Casares et al. (2023) model firms operating under monopolistic competition—where market power is solely derived from the elasticity of substitution across varieties, conflating market power with consumer preferences—our model attributes market power to ownership structure. This allows us to distinctly separate the concepts of consumer preferences and market power.

The paper is structured as follows. In section two, we discuss the theoretical model and the modifications made to the model of Azar and Vives (2021). In section three, we derive the equilibrium, and in part four, we solve the model numerically, comparing the results obtained from our model with data related to the U.S. economy.

2 Model setup

Consider an economy with a mass of $\mathbb{I}$ independent and identical islands, with no trade occurring between them. On each island, there are two types of people: owners and workers, both of whom live for two periods, and each type has a mass of N. There is a mass one of banks that can operate across all islands. In each islands there are N sectors, with each sector producing a distinct variety of consumption goods. Within each sector, there are J firms that produce the same good. To increase the number of sectors, the population must be proportionally increased[2].

Workers are endowed with an amount of time T, which they can allocate between leisure and labor. The utility function of worker i is given by

$$U^w(C_i^w, L_i) = C_i^w - \chi \frac{L_i^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \tag{1}$$

where

$$C_i^w = \left[\sum_{n=1}^{N} \left(\frac{1}{N} \right)^{\frac{1}{\theta}} (e_n)^{\frac{1}{\theta}} \left(c_{ni}^w \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where c_{ni}^w is the consumption of worker i of variety n, L_i is the amount of labor suplied by workers i, $\chi > 0$ is a parameter that weights labor disutility, $\theta > 1$ is the elasticity of substitution across variety, $\eta > 0$ is the elasticity of labor supply and e_n is an idiosyncratic shock of consumers' preference of variety n. We assume that shock is common across people and it is uniformly distributed between $[\underline{e}, \overline{e}]$.

Owners are endowed with the property of groups which hold the firms' share and their

[2] See Azar and Vives (2021) for more details.

only source of income comes from these holdings. They do not work and derive utility from consumption according to the following utility function $U^o(C_i^o) = C_i^o$. There is one group for each firm and, differently from Azar and Vives (2021) we allow for common ownership only within the sector. The ownership structure is built such that each group nj directly owns a share $(1 - \phi) \geq 0$ of firm nj and, an industry index of all firms in the same sector sector (ϕ/J). In this case $\phi \in [0, 1]$ [3] represents the level of portfolio diversification in the single sector. When ϕ equals zero, there is perfect competition among firms within each sector for sufficiently large J. Conversely, when ϕ equals one, the firms within the sector behave as one.

2.0.1 Agents behavior

Timing

The timing of our economy is the following:

- at t=0, firms decide how much to produce and workers decide how much labor to supply to finance consumption. Given production plans firms will borrow the financial resources needed to pay wages. The workers will receive the salaries and they will deposit them into the banks.
- at t=1, preference shocks occur and people decide, given the income obtained in the previous period, how much to consume of each variety. The firms that obtained sufficient revenue repaid their debt contracts and, the others were audited and liquidated.

[3] This model is equivalent to Azar and Vives (2021) multisectoral model where ϕ=0. Notice that their model ϕ represents the level of diversification in the whole economy while in this model represents the level of diversification in the industry index. The corresponding of my ϕ in their model is $\widetilde{\phi}$

The function that aggregates consumption across all sectors is homothetic, which implies that decisions regarding the allocation of consumption among different varieties and the total amount of consumption are separable. Consequently, the optimal choice at time 1 is independent of the optimal choice at time 0. Thus, workers determine their consumption levels after shocks have occurred, having already made the optimal allocation decision of time between leisure and work. This means that the optimal consumption choice is independent of the optimal labor supply decision. At time 0, workers anticipate their optimal choices at time 1 and decide how much time to allocate to work, taking into account their future consumption decisions. This problem is therefore solved using backward induction.

Workers problem at time 1

At time 1, workers take labor supply L_i, which they decided at time 0, and the labor income wL_i as given, therefore, worker i chooses how much to consume by solving the following problem

$$\max_{c^w_{ni}} \qquad C^w_i = \left[\sum_{n=1}^{N} \left(\frac{1}{N}\right)^{\frac{1}{\theta}} (e_n)^{\frac{1}{\theta}} \left(c^w_{ni}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

$$s.to.$$

$$(1+r_d)(wL_i) = \sum_{n=1}^{N} p_n c^w_{ni}. \tag{4}$$

Solving the above problem we get the optimal level of consumption of variety n

conditional to shock realization and aggregate level of consumption

$$c_{ni}^{w} = \frac{1}{N} e_n \left(\frac{p_n}{P}\right)^{-\theta} C_i^{w}. \tag{5}$$

From previous problem arise also that total expenditure is equal to price index multiplied the total level of consumption

$$PC_i^{w} = (wL_i)(1 + r_d). \tag{6}$$

The owners' demand conditional on their aggregate level of consumption is identical to those of workers. Integrating the individual demand for each agents we obtain the aggregate demand

$$\underbrace{\int_{I_w \bigcup I_o} c_{ni} di}_{c_n} = \frac{1}{N} e_n \left(\frac{p_n}{P}\right)^{-\theta} \underbrace{\int_{I_w \bigcup I_o} C_i di}_{C}. \tag{7}$$

Using this equation we can find the relative prices in a competitive equilibrium

$$\frac{p_n}{P} = \left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}} \left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}. \tag{8}$$

In a competitive equilibrium total production is equal to total consumption

$$c_n = \sum_{j=1}^{J} F(L_{nj}). \tag{9}$$

In this case the relative price of variety n relative to production plans is given by the following equation

$$\frac{p_n}{P} = \left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}} \left(\frac{\sum_{j=1}^{J} F(L_{nj})}{\left[\sum_{m=1}^{N} \left(\frac{1}{N}\right)^{\frac{1}{\theta}} (e_m)^{\frac{1}{\theta}} \left(\sum_{j=1}^{J} F(L_{mj})\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}} \right)^{-\frac{1}{\theta}} . \tag{10}$$

At time 1, production schedules are already fixed and the shocks' realization only affects relative prices. The relative price of sector n depends on the realization of the shock associated with variety n but, also on the realization of all $(N-1)$ shocks. As we show in the appendix, a positive shock of variety n produces an increase in the relative price of that specific variety and, at the same time, generates an increase in the price level that makes it possible for all other firms producing variety $m \neq n$ to increase their prices. The intensity of this relationship depends on the elasticity of substitution among varieties. At a higher level of the elasticity of substitution between varieties, the impact of a positive shock of variety n on the relative price of variety n and m decreases. This means that the impact on the relative price of variety n/m of a positive shock on variety n is zero for a sufficiently high value of θ. This is because when θ tends to infinite all products become close to perfect substitutes and it is like in the economy there is just one market and its relative price is equal to one.

Workers equilibrium at time 0

At time 0, workers choose how much to consume and how much labor to supply to maximize their expected utility. Workers maximize their expected utility taking into account the optimal choice that they make at time 1. The problem faced by each worker

i is

$$\max_{C_i^w, L_i} \qquad \mathbb{E}(U^w) = \mathbb{E}\left(C_i^w - \chi \frac{L_i^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right) \tag{11}$$

$$s.to.$$

$$PC_i^w = (wL_i)(1+r_d). \tag{12}$$

By defining P as the numeraire good and normalizing it to one, we have that $C_i^w = (wL_i)(1+r_d)$ and substituting it into $\mathbb{E}(U)$, we can rearrange the maximization problem such that it dependence only on L_i. The solution of this problem gives us the individual inverse labor supply as a function of wage

$$w = \chi L_i^{\frac{1}{\eta}} \frac{1}{(1+r_d)}. \tag{13}$$

Since workers are identical, the aggregate inverse labor supply is

$$w = \chi \left(\frac{L}{N}\right)^{\frac{1}{\eta}} \frac{1}{(1+r_d)}. \tag{14}$$

3 Firms' problem

Firms need a period to produce consumer goods, so they set their production schedule at time 0 before the shocks' realization. The profit of firm nj is

$$\Pi_{nj} = p_n F(L_{nj}) - wL_{nj}(1+r_l), \tag{15}$$

where $F(L_{nj}) = AL_{nj}^{\alpha}$ is the production function, A is the technology of production, w is the wage, p_n is the nominal price of variety n, $(1 + r_l)$ is the unity cost of credit and $\alpha \in [0, 1]$ is the output elasticity respect to labour.

Managers maximize the real value of shareholders' wealth. In this case, we assume that there is common ownership only within the sector. At time 1, the wealth of generic group nj is equal to

$$\Pi_{nj} + \lambda \sum_{k \neq j}^{J} \Pi_{nk}, \tag{16}$$

where λ represents the weights that each firm assign to the profit of other firms in the same sector respect to their own profit and it is equal to

$$\lambda = \frac{(2 - \phi)\phi}{(1 - \phi)^2 J + (2 - \phi)\phi}.$$

An higher level of λ indicate lower level of competition within sector lending to higher level of firms' market power. The value of lambda is a function of ϕ and J. When the level of portfolio diversification ϕ in the industry index tends to one the value of λ tends to one too and the economy converges to Dixits and Stiglitz outcome for if the number of sectors tend to infinity.

Since prices are affected by shocks' realization, the real profit of firm nj is a random variable so, the objective function that the manager of firm nj maximizes is the expected value of the function (17) conditional on success

$$pds \left(\mathbb{E}(\Pi_{nj}|succ) + \lambda \sum_{k \neq j}^{J} \mathbb{E}(\Pi_{nk}|succ) \right), \tag{17}$$

The problem faced by managers of firm nj is the following

$$\max_{L_{nj}} \ \mathbb{E}(p_n|succ)F(L_{nj}) - wL_{nj}(1+r_l) + \lambda \left[\sum_{k\neq j}^{J} \mathbb{E}(p_n|succ)F(L_{nk}) - wL_{nk}(1+r_l) \right] \tag{18}$$

The FOC of this problem is

$$\begin{aligned} &\mathbb{E}\left(p_n|suc\right)F'(L_{nj}) - w(1+r_l) - \frac{d\mathbb{E}(\omega)}{dL_{nj}} \left[L_{nj} + \lambda \sum_{k\neq j}^{J} L_{nk} \right] \\ &+ \frac{d\mathbb{E}\left(p_n|succ\right)}{dL_{nj}} \left[F(L_{nj}) + \lambda \sum_{k\neq j}^{J} F(L_{nk}) \right] = 0. \end{aligned} \tag{19}$$

Solving the FOC we obtain the inverse of labor demand as a function of expected markdown and expected prices

$$w = \frac{\mathbb{E}\left(p_n|succ\right)F'(\frac{L^d}{JN})}{(1+\mathbb{E}(\mu|succ))}. \tag{20}$$

3.0.1 Banks' problem

Banks compete each other à la Bertrand. They provide loans at the firms, asking a nominal interest rate equal to r_l and they collect deposit from workers paying a nominal interest rate equal to r_d. Since return is not freely verifiable, the optimal financing contract takes the form of standard debt contract. According this contract, banks receive r_l per unity of loan and, in case of bankruptcy they can achieve the whole cash flow. To undertake an audit process banks must bear a cost that is proportional to cash flow size. We assume $k \in [0,1]$ represents the fraction of cash flow that banks are able to extrapolate during the liquidation process. Banks participation constraint in the generic island i is

$$\mathbb{E}(vl) \ \geq (1+r_f)w\frac{L}{JN} \tag{21}$$

where $(1+r_f)$ is the return of free risk investment and vl is the value of loan and it is equal to

$$vl = \begin{cases} p_n \ A \ \left(\frac{L}{JN}\right)^\alpha \ k & \text{if } \widehat{e_n} > e_n \\ w \ (1+r_l) \ \frac{L}{JN} & \text{if } \widehat{e_n} \leq e_n \end{cases} \tag{22}$$

where $\widehat{e}_n$ is the realization of critical shock value such that firms n in a generic island make zero profit. Since banks can invest in all islands they can fully diversify their risk and will set their participation constraints at a global level. By the law of large numbers, although on each island the realization of the critical shock will be different, it will converge on the mean to its expected value. This means that by the law of large numbers the ex-post realization of $\widehat{e}_n$ converges in mean to its expected value. The expected value of the loan conditional on the probability of success or failure in aggregate will be equal to the value of the portfolio of banks that can invest in all islands. The aggregate conditional value of loan is

$$\mathbb{E}\left[p_n \ A \ \left(\frac{L}{JN}\right)^\alpha k|\widehat{e}_w > e_n\right] F(\widehat{e}_w) + w \ (1+r_l) \ \frac{L}{JN} \left(1 - F(\widehat{e}_w)\right) \tag{23}$$

Combining (23) and (21) with some algebraic rearrangement we obtain

$$(1+r_l) = \frac{(1+rf)}{pds} - \frac{pdf}{pds}\frac{A\mathbb{E}(p_n|fail)k(JN)^{1-\alpha}}{w(L)^{1-\alpha}} \tag{24}$$

where *pdf* is the probability of default and is equal to $F(\widehat{e}_w)$, pds is the probability of success and it is equal to $1 - pdf$ and $\mathbb{E}(p_n|fail)$ is the expected value of nominal price of variety n given default. The loss for the economy in this case is represented by:

$$loss = (1-k)\mathbb{E}(p_n|fail)A\left(\frac{L}{JN}\right)^{\alpha} \tag{25}$$

Notice that loss is increasing on total employment. Firms sign standard debt contracts to finance consumption so when the labor force asked to produce decreases there is also a decrease in financial resources that firms needed whit consequently a reduction in standard debt contract value. Obviously, when the shock critical value tends toward its minimum value the loss for the economy tends to be zero.

3.1 General Equilibrium

The conditions of Azar and Vives (2021) hold and it guarantees the existence of unique, symmetric and local stable equilibrium where:

Definition 1 *A competitive equilibrium relative to* $(L_1, ..., L_J)$ *is a price system and allocation* $[w, p_n, r_l, r_d\}; \{C_i, L_i\}_{i\in I_w}, \{C_i\}_{i\in I_O}]$ *such that the following statements hold:*

(i) For $i \in I_W, (C_i, L_i)$ *maximizes* $\mathbb{E}[(C_i, wL_i)]$ *subject to* $pC_i \leq wL_i(1 + r_d)$ *for* $i \in I_O, W_i/P$*;*

(ii) Labor supply equals labor demand by the firms: $\int_{i\in I_W} L_i di = \sum_{j=1}^{J} L_j$*;*

(iii) Total consumption equals total production: $\int_{i\in I_W\cup I_O} C_i di = \sum_{j=1}^{J} F(L_j)$*.*

(iv) Deposit rate is equal to free risk rate: $r_d = r_f$

The conditions of Azar and Vives (2021) hold and it guarantees the existence of unique, symmetric and local stable equilibrium where:

a) the total level of employment is

$$L^* = \left[\frac{1}{\chi} J^{1-\alpha} \frac{\mathbb{E}\left(p_n|succ\right)\alpha A}{\left(1+\mathbb{E}(\mu|succ)\right)} \frac{(1+r_d)}{(1+r_l)} \right]^{\frac{1}{\frac{1}{\eta}+1-\alpha}} N; \tag{26}$$

b) the value of expected real wage in equilibrium is

$$w = \left[\left(\chi \frac{1}{(1+r_f)} \right)^{\eta} \left(\frac{\mathbb{E}\left(p_n|succ\right)\alpha A}{\left(1+\mathbb{E}(\mu|succ)\right)(1+r_l)} \right)^{\frac{1}{1-\alpha}} J \right]^{\frac{1}{\eta+\frac{1}{1-\alpha}}}; \tag{27}$$

c) the expected markdown of real wage is

$$\mathbb{E}(\mu|succ) = \frac{\frac{1}{\eta} H_{labor} + 1}{1 - \frac{\tau\left(H_{product}\right)}{\mathbb{E}(p_n|succ)}} - 1. \tag{28}$$

d) the equilibrium cost of credit is

$$(1+r_l) = \frac{(1+rf)}{pds} \frac{1}{\left(1 + \frac{pdf}{pds} \frac{A\mathbb{E}(p_n|fail)k(JN)^{1-\alpha}}{w^{wf}(L^{wf})^{1-\alpha}}\right)} \tag{29}$$

where L^{wf} and w^{wf} represent the level of total employment and the wage in the frictionless economy, $H_{product} = \left[\frac{1}{J} + \lambda\frac{(J-1)}{J}\right]$ and $H_{labor} = \left[\frac{1}{JN} + \lambda\frac{(J-1)}{J}\right]$ are respectively the modify HHI index of labor market and product market.

At time 1, the firms' zero profit condition under symmetric equilibrium can be represented by the following equation:

$$p_n \; A \left(\frac{L^*}{JN}\right)^{\alpha} = w^* \; \frac{L^*}{JN} \; (1+r_l) \tag{30}$$

The left-hand side of the equation represents the income generated by the firm, while the right-hand side represents the cost of production. In a symmetric equilibrium, in which all enterprises produce the same quantity, the number of workers hired can be expressed in terms of total employment. To reach the zero profit condition, the revenue generated from the sale of the final good must equal the labor cost, represented by the standard debt contract. Since the relative prices of each variety depend on the outcome of various shocks, we aim to find the critical value of its specific variety shock such that the firm realizes zero profit. If the value falls below this threshold, the firm will have negative profits and will not be able to meet its debt obligations. Solving the above equation and taking the expectation we have

$$\mathbb{E}\left[\widehat{e_w}^{\frac{1}{\theta}}\left(\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\sum_{m\neq n} e_m^{\frac{1}{\theta}}+\widehat{e_w}^{\frac{1}{\theta}}\right)\right)^{\frac{1}{\theta-1}}\right]=\frac{1}{A}\left(\frac{L^{wf}}{JN}\right)^{1-\alpha} w^{wf} N^{\frac{1}{\theta}}. \qquad (31)$$

Due to the nature of the shocks' structure, we cannot solve this equation analytically. Therefore, in the next section, we will solve it numerically. Note that this equation does not depend on the friction structure, so the different type of friction or its intensity does not change the likelihood of default.

4 Numerical solution

In this section, we numerically solve the presented model and analyze how macroeconomic variables react to the intensity of financial frictions and market power. The model is solved using MATLAB, implementing a Monte Carlo simulation. The simulation considers a total of 10,000 islands. For each island, shocks are drawn from a uniform distribution, and the expected values are calculated as the means of the realizations of the various variables. According to the law of large numbers, the expected value approximates the mean of the realizations.

Following the calibration provided by Azar and Vives (2021), we set the values as follows: $N = 100$, $J = 5$, $A = 0.4976$, $\alpha = \frac{2}{3}$, $\theta = 3$, $\eta = 0.59$, and $\chi = 0.3827$. To ensure that the expected value of the shocks is equal to one, the lower and upper bounds of the uniform distribution are set to 0 and 2, respectively. Additionally, the risk-free interest rate is set to 0, and the weight of workers' utility in the welfare function is set to 0.89, reflecting the proportion of the US population that is composed of workers. We set the value of λ equal to 0.124 that is the value estimated by Azar and Vives (2021) for the year 2017.

4.1 Common ownership effects in an economy with financial frictions

In this section, we analyze the interaction between financial frictions and market power resulting from common ownership. Figure 1 illustrates the effect of increasing common ownership under varying levels of liquidation technology. When banks can fully recover firm's generated cash flow it is evident that increasing common ownership consistently has a negative effect on the economy. Specifically, a higher degree of common ownership reduces competition within the market, leading to a decline in all equilibrium variables. Given that there are no bankruptcy costs, a higher level of market power results in a lower cost of credit and, at the same time, a wage reduction, and consequently also the optimal level of employment decreases.

Table 1: Model Parameters

Parameter Symbol	**Description**	**Value**	**Source**
$\mathbb{I}$	Number of islands	10000	
N	Number of sectors	100	Azar and Vives (2021)
J	Number of firms within sector	5	Azar and Vives (2021)
A	Firm productivity	0.4976	Azar and Vives (2021)
α	Output elasticity	$\frac{2}{3}$	Azar and Vives (2021)
θ	Elasticity of substitution across varieties	3	Azar and Vives (2021)
λ	Edgeworth sympathy coefficients	0.124	Azar and Vives (2021)
η	Labour elasticity	0.59	Azar and Vives (2021)
χ	Disutility of labor	0.3827	Azar and Vives (2021)
$\underline{e}$	Shock lower bound	0	
$\overline{e}$	Shock upper bound	2	
r_f	Risk-free interest rate	0	
τ	Weight of workers' utility in the welfare function	0.89	

Table 2: Equilibrium values

Variable	**Value**	**Variable**	**Value**
ϕ	0.4144	pdf	0.2888
$\mathbb{E}(pn\|succ)$	1.0457	pds	0.7112
$\mathbb{E}(pn\|fail)$	0.6052	$(1 + r_l)$	1.0806
L^{wf}	114.7888	w	0.6214
w^{wf}	0.6630	L	110.4868
$Revenues$	0.1902	$Costs$	0.1484
$Profits$	14.5552	U^w	45.9622
U^o	14.5552	W	28.8645
$\mathbb{W}^w/\mathbb{W}^o$	3.1578	$loss$	90.2853
$Owners\ share$	0.1876	$Labor\ share$	0.8124

As the efficiency of liquidation technology decreases, Figure (1) shows that a higher level of common ownership has a positive effect on economic variables. An increase in common ownership reduces the probability of failure, thereby relaxing the financial constraints of firms and allowing them to increase production, which consequently raises the wage received by workers. In this case, we can see that the level of market power that maximizes aggregate welfare is increasing on financial frictions intensity. When the liquidation technology allows for only half of the generated cash flow to be recovered, the optimal level is $\lambda = 0.08$, whereas when it allows for no recovery, the optimal level is $\lambda = 0.46$.

Given that the social welfare function might be influenced by the weights assigned to various agents in the economy, I performed the same analysis for GDP. Figure (2) shows that this positive effect of increased common ownership structure is also present for aggregate income, yielding similar results. In this scenario, we observe similar effects to those we have seen on aggregate welfare.

The rise in common ownership has significant effects on the redistribution of income and resources. An increase in market power leads to a reduction in the labor share and a corresponding increase in the owner share, effectively reallocating resources from workers to owners. However, as depicted in Figure(3), an increase in market power subsequently results in a decrease in aggregate loss, both in terms of welfare and GDP, attributable to bankruptcy costs. Figure(3) also shows how this loss is distributed between workers and owners depending on financial frictions intensity. When there is no common corporate structure, workers incur more than 97.09% (99.49% if we consider welfare) of the loss. As corporate concentration increases, this share decreases to 86.69%

(97.46% if we consider welfare). Thus, as market power grows, the loss in terms of GDP and welfare diminishes, and the share of the loss absorbed by owners increases. It is important to note that despite the higher absorption percentage by owners, the social costs remain predominantly borne by workers. This might suggest that, in an oligopolistic corporate structure, workers are disproportionately affected by the negative impacts of bankruptcy.

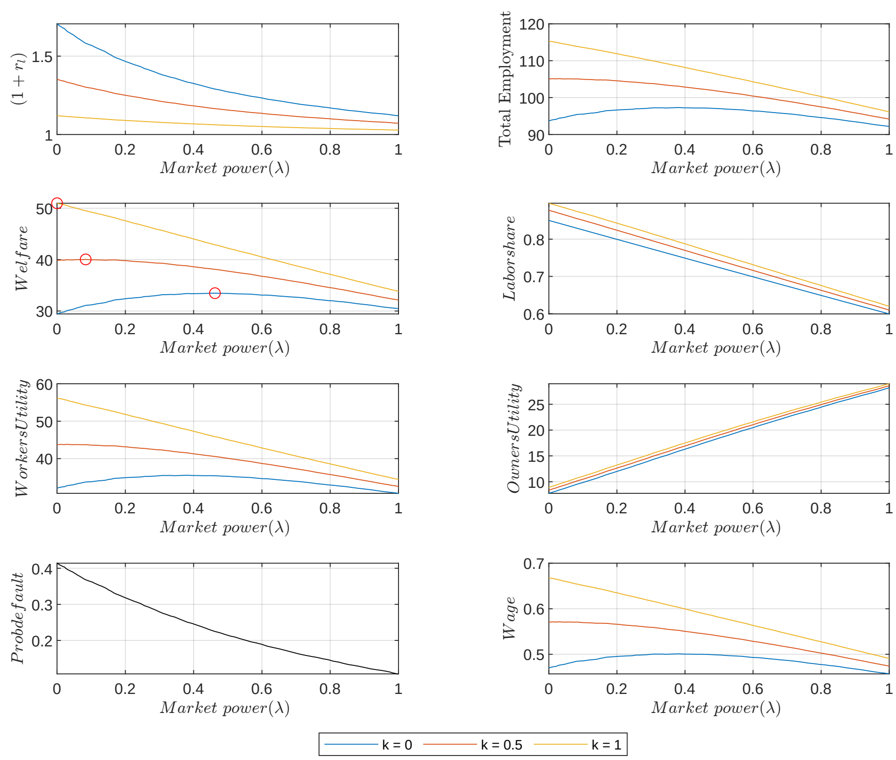

Figure 1: Equilibrium outcome under different levels of liquidation technology vs market power.

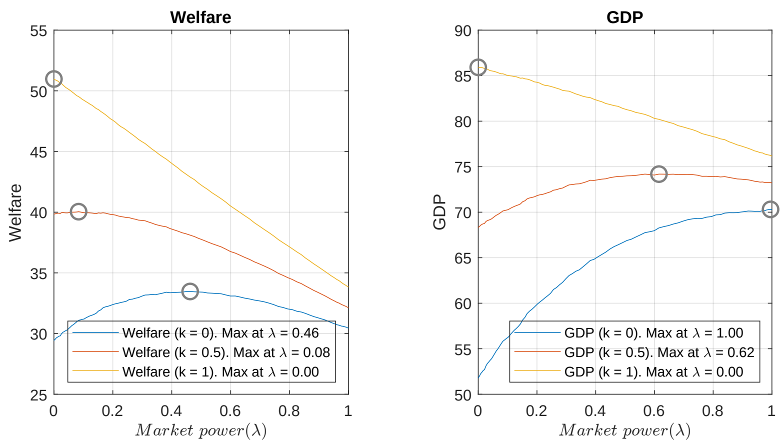

Figure 2: Optimal level of market power for Welfare and GDP under different level of efficiency of liquidation technology

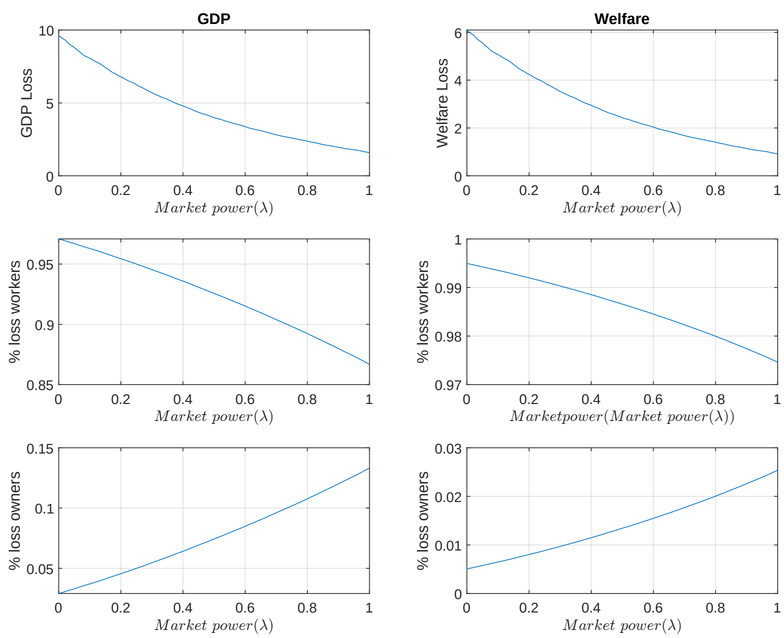

Figure 3: Loss due financial frictions on welfare and GDP vs market power

4.2 Financial frictions effects in an economy with common ownership

In this section, we examine the relationship between economic outcomes and financial frictions. Figure (4) shows how equilibrium variables change with the efficiency of the liquidation technology.

As the efficiency of the liquidation technology (k) increases, we observe a reduction in the interest rate $(1 + rl)$. This suggests that with better recovery technology, creditors demand a lower risk premium, reflecting a decreased likelihood of loan loss. Consequently, total employment rises as reducing financial frictions stimulates economic activity, allowing firms to expand production and hire more workers.

Social welfare also improves with the efficiency of the liquidation technology. Better liquidation technology implies low bankruptcy cost reducing the negative effects of financial frictions. Additionally, labor share shows a slight increase, indicating that a greater portion of the value added produced by firms is distributed to workers.

Workers' utility increases as well, with higher employment and wages contributing positively to their well-being. Owners' utility also rises, suggesting that both workers and owners benefit from better recovery technology, although to different extents. Aggregate income increases with the efficiency of the liquidation technology, as a more efficient financial environment allows for more productive use of capital and labor, boosting the total income of the economy.

As the efficiency of liquidation technology by banks increases, there is a shift in the allocation of resources within the economy, leading to an increase in the labor share and a corresponding decrease in the owner share. Figure(5) illustrates how bankruptcy losses vary with the intensity of financial frictions. Higher liquidation efficiency results in a reduction of costs, both in terms of welfare and GDP, approaching zero when efficiency is at its maximum. Even in this scenario, the burden of financial frictions predominantly falls on workers. However, the percentage they bear increased with reduced financial frictions, shifting from 94.26% (98.987 % in terms of welfare) when financial frictions are at their highest to 94.90% (99.01% in terms of welfare) when there are no financial frictions. This explains why the labor share increases as the intensity of financial frictions decreases. Since workers are the most affected by this negative effect, a more efficient recovery technology reduces loss. since this loss is proportionally greater for workers than for firms, this reduction results in an increase in the percentage of total income received by workers.

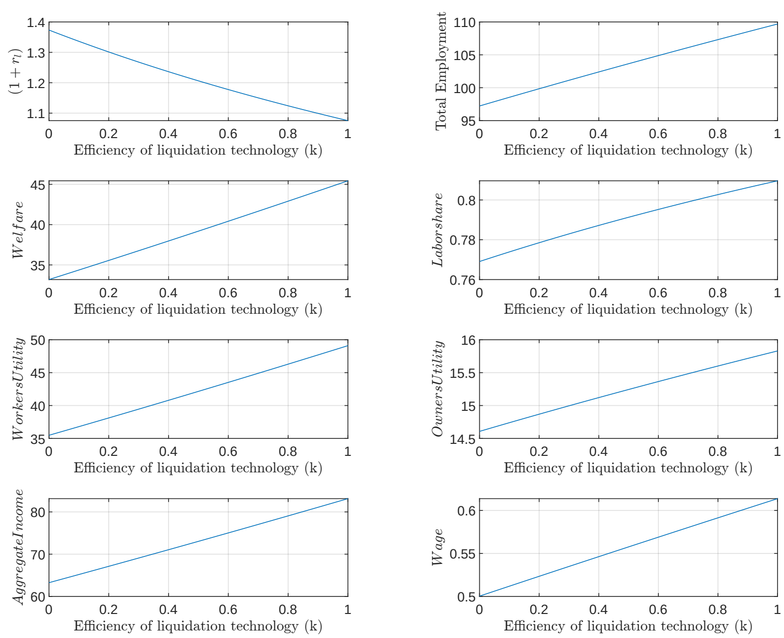

Figure 4: Equilibrium outcomes vs. efficiency of liquidation technology

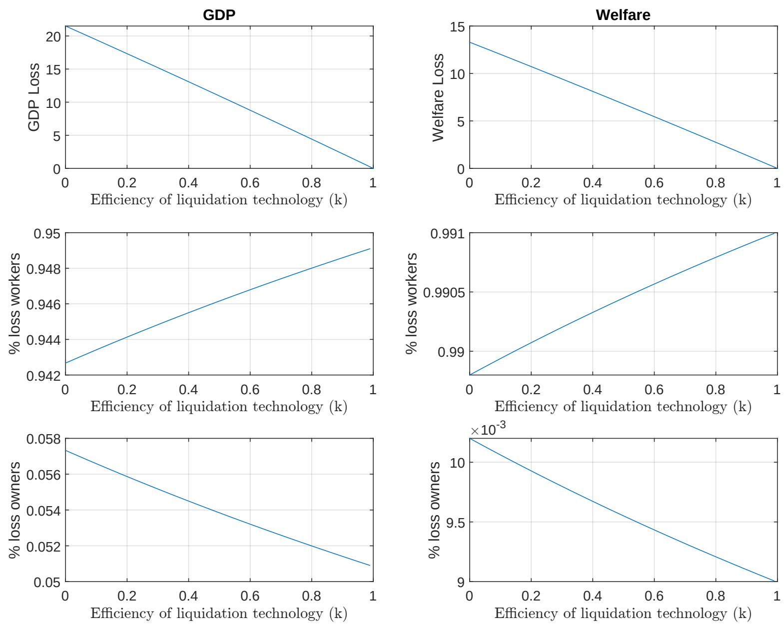

Figure 5: GDP and welfare loss vs efficiency of liquidation technology

4.3 Simulation using US data

To determine if this optimal level of market power is achieved in the U.S. economy, we examine the actual levels of common ownership and financial frictions present. Due to the lack of detailed data allowing for a precise estimation of common ownership levels in the U.S. market, we use data proposed by Azar and Vives (2021). In their paper, the authors estimate the evolution of lambda in the U.S. market. They estimate that for big firms, intra-group lambda values evolved from 0.41 in 1985 to 0.72 in 2017. Assuming other firms were not subject to a common ownership structure, they estimate a global evolution of lambda from 0.07 to 0.124 over the same period. For data regarding the efficiency of the liquidation technology we use the data regarding the percentage of loan

recovery given default published by S&P Global in their website [4]. Again, we do not have access to detailed data and the time window appears to be different here as well. According to this data, the average recovery level between 1990 and 2023 was 0.73.
In this section our purpose is not to quantify numerically the loss and gain from market power but only to try to understand whether there may have been scenarios in which the U.S. economy reached an optimal level of market power.

Figure(6) shows how the optimal lambda varies with the efficiency of liquidation technology. As liquidation efficiency increases, the level of market power that maximizes aggregate welfare decreases, approaching zero when efficiency is at its peak. The intersection line between the two areas indicates possible points where the level of market power, given the liquidation efficiency, was efficient. Considering the average loan recovery value of 73 percent we have that the level of market power resulting from the common corporate structure is always greater than its optimal. To try to get somewhat more precise estimates we selected a range of values in which the recovery rate might have moved. In the chart provided by S&P Global on their website, the recovery rate of loans given default ranges from a minimum of approximately 0.65 to a maximum of approximately 0.9. The figure (6) shows the level of optimal market power estimated by the model given a certain intensity of financial frictions. The points where the two areas join indicate the possible combination of market power and level of financial frictions that might have been realized in the U.S. economy. Again we see that the level of realized market power is always above the optimal level, which turns out to be zero in most of the selected air. When the recovery rate given default is between about 0.65

[4] https://www.spglobal.com/ratings/en/research/articles/231215-default-transition-and-recovery-u-s-recovery-study-loan-recoveries-persist-below-their-trend-12947167

and 0.68 there is an optimal positive level of market power that maximizes aggregate welfare but is still lower than any value of market power realized in the U.S. economy from 1985 to 2017. For the level of market power to be optimal for the economy the recovery rate of claims would have to be between 45 percent and 55 percent, 10 to 15 percentage points lower than those actually realized in the U.S. economy These results are consistent with Ederer and Pellegrino (2023) that estimates negative welfare effects of common corporate structures.

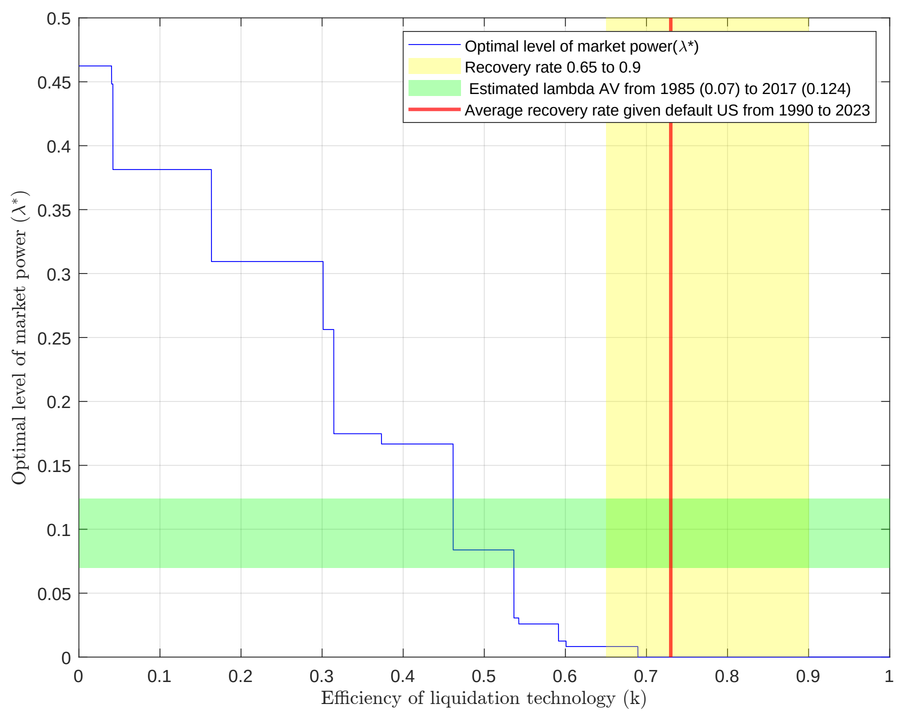

Figure 6: Optimal level of market power vs Recovery Rate (US Welfare)

5 Conclusion

In conclusion, using the framework by Azar and Vives (2021) and considering financial frictions like costly state verification, this paper has shown that increased market power from a more controlled corporate structure can positively impact welfare and aggregate income, especially when bankruptcy costs are high. However, as bankruptcy costs decrease, the optimal level of market power also decreases, eventually reaching zero if the bankruptcy cost is sufficiently low. Comparing these findings with empirical data from the American market reveals that the optimal level of market power is indeed zero. The negative effects noted in existing literature arise because market power exceeds this optimal level.

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Appendix

A Model solution

A.1 Workers' problem at time 1

At time 1 worker i solve the following problem

$$\max \quad C_i^w = \left[\sum_{n=1}^{N} \left(\frac{1}{N}\right)^{\frac{1}{\theta}} (e_n)^{\frac{1}{\theta}} \left(c_{ni}^w\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{A.1}$$

subject to

$$(1+r_d)wL_i = \sum_{n=1}^{N} p_n c_{ni}^w. \tag{A.2}$$

The Lagrangian is

$$\mathcal{L} = \left[\sum_{n=1}^{N} \left(\frac{1}{N}\right)^{\frac{1}{\theta}} (e_n)^{\frac{1}{\theta}} \left(c_{ni}^w\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} - \lambda \left(\sum_{n=1}^{N} p_n c_{ni}^w - wL_i(1+r_d)\right). \tag{A.3}$$

The FOC are

$$\frac{d\mathcal{L}}{dc_{ni}^w} = \left(\frac{1}{N}\right)^{\frac{1}{\theta}} \left(c_{ni}^w\right)^{\frac{\theta-1}{\theta}-1} \left[\sum_{n=1}^{N} \left(\frac{1}{N}\right)^{\frac{1}{\theta}} (e_n)^{\frac{1}{\theta}} \left(c_{ni}^w\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}-1} - \lambda p_n = 0; \tag{A.4}$$

$$\frac{d\mathcal{L}}{dc_{mi}^w} = \left(\frac{1}{N}\right)^{\frac{1}{\theta}} \left(c_{mi}^w\right)^{\frac{\theta-1}{\theta}-1} \left[\sum_{n=1}^{N} \left(\frac{1}{N}\right)^{\frac{1}{\theta}} (e_n)^{\frac{1}{\theta}} \left(c_{mi}^w\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}-1} - \lambda p_m = 0; \tag{A.5}$$

$$\frac{d\mathcal{L}}{d\lambda} = \sum_{n=1}^{N} p_n c_{ni}^w - wL_i = 0. \tag{A.6}$$

Taking the ratio between the first two equation, we obtain

$$\frac{p_n}{p_m} = \left(\frac{e_n}{e_m}\right)^{\frac{1}{\theta}} \left(\frac{c_{ni}^w}{c_{mi}^w}\right)^{-\frac{1}{\theta}}. \tag{A.7}$$

We solve for $c_n i$

$$c_{ni}^w = \frac{e_n}{e_m} \left(\frac{p_n}{p_m}\right)^{-\theta} c_{mi}^w. \tag{A.8}$$

Multiplying for p_n and adding up across n we obtain

$$\sum_n p_n c_{ni}^w = p_m^\theta c_{mi}^w \frac{1}{e_m} \sum_n p_n^{1-\theta} e_n. \tag{A.9}$$

We know price index is $P = \left[\sum_{n=1}^N \frac{1}{N} e_n \left(p_n\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$. It is easy to check that we can write it also as follow $NP^{1-\theta} = \left[\sum_{n=1}^N e_n \left(p_n\right)^{1-\theta}\right]$. So the previous equation is equal to

$$c_{mi}^w = \frac{1}{N} \left(\frac{p_m}{P}\right)^{-\theta} e_m \frac{1}{P} \sum_{n=1}^{N} p_n c_{ni}^w. \tag{A.10}$$

If we replace c_{mi}^w into the equation of C_i^w we can show that $\sum_{n=1}^N p_n c_{ni}^w = PC_i^w$. Using this relation we find the optimal choice of variety m consumption as a function of total consumption C_i^w.

$$c_{ni}^w = \frac{1}{N} e_n \left(\frac{p_n}{P}\right)^{-\theta} C_i^w \tag{A.11}$$

B Multiple sector with financial needs and financial friction: workers' problem at time 0

The problem face by each worker is

$$\max_{C_i^w, L_i} \; w = \mathbb{E}\left(C_i^w - \chi \frac{{L_i}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right); \tag{B.1}$$

$$s.to. \tag{B.2}$$

$$PC_i^w = wL_i(1+r_d). \tag{B.3}$$

Setting $P = 1$ and replacing $C_i^w = wL_i(1 + r_d)$ into $\mathbb{E}(U^w)$ we obtain that objective function depends only on L_i. The maximization problem becomes

$$\max_{L_i} \mathbb{E}(U^w) = (1+r_d)\mathbb{E}(\omega) L_i - \chi \frac{{L_i}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}. \tag{B.4}$$

The FOC is

$$\frac{d\mathbb{E}(U^w)}{dL_i} = (1+r_d)w - \chi L_i^{\frac{1}{\eta}} = 0. \tag{B.5}$$

Solving for w we obtain

$$w = \chi L_i^{\frac{1}{\eta}} \frac{1}{(1+r_d)} \tag{B.6}$$

Since the workers are homogeneous, we know that $\int_0^N L_i di = NL_i$ so $NL_i = L$. Replacing it in the individual inverse labor supply we obtain the aggregate labor supply

$$w = \chi \left(\frac{L}{N}\right)^{\frac{1}{\eta}} \frac{1}{(1+r_d)}. \tag{B.7}$$

C Multiple sector with financial needs and financial friction: firms' problem

At time 0 each firms have to choose L_{nj} to maximize the following function

$$\max_{L_{nj}} \left[\mathbb{E}\left(p_n|succ\right) F(L_{nj}) - wL_{nj}(1+r_l) + \lambda\left(\sum_{k\neq j}^{J} \mathbb{E}\left(p_n|succ\right) F(L_{nk}) - wL_{nk}(1+r_l)\right)\right] pds. \tag{C.1}$$

The FOC of this problem is

$$\begin{aligned} \mathbb{E}\left(p_n|succ\right) F'(L_{nj}) - w(1+r_l) \quad & - (1+r_l)\frac{dw}{dL_{nj}}\left[L_{nj} + \lambda\sum_{k\neq j}^{J} L_{nk}\right] \\ + \frac{d\mathbb{E}\left(p_n|succ\right)}{dL_{nj}} & \left[F(L_{nj}) + \lambda\sum_{k\neq j}^{J} F(L_{nk})\right] = 0. \end{aligned} \tag{C.2}$$

Now we have to calculate $\frac{d\mathbb{E}(p_n|succ)}{dL_{nj}}$. We know that

$$\mathbb{E}\left(p_n|succ\right) = \mathbb{E}\left(\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}} \left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}} |succ\right). \tag{C.3}$$

Using a Leibniz's rule we can see that

$$\frac{d}{dL_{nj}}\mathbb{E}\left(p_n|succ\right) = \mathbb{E}\left(\frac{d}{dL_{nj}} p_n|succ\right). \tag{C.4}$$

Hence

$$\frac{\mathbb{E}\left(p_n|succ\right)}{dL_{nj}} = \mathbb{E}\left(-\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}-1}\left(\frac{F'(L_{nj})C - \frac{\theta-1}{\theta}c_n^{\frac{\theta-1}{\theta}-1}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\frac{\theta}{\theta-1}\frac{C}{C^{\frac{\theta-1}{\theta}}}F'(L_{nj})c_n}{C^2}\right)|succ\right);$$

$$= \mathbb{E}\left(-\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\left(1-\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\frac{c_n}{C}\right)\frac{F'(L_{nj})}{c_n}|succ\right);$$

$$= \mathbb{E}\left(-\frac{1}{\theta}p_n\left(1-p_n\frac{c_n}{C}\right)\frac{F'(L_{nj})}{c_n}|succ\right). \tag{C.5}$$

We define $s^L_{nj} = \frac{L_{nj}}{L}$, $s^L_{n-j} = \frac{\sum_{k\neq j} L_{nk}}{L}$, and $s_{nj} = \frac{F(L_{nj})}{c_n}$. The FOC becomes

$$\mathbb{E}\left(p_n|succ\right)F'(L_{nj}) - (1+r_l)w - (1+r_l)\frac{dw}{dL_{nj}}L\left[s^L_{nj} + \lambda s^L_{n-j}\right] + \frac{d\mathbb{E}(p_n|succ)}{dL_{nj}}c_n\left[s_{nj} + \lambda(1-s_{nj})\right]. \tag{C.6}$$

We dividing both side for the expected value of real wages and for $(1+r_l)$

$$\mathbb{E}(\mu|succ) = \frac{1}{\eta}\left[s^L_{nj} + \lambda s^L_{n-j}\right] + \frac{1}{w(1+r_l)}\frac{d\mathbb{E}(p_n|succ)}{dL_{nj}}c_n\left[s_{nj} + \lambda(1-s_{nj})\right], \tag{C.7}$$

where $\mathbb{E}(\mu|succ) = \frac{\mathbb{E}(p_n|succ)F'(L_{nj})-(1+r_l)w}{(1+r_l)w}$ is the expected value of markdown of real wages and $\frac{1}{\eta}$ is the elasticity of labor supply respect to expected real wages.
We want characterize a symmetric equilibrium where all firms produce the same quantity. In this case $s^L_{nj} = \frac{L_{nj}}{L} = \frac{1}{JN}$, $s^L_{n-j} = \frac{\sum_{m\neq n} L_{nk}}{L} = \frac{(J-1)}{JN}$, and $s_{nj} = \frac{F(L_{nj})}{c_n} = \frac{1}{J}$.
The FOC becomes

$$\mathbb{E}(\mu|succ) = \frac{1}{\eta}\left[\frac{1}{JN} + \lambda\frac{(J-1)}{J}\right] + \frac{1}{w(1+r_l)}\frac{d\mathbb{E}\left(p_n|succ\right)}{dL_{nj}}c_n\left[\frac{1}{J} + \lambda\frac{(J-1)}{J}\right]. \tag{C.8}$$

We define $H_{product} = \left[\frac{1}{J} + \lambda\frac{(J-1)}{J}\right]$ and $H_{labor} = \left[\frac{1}{JN} + \lambda\frac{(J-1)}{J}\right]$. Replacing into FOC we obtain

$$\mathbb{E}(\mu|succ) = \frac{1}{\eta}H_{labor} + \frac{1}{w}\frac{d\mathbb{E}\left(p_n|succ\right)}{dL_{nj}}c_nH_{product}. \tag{C.9}$$

Now we have to evaluate $\frac{d\mathbb{E}(p_n|succ)}{dL_{nj}} c_n$ in a symmetric equilibrium.

$$\frac{d\mathbb{E}\left(p_n|succ\right)}{dL_{nj}} c_n = \mathbb{E}\left(-\frac{1}{\theta} p_n \left(1 - p_n \frac{c_n}{C}\right) F'(L_{nj})|succ\right) = \tau F'(L_{nj}); \tag{C.10}$$

where $\tau = \mathbb{E}\left(-\frac{1}{\theta} p_n \left(1 - p_n \frac{c_n}{C}\right)|succ\right)$. Using the definition of $\mathbb{E}(\mu|succ)$ we can write $\frac{F'(L/JN)}{w(1+r_l)} = \frac{\mathbb{E}(\mu|succ)+1}{w}$.

Hence the FOC is

$$\mathbb{E}(\mu|succ) = \frac{1}{\eta} H_{labor} + \frac{\mathbb{E}(\mu|succ)+1}{\mathbb{E}\left(p_n|succ\right)} \tau H_{product} \tag{C.11}$$

$$\mathbb{E}(\mu|succ) = \frac{\frac{1}{\eta} H_{labor} + 1}{1 - \frac{\tau H_{product})}{\mathbb{E}(p_n|succ)}} - 1 \tag{C.12}$$

Using the definition of $\mathbb{E}(\mu|succ)$ we can find the aggregate demand

$$w = \frac{\mathbb{E}\left(p_n|succ\right) F'(\frac{L^d}{JN})}{\left(1+\mathbb{E}(\mu|succ)\right)\left(1+r_l\right)}. \tag{C.13}$$

C.1 General Equilibrium

Matching aggregate supply and demand we find total employment in equilibrium

$$L^* = \left[\chi \; J^{1-\alpha} \frac{\mathbb{E}\left(p_n|succ\right)\alpha A}{\left(1+\mathbb{E}(\mu|succ)\right)} \frac{\left(1+r_d\right)}{\left(1+r_l\right)}\right]^{\frac{1}{\frac{1}{\eta}+1-\alpha}} N. \tag{C.14}$$

To find w we solve (C.13) and (B.7) for L obtaining

$$L^d = \left[\frac{\mathbb{E}\left(p_n|succ\right)\alpha A (JN)^{1-\alpha}}{w(1+r_l)\left(1+\mathbb{E}(\mu|succ)\right)}\right]^{\frac{1}{1-\alpha}} \tag{C.15}$$

$$L^s = \left(\frac{(1+r_d)}{\chi}\right)^{\eta} N w^{\eta} \tag{C.16}$$

Matching demand and supply we get

$$\left(\frac{(1+r_d)}{\chi}\right)^{\eta} N w^{\eta} = \left[\frac{\mathbb{E}\left(p_n|succ\right)\alpha A (JN)^{1-\alpha}}{w(1+r_l)\left(1+\mathbb{E}(\mu|succ)\right)}\right]^{\frac{1}{1-\alpha}} \tag{C.17}$$

$$w = \left[\left(\frac{\chi}{(1+r_d)}\right)^{\eta}\left(\frac{\mathbb{E}\left(p_n|succ\right)\alpha A}{\left(1+\mathbb{E}(\mu|succ)\right)(1+r_l)}\right)^{\frac{1}{1-\alpha}} J\right]^{\frac{1}{\eta+\frac{1}{1-\alpha}}} \tag{C.18}$$

To find the critical value of e_n we use the zero profit condition

$$p_n A \left(\frac{L^*}{JN}\right)^{\alpha} - (w^*)\frac{L^*}{JN}(1+r_l) = 0. \tag{C.19}$$

Replacing the value of p_n

$$\left(\frac{1}{N}\right)^{\frac{1}{\theta}} \widehat{e}_n^{\frac{1}{\theta}} \left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}} = w^* \left(\frac{L^*}{JN}\right)^{1-\alpha}(1+r_l) \tag{C.20}$$

$$\left[\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\sum_{m\neq n} e_m^{\frac{1}{\theta}} + \widehat{e}_n^{\frac{1}{\theta}}\right)\right]^{\frac{1}{\theta-1}} \widehat{e}_n^{\frac{1}{\theta}} = w^* \left(\frac{L^*}{JN}\right)^{1-\alpha}\frac{1}{A}(1+r_l)N^{\frac{1}{\theta}} \tag{C.21}$$

Taking the expected value we obtain

$$\mathbb{E}\left[\left[\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\sum_{m\neq n} e_m^{\frac{1}{\theta}} + \widehat{e}_n^{\frac{1}{\theta}}\right)\right]^{\frac{1}{\theta-1}} \widehat{e}_n^{\frac{1}{\theta}}\right] = w^* \left(\frac{L^*}{JN}\right)^{1-\alpha}\frac{1}{A}(1+r_l)N^{\frac{1}{\theta}} \tag{C.22}$$

We define total employment and the expected real equilibrium wage as

$$L^* = L^{wf}\left(\frac{(1+r_d)}{(1+r_l)}\right)^{\frac{1}{\frac{1}{\eta}+1-\alpha}}. \tag{C.23}$$

$$w^* = w^{wf}\left(\frac{1}{(1+r_d)}\right)^{\frac{(\eta-\alpha\eta)}{(\eta+1-\eta\alpha)}}\left(\frac{1}{(1+r_l)}\right)^{\frac{1}{(\eta+1-\eta\alpha)}} \tag{C.24}$$

By substituting the equilibrium values into the previous equation, we obtain

$$\mathbb{E}\left[\left[\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\sum_{m\neq n} e_m^{\frac{1}{\theta}}+\hat{e}_n^{\frac{1}{\theta}}\right)\right]^{\frac{1}{\theta-1}}\hat{e}_n^{\frac{1}{\theta}}\right]=w^{wf}\left(\frac{L^{wf}}{JN}\right)^{1-\alpha}\frac{1}{A}N^{\frac{1}{\theta}} \tag{C.25}$$

D Mathematical proofs

We want to show that $\frac{d\left(\frac{p_n}{P}\right)}{de_n}$ and $\frac{d\left(\frac{p_n}{P}\right)}{de_m}$ are greater than zero

$$\begin{aligned}
\frac{d\left(\frac{p_n}{P}\right)}{de_n} &= \frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}-1}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}} - \frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}-1}\left(\frac{-\frac{1}{\theta}c_n^{\frac{\theta-1}{\theta}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}-1}\frac{\theta}{\theta-1}\frac{C}{C^{\frac{\theta-1}{\theta}}}c_n}{C^2}\right); \\
&= \frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}} + \frac{1}{\theta(\theta-1)}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{-1}\left(\frac{c_n}{C}\right); \\
&= \frac{1}{\theta}\left(\frac{p_n}{P}\right)\frac{1}{e_n} + \frac{1}{\theta(\theta-1)}\left(\frac{p_n}{P}\right)^2\left(\frac{c_n}{C}\right)\frac{1}{e_n}; \\
&= \frac{1}{\theta}\left(\frac{p_n}{P}\right)\frac{1}{e_n}\left(1+\frac{1}{(\theta-1)}\left(\frac{p_n}{P}\right)\left(\frac{c_n}{C}\right)\right) > 0.
\end{aligned} \tag{D.1}$$

$$\begin{aligned}
\frac{d\left(\frac{p_n}{P}\right)}{de_m} &= -\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}-1}\left(\frac{-\frac{1}{\theta}c_m^{\frac{\theta-1}{\theta}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_m^{\frac{1}{\theta}-1}\frac{\theta}{\theta-1}\frac{C}{C^{\frac{\theta-1}{\theta}}}c_n}{C^2}\right); \\
&= \frac{1}{\theta(\theta-1)}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_m^{-\frac{1}{\theta}}\left(\frac{c_m}{C}\right)^{-\frac{1}{\theta}}\left(\frac{c_m}{C}\right); \\
&= \frac{1}{\theta(\theta-1)}\left(\frac{p_n}{P}\right)\left(\frac{p_m}{P}\right)\left(\frac{c_m}{C}\right)\left(\frac{1}{e_m}\right) > 0.
\end{aligned} \tag{D.2}$$

Under symmetric equilibrium

$$\begin{aligned}
\mathbb{E}\left(\frac{c_n}{C}\right) &= \mathbb{E}\left(\frac{\sum_{j=1}^{J} F(L^*/NJ)}{\left[\sum_{n=1}^{N}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\sum_{j=1}^{J} F(L^*/NJ)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}}\right) \\
&= \mathbb{E}\left(\frac{JF(L^*/NJ)}{JF(L^*/NJ)\left[\sum_{n=1}^{N}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\right]^{\frac{\theta}{\theta-1}}}\right) \\
&= \mathbb{E}\left(\frac{1}{\left[\sum_{n=1}^{N}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\right]^{\frac{\theta}{\theta-1}}}\right)
\end{aligned} \tag{D.3}$$

$$\begin{aligned}
\mathbb{E}\left(p_n|succ\right) &= \mathbb{E}\left(\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}|succ\right) \\
&= \mathbb{E}\left(\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left(\frac{1}{\left[\sum_{n=1}^{N}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\right]^{\frac{\theta}{\theta-1}}}\right)^{-\frac{1}{\theta}}|succ\right) \\
&= \mathbb{E}\left(\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left[\sum_{n=1}^{N}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\right]^{\frac{1}{\theta-1}}|succ\right)
\end{aligned} \tag{D.4}$$

$$\begin{aligned}
\tau &= \mathbb{E}\left(-\frac{1}{\theta}(p_n|succ)\left(1-(p_n|succ)\frac{c_n}{C}\right)\right) \\
&= \mathbb{E}\left(\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\left[\sum_{n=1}^{N}\left(\frac{1}{N}\right)^{\frac{1}{\theta}} e_n^{\frac{1}{\theta}}\right]^{\frac{1}{\theta-1}} \frac{e_n^{\frac{1}{\theta}}}{\sum_{m\neq n} e_m^{\frac{1}{\theta}}}\right)
\end{aligned} \tag{D.5}$$

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