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INFERENCE FOR DEPRIVATION PROFILES IN A BINARY SETTING

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Inference for deprivation profiles in a binary setting

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Abstract
The paper addresses the issue of comparing deprivation distributions, when poverty is measured by a sum of binary variables. To accomplish this task, it provides a graphical device, the Three I's of Deprivation (TID) curve, that summarizes incidence, intensity and inequality aspects of deprivation in a society, and it is the natural counterpart of the TIP curve widely used in income poverty analysis. Uncertainty around the estimated deprivation curves is evaluated through simultaneous confidence bands. An hypothesis test of dominance is presented to facilitate the comparison and the ordering of deprivation curves across groups and over time. An extension of the Sen-Shorrocks poverty index that summarizes the three I's of deprivation is characterized and confidence intervals are developed. As a substantive illustration the evolution of material and social deprivation across European countries over the period of the outbreak of the pandemic is analysed.

Keywords: Deprivation curves, Stochastic dominance, Binary variables, EU-SILC data.
Jel Classification: D31; D63; I31.

1 Introduction

Most of policy designs for alleviating poverty focus on head-count measures of monetary-based poverty. In the literature, however, it is commonly agreed that two important aspects of poverty – intensity and inequality – should be considered in practice, in addition to incidence. Whether a measure captures these three aspects of poverty has large consequences for the incentives of a policy maker (Alkire and Seth, 2014). The 'Three Is of Poverty' (TIP) curve (Jenkins and Lambert, 1997), also known as Poverty Profile curve or Poverty Gap Profile (Barrett et al., 2016), is a valuable graphical device, because of its ability to simultaneously represent the poverty head-count (Incidence), the average poverty income-gap, *i.e.* the average distance of individual incomes from the poverty line (Intensity) and the within-poor distribution (Inequality). With individuals ranked in ascending order of income, a TIP curve plots the cumulative poverty income-gap per person against the corresponding cumulative proportion of individuals.

At the same time, there is a growing consensus that measurement of poverty should not be confined to a monetary variable, such as income, because poverty has multiple facets, which, in practice, are often binary or ordinal in nature. Within this discrete response framework, poverty is defined as a situation of deprivation that reflects enforced lack of material, social, political, health benefits considered to be basic needs in a modern society.

Although the extent to which an individual is deprived cannot be directly observed, it can be inferred from observed behaviour representing deprivation observable expressions. Therefore, to measure deprivation in its broad sense, questionnaires for surveys on poverty and social exclusion are typically composed by a series of items scored on a binary scale. Binary scoring are typical of yes/no, revealing either the presence (yes) or the absence (no) of specific deprivations due to lack of resources, access or freedom. Each individual is characterized by a score equal to the total number of questions that are answered affirmatively, ranging in between 0 and the total number of considered items. Individuals are then classified as either no deprived or in different levels of deprivation if their deprivation score is equal or higher than a certain threshold or cut-off.

The present paper aims at translating the 'three I's' properties of TIP curves into a binary setting, where deprivation scores are countable but not continuous. Since the diagram bears a close resemblance to the TIP curve, it is described here as a 'Three

I's of Deprivation' (TID). The TID curve presented here generalizes deprivation profiles derived from deprivation counts, first introduced by Lasso de la Vega (2010) and Espinoza-Delgado and Silber (2024). While statistical inference for TIP curves has been proposed by Thuysbaert (2008), Barrett et al. (2016), and recently by Fourrier-Nicolaï and Lubrano (2020) in a Bayesian framework, it has not been developed for TID curves so far. At the same time, the methodology implemented for TIP curves cannot be straightforwardly applied, because of the discrete nature of our data. Therefore, we provide a procedure to obtain simultaneous confidence bands to evaluate uncertainty around the estimated TID curves. An hypothesis test of stochastic dominance is also illustrated, to make it possible comparison and ordering of deprivation curves across groups and over time. We also show that the TID curve is related to the (censored) generalized Lorenz curve, so red that dominance criterion can be used to rank deprivation distributions by all those measure meeting a given set of axioms. In line with Espinoza-Delgado and Silber (2024), an extension of the Sen-Shorrocks index in a binary setting that satisfies the given set of axioms is introduced and discussed. Statistical inference for the index is also provided.

The empirical motivation of this study is to look at the evolution of material and social deprivation (Guio et al., 2016) across European countries, to compare countries' deprivation profiles before and after the pandemic outbreak. Data come from the latest 2022 wave of the European Union Statistics on Income and Living Conditions (EU-SILC). However, the TID curves can be relevant in other related contexts, whenever bounded count data are encountered. For instance, to analyse experience-based food security (Bickel et al., 2000), job satisfaction (Stride et al., 2008), or several health outcomes (Mullahy, 2023).

The paper is organised as follows. In section 2 we introduce notation and basic definitions. Section 3 formally describes the TID curve. It provides an estimator of the curve along with a procedure to build a confidence band. Section 4 proposes a dominance test for comparing TID curves. It also shows the implications of TID dominance to poverty aggregate measures, and specifically to the extension in the binary setting of the Sen-Shorrocks index. Section 6 provides the main results of our temporal analysis of the European countries' deprivation profiles. Section 7 is devoted to conclusions.

2 Deprivations and achievements: the framework

Each individual is assigned a vector of several attributes that represent different aspects of living conditions. To measure deprivation, it becomes necessary to check whether an individual has a minimum acceptable number of these attributes. These acceptable quantities of attributes represent the individual threshold limits necessary for an adequate standard of living. The ability to deal with such discrete response data is fundamental, since only few variables measuring individual well-being are numerical in nature. This is the case of material deprivation, where each attribute represents a deprivation item, and an affirmative response indicates a basic necessity failure.

Given a population of N individuals, and D attributes/items of deprivation, define d_{is} as:

$$d_{is} = \begin{cases} 1, & \text{if individual } i \text{ is deprived in item } s \\ 0, & \text{otherwise} \end{cases}$$

$i \in \{1, ..., N\}$ and $s \in \{1, ..., D\}$.

Within this binary framework, the formal representation is a deprivation matrix whose rows denote a pattern of zeros and ones for each individual. The value one, corresponding to an affirmative response, identifies a basic necessity failure, while the value zero corresponds to a basic need achievement. For each individual i, deprivation assessment is the sum of his/her positive items (deprivations), the deprivation raw score:

$$RS_i = \sum_{s=1}^{D} d_{is},\ i = 1, \ldots, N.$$

Symmetrically, the achievement raw score of individual i is equal to

$$AS_i = \sum_{s=1}^{D} (1 - d_{is}) = D - RS_i,\ i = 1, \ldots, N,$$

indicating the sum of items the individual i has access to.

Deprived individuals are defined as those lacking at least a certain number of items. This definition requires the identification of a threshold or cutoff. Such a threshold is a positive integer k and denotes the minimum number of items that an individual cannot afford in order to be classified as deprived/poor. Having fixed k, each individual is either deprived/poor or not deprived/poor according to the following crisp identification:

$$\text{deprived}_i^{(k)} = \begin{cases} 1 & \text{if } RS_i \geq k, \\ 0 & \text{if } RS_i < k. \end{cases}$$

Potentially, the deprivation threshold k may take all values in the range $k = 1, \cdots, D$. Extreme cases are when individuals lack all the D items ($k = D$), and when individuals lack at least one item ($k = 1$).[1]

For a given deprivation cutoff k, the proportion of population that fails to meet the minimum standard k is equivalently referred to as the incidence of deprivation, deprivation rate, or deprivation head-count ratio:

$$H_k = \frac{\sum_{i=1}^{N} \text{deprived}_i^{(k)}}{N} = \frac{q_k}{N}. \tag{1}$$

The head-count is usually taken as the unique measure of deprivation, ignoring intensity and shape of the deprivation distribution among the poor.

Related to the threshold k, is the concept of deprivation gap, which is defined as the number of deprivations individuals need to convert into achievements to escape poverty:

$$x_i = \begin{cases} RS_i - (k-1) & \text{if } RS_i \geq k \\ 0 & \text{if } RS_i < k. \end{cases} \tag{2}$$

For example, suppose to measure the deprivation of a phenomenon with $D = 12$ items and that the threshold is fixed at $k = 4$. If individuals have a raw score of 9 (they are deprived in 9 items out of 12), their gap score is equal to 6 since they have to convert at least 6 deprivations into achievements to reach the status of non deprived. When individuals report $D = 12$ affirmative items they need to convert $D^* = 9$ items out of 12 to be considered out of poverty.[2]

The average of the deprivation gaps x_i among the poor is the intensity of deprivation

$$I_q = \frac{\sum_{i=1}^{q_k} x_i}{q_k}. \tag{3}$$

The average of the deprivation gaps x_i among the total population is the adjusted head-count ratio (Alkire and Foster, 2011) since it can be expressed as:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} = H_k \times I_q. \tag{4}$$

[1] In the multidimensional poverty literature, these two special cases correspond to the intersection approach in which individuals are multidimensional poor if they are poor in all the poverty dimensions (Atkinson, 2003) and to the union approach which establishes that individuals are poor if they are poor in at least one poverty dimension. Alkire and Foster (2011) provide the intermediate solution, based on a 'suitably fixed minimal number' of dimensions.

[2] Its relative formulation is $x_i^* = \frac{x_i}{D^*}$ where $D^* = D - (k-1)$ represents the maximum number of deprivations individuals need to transform into achievements to escape deprivation. A relative formulation is useful when we want to compare two groups with a different number of deprivation items.

A complementary portrait of deprivation incidence is given by information on intensity and inequality, representing different aspects of deprivation. To develop measures that account for these distributional aspects, we need to combine information on deprivation suffered by poor people. Based on that, we develop deprivation curves able to hierarchically compare different population groups or countries, and eventually able to derive dominance relationship.

3 Setting up a deprivation profile

3.1 Deprivation curves

Deprivation curves or deprivation profiles provide a graphical device that plays a similar role in deprivation analysis as the Lorenz curve plays in income inequality. Beyond a graphical perspective, deprivation curves have useful implications in deriving measures that go beyond the simple head-count ratio.

In the monetary poverty literature, Jenkins and Lambert (1997) demonstrated that three important dimensions of poverty can be summarized by plotting the cumulative poverty gap per capita against the corresponding cumulative proportion of people. In this way, the 'Three I's of Poverty' can be simultaneously represented in the TIP curve. These I's are: I1. Incidence of poverty, as captured by the head-count poverty measure; I2. Intensity, as measured by the income gap, the average distance of the incomes of the poor from the poverty line; I3. Inequality of poverty within the poor group, capturing how far the incomes of the poorest differ from those closer to the poverty line.

Due to the importance of simultaneously representing these three different aspects of poverty, we formally derive a deprivation curve or deprivation profile for binary items that mimics the TIP curve.

Let X be a discrete random variable (r.v.) representing the deprivation gap for a given threshold k, taking values in the finite set $\{0, 1, \ldots, x, \ldots, D^*\}$, with probability function $p_X(x) = \Pr(X = x)$. Its survival function

$$S_X(x) = \Pr(X \geq x), \quad x = 0, 1, \ldots, D^* \tag{5}$$

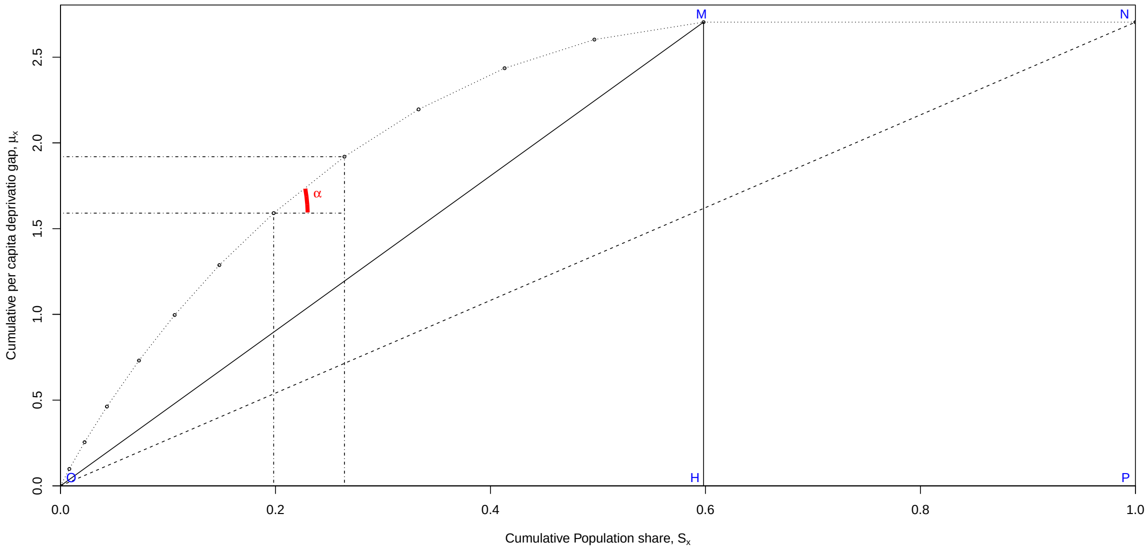
is a jump function and represents the cumulative share of individuals with a gap at least equal to x. The corresponding cumulative per capita deprivation gap is given by

$$\mu_X(x) = \sum_{j=x}^{D^*} j \cdot p_X(j). \tag{6}$$

and the "total" mean $\mu_X(0)$ represents the overall per capita deprivation gap. Therefore, the generic $\mu_X(x)$ can be interpreted as the "incomplete mean" of deprivation gap, namely the contribution to the overall mean from those with a gap at least equal to x.

The deprivation curve is a piecewise linear function defined on $[0,1]$ that connects points $[(0,0),(S_X(x),\mu_X(x))]$, $x = D^*, D^*-1,\cdots,0$, in descending order and is represented in Figure 1. At each point $S_X(x)$ the slope α of the curve changes and the slope corresponding to the breakpoint $S_X(x)$ is equal to $\tan(\alpha) = (x-1)$ (see Figure 1).[3]

Figure 1: Deprivation (TID) curve for a fixed threshold



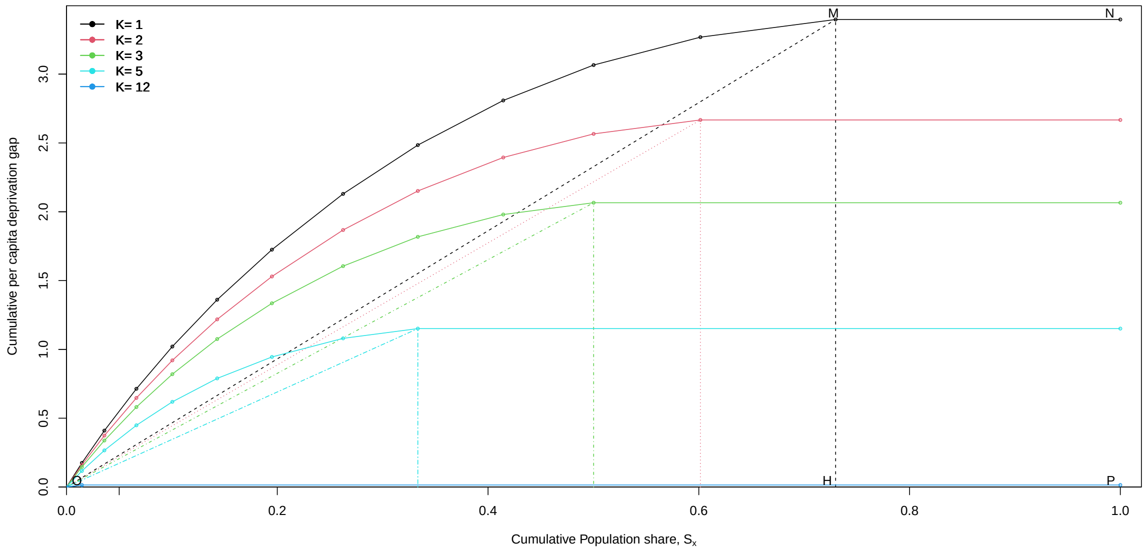
Segment $\overline{OH}$ on the horizontal axis, corresponding to $S_X(1)$, is the share of population having a deprivation gap at least equal to one. Therefore, it represents the deprivation head-count ratio H_k. At point H the deprivation curve saturates and becomes flat at value $\mu_X(1)$, being the remaining share of population not deprived with zero deprivation gap. The equal segments $\overline{MH}$ and $\overline{NP}$, corresponding to $\mu_X(1)$ and $\mu_X(0)$ respectively, represent the average deprivation gap for the whole population, M_k. The slope of the triangle MOH ($\overline{MH}/\overline{OH}$) is the average deprivation gap of deprived individuals A_k, and it represents the intensity of deprivation among the poor. Segment $\overline{OM}$ can be interpreted as a 'deprivation equality line' being the deprivation curve when each deprived individual has the same deprivation gap. Therefore, the area

[3]The slope is simply given by the ratio between $(\mu_X(x-1)-\mu_X(x)) = (x-1)p_X(x-1)$ and $(S_X(x-1)-S_X(x)) = p_X(x-1)$.

between segment $\overline{OM}$ and curve OM represents the extent of inequality among the deprived. As in the monetary poverty approach, this deprivation curve emphasizes the T(hree) I('s) of D(eprivation): I1 Incidence; I2 Intensity; I3 Inequality. For this reason, the deprivation curve is named TID curve.

While Figure 1 depicts the deprivation profile for a fixed threshold k, a more general view is shown in Figure 2, where TID curves for different values of k are depicted. The deprivation curves in Figure 2 are based on a $D = 12$ item list and the threshold assumes values in the range $k = 1$ to $k = 12$. The size of the three I's of deprivation changes according to the threshold, but the meaning remains the same.

Figure 2: Deprivation (TID) curves for different thresholds k



In the extreme case where $k = D$, individuals are deprived only when they failure in all D items. Then, the deprivation gap of individual i becomes a binary variable:

$$x_i = \begin{cases} 1 & \text{if } i \text{ is deprived in all } D \text{ items,} \\ 0 & \text{otherwise.} \end{cases}$$

and $\mu_X(1)$ is equal to the head-count ratio $S_X(1)$. The deprivation curve is composed by a linear segment with unit slope from the origin to percentile $S_X(1)$, and at value $\mu_X(1)$ it becomes flat (see the bottom blue curve in Figure 2, corresponding to $k = 12$).[4]

The other extreme case is when individuals are deprived if they failure in at least one item (see the black curve in Figure 2, corresponding to $k = 1$). When $k = 1$, the

[4]In this case the head-count ratio is a sufficient statistics for the assessment of deprivation (Seth and Yalonetzky, 2020).

deprivation gap becomes equal to the deprivation raw score:

$$x_i = RS_i \quad \forall i.$$

Segment $\overline{OH}$ on the horizontal axis still corresponds to the proportion of deprived population and measures the incidence of deprivation, H_1. Segments $\overline{MH}$ and $\overline{NP}$ in Figure 2 now represent the average number of deprivations suffered by the whole population M_1. The ratio $\overline{MH}/\overline{OH}$ is the average number of deprivations suffered by the deprived, and it still measures the intensity of deprivation A_1. Segment $\overline{OM}$ is still a 'deprivation equality line', and the area between segment $\overline{OM}$ and curve OM is the extent of inequality among the deprived. The TID curve for $k = 1$ is similar to the 'PUB curve' presented by Espinoza-Delgado and Silber (2024), which takes simultaneously into account the 'prevalence' (incidence), the 'unevenness' (inequality), and the 'breadth' (intensity) of deprivation. The TID curve is also similar to the 'SD curve' introduced by Lasso de la Vega (2010) although she focused on the adjusted head-count ratio. Both approaches do not address the issue of statistical inference of the curves. In the following sections, attention is devoted to provide statistical inference for TID curves and characterization of TID dominance. For the purpose of inference, an alternative parametrization of the deprivation curve in terms of achievements can be convenient and it is explained in section 3.2.

3.2 An alternative specification of deprivation curves

A deprivation curve can also be specified for convenience in terms of achievements. Given the deprivation gap X for a fixed threshold k, define $Y = D^* - X$ with

$$y_i = D^* - x_i = \begin{cases} AS_i & \text{if } RS_i \geq k \\ D^* = D - (k-1) & \text{if } RS_i < k, \end{cases} \tag{7}$$

for $i = 1, \cdots, n$.

The variate Y counts the number of achievements for deprived individuals, and it is equal to D^* for not deprived individuals, being D^* the minimum number of achievements to be out of poverty according to a fixed threshold k.

The r.v. Y is of course discrete, and takes values in the finite set $\{0, 1, \ldots, D^*\}$. Its probability function is given by $\Pr(Y = y) = p_Y(y) = p_X(D^* - y)$. If $y = D^* - x$, the following relations hold:

$$S_X(x) = \Pr(Y \leq D^* - x) = F_Y(y), \tag{8}$$

$$\mu_X(x) = \sum_{j=0}^{y}(D^* - j)p_Y(j) = D^*F_Y(y) - \mu_Y(y), \tag{9}$$

where the incomplete mean of Y is defined as

$$\mu_Y(y) = \sum_{j=0}^{y} j \cdot p_Y(j) = \sum_{j=0}^{y} j \cdot [F_Y(j) - F_Y(j-1)], \;\; y = 0, 1, \cdots, D^*. \tag{10}$$

Therefore points $(S_X(x), \mu_X(x))$ are equivalent to points $(F_Y(y), D^*F_Y(y) - \mu_Y(y))$.

Note that $F_Y(0) = 0$, $\mu_y(0) = 0$, and $F_Y(D^*) = 1$, $\mu_Y(D^*) = D^* - \mu_X(1)$.

Consider next the quantile function of Y, $Q_Y(p) = \inf\{y : F_y(y) \geq p\}, \;\; 0 \leq p \leq 1$. In explicit terms, $Q_Y(p)$ is equal to

$$Q_Y(p) = \begin{cases} 0 & \text{if } 0 \leq p \leq F_Y(0) = p_Y(0), \\ y & \text{if } F_Y(y-1) \leq p \leq F_Y(y), \;\; y = 1, \ldots, D^*. \end{cases}$$

Using the symbol $H_Y(p)$, $0 \leq p \leq 1$ to denote the deprivation curve, it can be written as

$$H_Y(p) = D^*p - \int_0^p Q_Y(u)du = D^*p - G_Y(p), \;\;\; 0 \leq p \leq 1 \tag{11}$$

where $G_Y(p) = \int_0^p Q_Y(u)du$, is the generalized concentration curve of Y. Since $G_Y(p)$ is a convex function, $H_Y(p)$ is concave and it is easy to shown that $H_Y(0) = 0$ and $H_Y(1) = D^* - \mu_Y(D^*)$.

From (11), it is clear that the relationships $D^*F_Y(y) - \mu_Y(y) = H_Y(F_Y(y))$ hold.

3.3 Estimation and uncertainty of TID curves

Estimation of the deprivation profile in a population requires estimation of $S_X(x)$ and $\mu_X(x)$, or, equivalently (cfr. eqns.(8) and (9)), of $F_Y(y)$ and $H_Y(F(y))$.

Let $X_1, \ldots, X_n$ be a random sample of size n of deprivations from a population, composed by independent and identically distributed (*i.i.d.*) r.v.s with probability function $p_X(\cdot)$, and let $Y_1, \ldots, Y_n$ be the corresponding random sample of achievements, with $Y_i = D^* - X_i$, $i = 1, \ldots, n$. To simplify the notation, define $p_y = p_Y(y)$, $F_y = F_Y(y)$, $H_y = H_Y(F(y))$, and $\mu_y = \mu_Y(y)$. Define further the indicator function

$$\mathbb{1}_{(Y_i=y)} = \begin{cases} 1 & \text{if } Y_i = y \\ 0 & \text{if } Y_i \neq y \end{cases}$$

The r.v. $(\mathbb{1}_{(Y_i=0)}, \ldots, \mathbb{1}_{(Y_i=D^*)})$ possesses Multinomial distribution with parameters 1 and $p_1, \ldots, p_{D^*}$. Hence, by standard computations, the Maximum Likelihood Estimator (MLE, for short) of p_y is

$$\widehat{p}_y = \sum_{i=1}^{n} \mathbb{1}_{(Y_i=y)}.$$

The corresponding MLE of the cumulative function $F_y = F_Y(y)$ is then, by the invariance property,

$$\widehat{F}_y = \sum_{j=0}^{y} \widehat{p}_j,$$

and the MLE of the incomplete mean $\mu_y = \mu_Y(y)$ is

$$\widehat{\mu}_y = \sum_{j=0}^{y} j\,\widehat{p}_j = \sum_{j=0}^{y} j\left[\widehat{F}_j - \widehat{F}_{j-1}\right].$$

Finally, again by the invariance property, the MLE of $H_y = H_Y(F(y))$ is therefore

$$\widehat{H_y} = D^*\widehat{F}_y - \widehat{\mu}_y.$$

The above MLEs have desirable properties, such as consistency, asymptotic normality, and asymptotic efficiency. Their main properties are summarized in Proposition 1.

Proposition 1 *Define the D^*-dimensional vectors*

$$\begin{aligned}\mathbf{p} = [p_0, p_1, \cdots, p_{D^*-1}]^T,\ \ \widehat{\mathbf{p}} = [\widehat{p}_0, \widehat{p}_1, \cdots, \widehat{p}_{D^*-1}]^T;\\ \mathbf{F} = [F_0, F_1, \cdots, F_{D^*-1}]^T,\ \ \widehat{\mathbf{F}} = [\widehat{F}_0, \widehat{F}_1, \cdots, \widehat{F}_{D^*-1}]^T;\\ \mathbf{H} = [H_0, H_1, \cdots, H_{D^*-1}]^T,\ \ \widehat{\mathbf{H}} = [\widehat{H}_0, \widehat{H}_1, \cdots, \widehat{H}_{D^*-1}]^T;\end{aligned}$$

and let $\mathcal{T}_n = \sqrt{n}(\widehat{\mathbf{p}} - \mathbf{p})$, $\mathcal{W}_n = \sqrt{n}(\widehat{\mathbf{F}} - \mathbf{F})$, $\mathcal{V}_n = \sqrt{n}(\widehat{\mathbf{H}} - \mathbf{H})$. The following three statements hold. In the sequel, the symbol $\stackrel{d}{\to}$ ($\stackrel{p}{\to}$) will denote convergence in distribution (probability).

1. *As $n \to \infty$, $\mathcal{T}_n$ tends in distribution to a Multinormal r.v. with null mean vector and covariance matrix*

$$\mathbf{\Sigma}_T = [\sigma_{jk}] = \begin{cases} p_j(1-p_j) & \text{if } k = j, \\ -p_j p_k & \text{if } k \neq j. \end{cases}$$

   In symbols: $\mathcal{T}_n \stackrel{d}{\to} \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_T)$ as $n \to \infty$.

2. *$\mathcal{W}_n \stackrel{d}{\to} \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_W)$ as $n \to \infty$, where $\mathbf{\Sigma}_W = \mathbf{L}\mathbf{\Sigma}_T\mathbf{L}^T$, and L is the lower triangular matrix (A.1) .*

3. $\mathcal{V}_n \xrightarrow{d} \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_V)$ *as* $n \to \infty$, *where* $\mathbf{\Sigma}_V = \mathbf{B}\mathbf{\Sigma}_W\mathbf{B}^T$, *and* B *is the lower triangular matrix* $(A.2)$.

Proof. See Appendix.

Results in Proposition 1 can be used to construct a (conservative) confidence band for the deprivation curve. First of all, due to its piecewise linearity, the problem is equivalent to construct simultaneous confidence intervals for the D^* points (F_y, H_y), $0 \le y \le D^* - 1$. To this purpose, from Proposition 1 and the continuous mapping theorem,it is easy to see that

$$\max_{0 \le y \le D^*-1} \sqrt{n}|\widehat{F_y} - F_y| \xrightarrow{\mathrm{d}} \max_{0 \le y \le D^*-1} |\mathcal{W}_{(}y)| = |\mathcal{W}|^{\max};$$
$$\max_{0 \le y \le D^*-1} \sqrt{n}|\widehat{H_y} - H_y| \xrightarrow{\mathrm{d}} \max_{0 \le y \le D^*-1} |\mathcal{V}_{(}y)| = |\mathcal{V}|^{\max}.$$

Since the distributions of $|\mathcal{W}|^{\max}$ and $|\mathcal{V}|^{\max}$ are absolutely continuous, there exist unique percentiles $d_{\mathcal{W},1-\alpha}$ and $d_{\mathcal{V},1-\alpha}$ such that for each $0 < \alpha < 1$:

$$\Pr\left(|\mathcal{W}|^{\max} \le d_{\mathcal{W},1-\alpha}\right) = 1 - \alpha; \tag{12}$$
$$\Pr\left(|\mathcal{V}|^{\max} \le d_{\mathcal{V},1-\alpha}\right) = 1 - \alpha. \tag{13}$$

An overall continuous confidence band for the deprivation profile $H_Y(p)$ requires definition of a lower curve $H^-(p)$ and an upper curve $H^+(p)$ for each $p \in [0,\,1]$ such that:

$$\Pr\left(H^-(p) \le H_Y(p) \le H^+(p)\right) \ge 1 - \alpha, \ \ \forall p. \tag{14}$$

In particular, in the present case the lower and upper curve are constructed as follows.

1. The lower curve $H^-(p)$, is a piecewise linear and continuous curve, joining the points

$$\left(\widehat{F_y} + \frac{d_{\mathcal{W},1-\alpha/2}}{\sqrt{n}}, \widehat{H_y} - \frac{d_{\mathcal{V},1-\alpha/2}}{\sqrt{n}}\right), \quad y = 0,\,1,\,\ldots,\,D^* - 1.$$

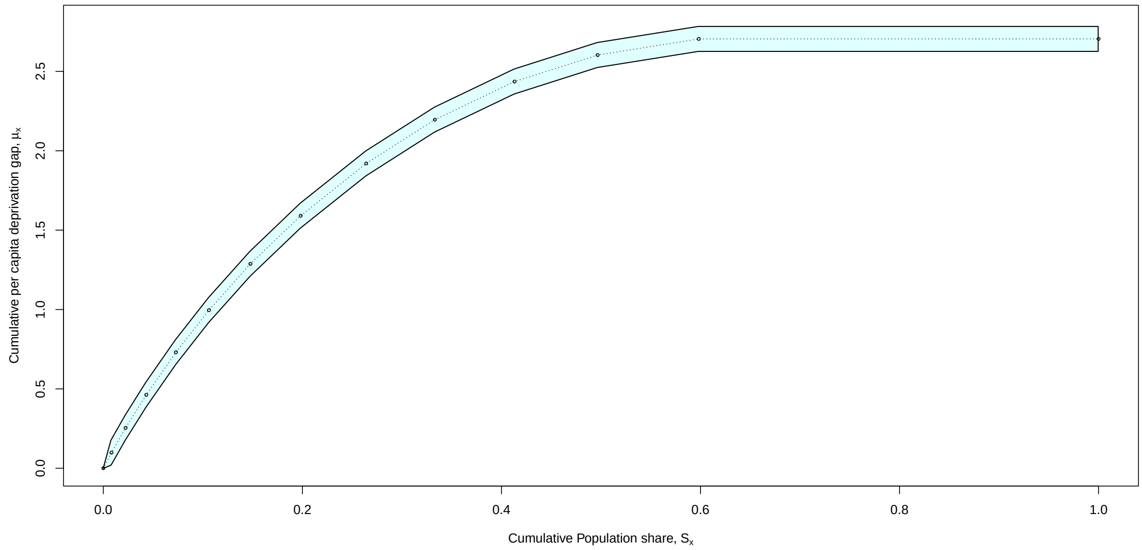
2. The lower curve $H^+(p)$, is a piecewise linear and continuous curve, joining the points

$$\left(\widehat{F_y} - \frac{d_{\mathcal{W},1-\alpha/2}}{\sqrt{n}}, \widehat{H_y} + \frac{d_{\mathcal{V},1-\alpha/2}}{\sqrt{n}}\right), \quad y = 0,\,1,\,\ldots,\,D^* - 1.$$

Proposition 2 *As* $n \to \infty$, *the confidence band* $(H^-(p),\, H^+(p);\, p \in [0,\,1])$ *possesses asymptotic level* $\ge 1 - \alpha$.

Proof. See Appendix.

Figure 3: Deprivation curve confidence band at level ≥95%



The simultaneous continuous confidence band at level ≥95% is shown in Figure 3. Quantiles $d_{\mathcal{W},1-\alpha}$ and $d_{\mathcal{V},1-\alpha}$ are estimated using a parametric bootstrap procedure with 1000 replications, under the assumption of a multinomial distribution.[5]

4 Comparing TID curves: testing for stochastic dominance

Let A, B be two populations, with corresponding TID curves $H_A(\cdot)$, $H_B(\cdot)$, respectively. Distribution A TID dominates distribution B if the deprivation curve of population A lies below the deprivation curve of population B over its support, namely $H_A(p) \leq H_B(p)$ for each $p \in [0, 1]$. Therefore, as pointed out by Fourrier-Nicolaï and Lubrano (2020) for income distributions, TID dominance means less deprivation.

Statistical tests of stochastic dominance for numerical variables distinguish between the null hypothesis of dominance and the null hypothesis of non-dominance. In the first case the null of dominance is rejected only when there is clear evidence against it, while in the second case the alternative hypothesis of dominance is accepted only when there is clear evidence in its favour (Davidson and Duclos, 2000; Davidson, 2006). Following the most popular approach of testing the null hypothesis of dominance, we

[5]Functions for all the computing are written in R (Team, 2021).

extend the procedure typically adopted for numerical variables to the case of binary variables and piecewise linear curves.

Let us consider two (independent) populations A, B, with achievements probabilities $p_{y,A}$, $p_{y,B}$ and d.f.s $F_A(y)$, $F_B(y)$, respectively, $y = 0, 1, \dots, D^*$. Furthermore, let $H_A(p)$, $H_B(p)$ be the corresponding deprivation curves, and $H_{y,A} = H_A(F_A(y))$, $H_{y,B} = H_A(F_B(y))$, as defined in Section 3.3. The main goal of the present section is to develop a test for the hypothesis problem

$$\begin{cases} H_0 : H_A(p) \leq H_B(p) \ \forall p \in [0,1] \\ H_1 : H_A(p) > H_B(p) \ \text{ for some } p \in [0,1]. \end{cases} \tag{15}$$

Since the cumulative deprivation share has support in $[0,1]$, our approach compares the object at all points in the support, a considerable property as highlighted by Barrett and Donald (2003). We focus on a situation where the available data correspond to two independent samples with possibly different sizes, n_A and n_B, coming from two populations with deprivation (TID) curves $H_A(p)$ and $H_B(p)$, respectively.

Define

$$\Delta(p) = H_A(p) - H_B(p), \ p \in [0,\ 1]. \tag{16}$$

Then, a grid of (equally spaced) values u_j, $j = 1, \dots, J$ is selected, and the hypothesis problem (15) is transformed into

$$\begin{cases} H_0 : \quad \Delta(u_j) \leq 0 \qquad \forall\, j = 1, \dots, J \\ H_1 : \quad \Delta(u_j) > 0 \quad \text{for some } j = 1, \dots, J \end{cases} \tag{17}$$

The adopted testing procedure is based on constructing simultaneous confidence regions for $\Delta(u_1), \dots, \Delta(u_J)$, and in rejecting H_0 whenever for at least one $\Delta(u_j)$ the corresponding confidence region does not intersect the negative half-line.

Let $Y_{A,1}, \dots, Y_{A,n_A}, Y_{B,1}, \dots, Y_{B,n_B}$ be two independent samples of size n_A, n_B of achievements from populations A, B, respectively. If $\widehat{H}_A(p)$, $\widehat{H}_B y(p)$ are the MLEs of $H_A(p)$, $H_B(p)$ obtained from the two samples, define $\widehat{\Delta}(u_j) = \widehat{H}_A(u_j) - \widehat{H}_B(u_j)$, $j = 1, \dots, J$. Consider the J-dimensional vectors of equally spaced points $\mathbf{u} = [u_1, \dots, u_J]^T$ in the interval $[0,1]$, and define further the vectors

$$\Delta(\mathbf{u}) = \begin{bmatrix} \Delta(u_1) \\ \cdots \\ \Delta(u_J) \end{bmatrix}, \quad \widehat{\Delta}(\mathbf{u}) = \begin{bmatrix} \widehat{\Delta}(u_1) \\ \cdots \\ \widehat{\Delta}(u_J) \end{bmatrix}, \quad \mathcal{U}_{n_A,n_B}(\mathbf{u}) = \sqrt{\frac{n_A n_B}{n_A + n_B}} \left(\widehat{\Delta}(\mathbf{u}) - \Delta(\mathbf{u}) \right).$$

Proposition 3 *Assume that, as n_A, n_B increase,* $\frac{n_A}{n_A+n_B} \to \gamma \in (0,1)$, i.e. *the ratio of the sample sizes is finite and bounded away from zero. The following two statements hold.*

1. *As n_A, n_B go to infinity, $\mathcal{U}^{max}_{n_A,n_B} = \max_j \mathcal{U}_{n_A,n_B}(u_j)$ tends in distribution to an absolutely continuous r.v. $\mathcal{U}^{max}$.*

2. *If $d_{\mathcal{U},1-\alpha}$ is the $(1-\alpha)$-quantile of $\mathcal{U}^{max}$, the confidence regions*

$$\left[\widehat{\Delta}(u_j) - \sqrt{\frac{n_A+n_B}{n_A n_B}} d_{\mathcal{U},1-\alpha},\ +\infty\right),\ = 1,,\ \ldots,\ J \tag{18}$$

   have simultaneous asymptotic confidence level $1-\alpha$.

3. *The testing procedure consisting in rejecting H_0 in (17) whenever*

$$\widehat{\Delta}(u_j) - \sqrt{\frac{n_A+n_B}{n_A n_B}} d_{\mathcal{U},1-\alpha} > 0 \text{ for some } j = 1,\ \ldots,\ J \tag{19}$$

   possesses asymptotic significance level α.

Proof. See Appendix.

To make the results in Proposition 3 operational, the quantile $d_{\mathcal{U},1-\alpha}$ can be estimated using a parametric bootstrap procedure consisting in replicating M times the subsequent steps (ii)-(iv).

(i) Estimate the relative frequencies in the original samples with sizes n_A and n_B: $\widehat{p}_{y,A}$ and $\widehat{p}_{y,B}$, $y = 0, 1, \cdots, D^*$.

(ii) Draw two independent samples of size n_A, n_B from two multinomial distributions with parameters $(n_A; \widehat{p}_{y,A})$ and $(n_B; \widehat{p}_{y,B}), y = 0, 1, \cdots, D^*$ respectively. They are the mth replicate ($m = 1,\ \ldots,\ M$).

(iii) For each replicate m, estimate the piecewise TID curves $\widehat{H}^m_A(p)$ and $\widehat{H}^m_B(p)$ connecting points $[(0,0), (\widehat{F}^m_{y,A}, \widehat{H}^m_{y,A})]$ and points $[(0,0), (\widehat{F}^m_{y,B}, \widehat{H}^m_{y,B})]$, $y = 0, 1, \cdots, D^*$, respectively.

(iv) For each replicate m, compute $\widehat{\Delta}^m(u_j) = \widehat{H}^m_A(u_j) - \widehat{H}^m_B(p_j)$ at each point p_j over the equally spaced grid, and consider

$$\mathcal{U}^{\max,m}_{n_a,n_B} = \sqrt{\frac{n_A n_B}{n_A+n_B}} \max_{1\le j\le J} \left(\widehat{\Delta}^m(p_j) - \widehat{\Delta}(p_j)\right).$$

(v) Compute the empirical cumulative distribution of the $\mathcal{U}^{\max,m}_{n_A,n_B}$ statistics over the M bootstrap replicates, namely:

$$\widehat{R}_{n_A,n_B,M}(x) = \frac{1}{M}\sum_{m=1}^{M} \mathbb{1}\left(\mathcal{U}^{\max,m}_{n_a,n_B} \le x\right), x \in \mathbb{R}.$$

Estimate the percentile $\widehat{d}_{\mathcal{U},1-\alpha} = \inf\{x : \widehat{R}_{n_A,n_B,M}(x) \ge 1-\alpha\}$ for a fixed level of significance α.

Proposition 4 *$\widehat{d}_{\mathcal{U},1-\alpha}$ converges almost surely to $d_{\mathcal{U},1-\alpha}$, as n_A, n_B, M tend to infinity.*

Proof. See Appendix.

From Propositions 3, 4, it follows that the testing procedure consisting in rejecting H_0 whether

$$\widehat{\Delta}(u_j) - \sqrt{\frac{n_A + n_B}{n_A n_B}} \widehat{d}_{\mathcal{U},1-\alpha} > 0 \text{ for some } j = 1, \ldots, J \tag{20}$$

possesses asymptotic significance level α.

5 TID dominance and deprivation measures

5.1 A deprivation index: descriptive aspects

We have shown that TID curves are related to the generalized Lorenz curve through eqn.(11). Therefore, TID dominance is equivalent to second-order stochastic dominance, and can be used to check deprivation orderings by all aggregate deprivation measures satisfying a set of properties, as for the TIP dominance (Jenkins and Lambert, 1998). Lasso de la Vega (2010) declined into the counting approach this set of properties traditionally used in the monetary poverty measurement.[6]

Following this reasoning, TID dominance of population A over population B implies that all deprivation indexes in B are larger than in A as long as the following axioms are satisfied:

(i) Focus. A deprivation measure should not change when the deprivation score of a non-deprived individual changes, as long as the individual remains non-deprived.

(ii) Symmetry. The measure should not change if two individuals switch their deprivation scores.

(iii) Replication invariance. The deprivation measure should not be affected by the pooling of identical populations.

(iv) Monotonicity. When the deprivation score of a deprived individual decreases, the aggregate measure of deprivation should decrease, as well.

(v) Distribution sensitivity. The reduction of deprivation measure due to a decrease in the deprivation score of an individual is inversely related to the score of the individual itself.

[6]See the basic papers of Atkinson (1987) and Foster and Shorrocks (1988). Zheng (2000) and Zheng (2023) for exhaustive and updated reviews.

An index of deprivation that satisfies the above axioms is the Multiple Deprivation Index (MDI), originally introduced by Espinoza-Delgado and Silber (2024). The MDI index represents an extension of the Sen-Shorrocks poverty index (Sen, 1976; Shorrocks, 1995) and it summarizes in a single measure the three I's of the TID curve. The index can be formalized as follows:

$$MDI = \frac{1}{N^2} \sum_{i=1}^{N} (2N - 2i + 1) x_i, \tag{21}$$

where i is the position of individual i in the distribution of the raw scores decreasingly ordered. The Multiple Deprivation Index can be also written as a function of the average deprivation gap of the population $\bar{x}$, and the Gini index of the censored distribution of the deprivation scores G_x:

$$MDI = \bar{x}(1 + G_x). \tag{22}$$

Equation (22) is derived from equation (21) since the Gini index G_x is equal to:

$$G_x = 1 - \left(\frac{1}{N^2 \bar{x}} \right) \left[2 \sum_{i=1}^{N} i x_i - N\bar{x} \right]. \tag{23}$$

If the sub-population of the q_k deprived individuals is considered, the Gini index G_x can be also written as:

$$G_x = 1 - H_k(1 - G_q),$$

where H_k is the incidence of deprivation (see eqn. (1)), and G_q is the Gini inequality of the deprivation gaps of the poor.

Finally, MDI can be expressed in terms of incidence of deprivation H, intensity I, and inequality among the deprived, similarly to what shown in Shorrocks (1995, p. 1228):

$$MDI = H_k I_q [2 - H_k(1 - G_q)]. \tag{24}$$

The index ranges from 0 (minimum deprivation) to $D^* = D - (k - 1)$.[7] When $k = D$ the index reduces to $MDI = H_D(2 - H_D)$, while when $k = 1$ the average deprivation gap I_q corresponds to the average number of deprivations among the poor.

The index MDI can be also written in a different way, useful for statistical inference purposes. Denote by N_j ($j = 0, 1, \ldots, D^*$) the number of individuals in the population

[7] When the deprivation gaps x_i are normalised, the index ranges from 0 to 1.

with j deprivation gaps, and by $N_{surv,j} = N_j + N_{j+1} + \cdots + N_{D^*}$ $(j = 0, 1, \ldots, D^*)$ the corresponding inverse cumulative sums; denote further by $\overline{x}_N = \sum x_i/N$ the finite population mean. Since $\overline{x}_N/N = O(N^{-1})$, the index MDI can be then written as

$$\begin{aligned} MDI &= \frac{1}{N^2}\sum_{j=0}^{D^*} j\left\{\sum_{i=N_{surv,j+1}+1}^{N_{surv,j}} 2(N-i)+1\right\} \\ &= 2\sum_{j=0}^{D^*} j\left\{\frac{N_j}{N} - \left(\frac{N_{surv,j+1}}{N}\frac{N_j}{N} + \frac{1}{2}\left(\frac{N_j}{N}\right)^2\right)\right\} + \frac{2}{N}\overline{x}_N \\ &= 2\sum_{j=0}^{D^*} j\left\{1 - \frac{N_{surv,j+1}}{N}\frac{N_j}{N} - \frac{1}{2}\frac{N_j}{N}\right\}\frac{N_j}{N} + O(N^{-1}). \end{aligned} \tag{25}$$

At an "infinite population" level, similarly to Section 3.2, let X be the r.v. "deprivation gap", taking value j $(= 0, 1, \ldots, D^*)$ with probability $p_X(x)$, and let $S_X(x) = 1 - F_X(x-1) = p_X(x) + p_X(x+1) + \cdots + p_X(D^*)$ be the corresponding survival function. The index MDI can be then written as

$$\begin{aligned} MDI &= 2\sum_{j=0}^{D^*} j\left\{1 - S_X(j+1) - \frac{p_X(j)}{2}\right\}p_X(j) \\ &= 2\sum_{j=0}^{D^*} j\left\{F_X(j) - \frac{p_X(j)}{2}\right\}p_X(j). \end{aligned} \tag{26}$$

The subsequent section is devoted to statistical inference for the deprivation measure (26).

5.2 Inference on the deprivation index

Let $X_1, \ldots, X_n$ be a random sample of size n, composed by *i.i.d.* r.v.s with $P(X_i = x) = p_X(x)$, $x = 0, 1, \ldots, D^*$. To simplify the notation, define

$$\pi_x = p_X(x),\ x = 0, 1, \ldots, D^*,$$
$$\Pi_x = \pi_0 + \cdots + \pi_x,\ x = 0, 1, \ldots, D^*.$$

The index MDI (26) can be then written as

$$MDI = 2\sum_{j=0}^{D^*} j\left\{\Pi_j - \frac{\pi_j}{2}\right\}\pi_j. \tag{27}$$

A natural estimator of MDI, which is by invariance a MLE, is:

$$\widehat{MDI} = 2\sum_{j=0}^{D^*} j\left\{\widehat{\Pi}_j - \frac{\widehat{\pi}_j}{2}\right\}\widehat{\pi}_j \tag{28}$$

where

$$\widehat{\pi}_j = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{(X_i=j)},\ \widehat{\Pi}_j = \widehat{\pi}_0 + \cdots + \widehat{\pi}_j;\ \ j = 0,\, 1,\, \ldots,\, D^*.$$

Let us now consider the $D^* + 1$-dimensional vectors

$$\begin{aligned}\boldsymbol{\pi} &= [\pi_0, \pi_1, \ldots, \pi_{D^*}]^T,\ \boldsymbol{\Pi} = [\Pi_0, \Pi_1, \ldots, \Pi_{D^*}]^T;\\ \widehat{\boldsymbol{\pi}} &= [\widehat{\pi}_0, \widehat{\pi}_1, \ldots, \widehat{\pi}_{D^*}]^T,\ \widehat{\boldsymbol{\Pi}} = [\widehat{\Pi}_0, \widehat{\Pi}_1, \ldots, \widehat{\Pi}_{D^*}]^T.\end{aligned}$$

The $D + 1$-variate r.v. $n\widehat{\boldsymbol{\pi}}$ has a Multinomial distribution with parameters n, $\pi_0, \ldots, \pi_D$. As well-known, from the Central Limit Theorem it follows that $\sqrt{n}(\widehat{\boldsymbol{\pi}} - \boldsymbol{\pi})$ tends in distribution, as $n \to \infty$, to a singular Multinormal distribution, with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}_\pi$ having diagonal elements $\pi_j(1 - \pi_j)$ and extra-diagonal elements $-\pi_j\pi_h$ ($j,\, h = 0,\, 1,\, \ldots,\, D^*$).

Proposition 5 *Let L be a $(D^* + 1) \times (D^* + 1)$ lower triangular matrix as in (A.1); and define the $(D^* + 1)$-dimensional vectors:*

$$\begin{aligned}\boldsymbol{a}_1 &= \begin{bmatrix} 0 \\ \Pi_1 - \frac{\pi_1}{2} \\ 2\left(\Pi_2 - \frac{\pi_2}{2}\right) \\ \cdots \\ D^*\left(\Pi_{D^*} - \frac{\pi_{D^*}}{2}\right)\end{bmatrix},\ \ \boldsymbol{a}_2 = \begin{bmatrix} 0 \\ \pi_1 \\ 2\pi_2 \\ \cdots \\ D^*\pi_{D^*}\end{bmatrix};\\ \boldsymbol{c}^T &= 2\boldsymbol{a}_1^T - 2\boldsymbol{a}_2^T\boldsymbol{L} - \boldsymbol{a}_2^T.\end{aligned}$$

As $n \to \infty$, the r.v. $\sqrt{n}(\widehat{MDI} - MDI)$ tends in distribution to a Normal variate with expectation 0 and variance $\mathbf{c^T}\boldsymbol{\Sigma}_\pi\mathbf{c}$. In symbols:

$$\sqrt{n}(\widehat{MDI} - MDI) \xrightarrow{d} \mathcal{N}(\mathbf{0}, \mathbf{c^T}\boldsymbol{\Sigma}_\pi\mathbf{c}).$$

Proof. See Appendix.

Proposition 5 allows us to construct a confidence interval for MDI. Define the vectors $\widehat{\boldsymbol{a}}_1$, $\widehat{\boldsymbol{a}}_2$, $\widehat{\boldsymbol{\Sigma}}_\pi$ exactly as $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{\Sigma}_\pi$, respectively, but with π_x, Π_x replaced by their MLEs $\widehat{\pi}_x$, $\widehat{\Pi}_x$. Define further $\widehat{\boldsymbol{c}} = 2\widehat{\boldsymbol{a}}_1 - 2\boldsymbol{L}^T\widehat{\boldsymbol{a}}_2 - \widehat{\boldsymbol{a}}_2$. Then, as a consequence of the above results and the continuous mapping theorem, it is easy to see that $\widehat{\boldsymbol{c}}^T\widehat{\boldsymbol{\Sigma}}_\pi\widehat{\boldsymbol{c}}$ tends in probability, as n increases, to $\boldsymbol{c}^T\boldsymbol{\Sigma}_\pi\boldsymbol{c}^T$. In symbols:

$$\widehat{\boldsymbol{c}}^T\widehat{\boldsymbol{\Sigma}}_\pi\widehat{\boldsymbol{c}} \xrightarrow{p} \boldsymbol{c}^T\boldsymbol{\Sigma}_\pi\boldsymbol{c}^T \ \text{ as } n \to \infty.$$

Hence, the interval

$$\left[\widehat{MDI} - \frac{1}{\sqrt{n}} z_{\alpha/2} \sqrt{\widehat{\boldsymbol{c}}^T \widehat{\boldsymbol{\Sigma}}_\pi \widehat{\boldsymbol{c}}},\ \widehat{MDI} + \frac{1}{\sqrt{n}} z_{\alpha/2} \sqrt{\widehat{\boldsymbol{c}}^T \widehat{\boldsymbol{\Sigma}}_\pi \widehat{\boldsymbol{c}}}\right] \tag{29}$$

is a confidence interval for MDI with asymptotic level $1 - \alpha$.

6 How has the pandemic affected the downward trend in material deprivation in the EU?

The official EU deprivation scale provides a measure related to the (in)ability of individuals to afford a set of thirteen predefined items (Guio et al., 2017), that are considered to be desirable or even necessary to experience an adequate quality of life. The items, seven related to the household and six related to the individual, are listed in Table 1.

Table 1: The thirteen items for measuring material and social deprivation in the EU.

Item description	**Measurement level**
Capacity to face unexpected expenses	Household
Capacity to afford paying for one week annual holiday away from home	Household
Capacity to being confronted with payment arrears (mortgage or rental payments, utility bills, hire purchase installments or other loan payments)	Household
Capacity to afford a meal with meat, chicken, fish or a vegetarian equivalent every second day	Household
Ability to keep home adequately warm	Household
Have access to car-van for personal use	Household
Replace worn-out furniture	Household
Having internet connection	Individual
Replacing worn-out clothes by some new ones	Individual
Having two pairs of properly fitting shoes (including a pair of all-weather shoes)	Individual
Spending a small amount of money each week on him/herself	Individual
Having regular leisure activities	Individual
Getting together with friends/family for a drink/meal at least once a month	Individual

The material and social deprivation rate is defined as the proportion of the population that is unable to afford five or more out of this list of thirteen items. The severe material and social deprivation rate is defined as the proportion of the population that is unable to afford seven or more of the above-mentioned items. As reported in Table 1, some items are recorded at household level. However, the unit of our analysis is the individual, and household failures are attributed to each household member. We analyze the distribution of achievement failures among individuals.

The indicators come from the European Union Survey on Income and Living Conditions (EU-SILC).[8] The survey is employed by European Union member states and the Commission to monitor national and EU progress toward key objectives for the social inclusion process since the launch of the Europe 2020 strategy and included in the 2030 Agenda for Sustainable Development.

Since 2012, incidence of deprivation has continuously declined in almost all the EU-27 countries. Notably, the share of population suffering from material and social deprivation (cutoff fixed at $k = 5$ items) was reduced by more than one third between 2014 and 2019. The pandemic ended this trend, as deprivation surged in 2020. The most recent release of EU-SILC 2022 cross-sectional data[9] allows to understand what happened after the pandemic to the downward trend of material and social deprivation experienced until 2019 in the EU. According to the Eurostat statistics, the material and social deprivation rate in the EU was 12.7% in 2022, slightly above the 12.5% reached in 2019. However, changes in incidence of deprivation largely differ across the 27 countries. The TID curves complement the picture of deprivation incidence with information also about deprivation intensity and inequality. The deprivation profiles are estimated by considering the list of 13 items and a threshold equal to $k = 5$ in accordance to the official EU definition of material and social deprivation.

In the majority of countries,[10] a reduction in the deprivation rate is reported (Eurostat, 2023). For most of them, deprivation is unambiguously lower in 2022 since the TID 2022 curve lies wholly below the TID 2019 curve and it flattens sooner, revealing a reduction of the prevalence rate along with a clear dominance of the TID 2022 curve. As an illustrative case, Figure 4 shows the estimated deprivation profiles for Poland in 2019 (red) and in 2022 (blue), along with the TID dominance test results. The sample sizes are $n_{19} = 41,622$ and $n_{22} = 33,893$, respectively. The upper panel of Figure 4 shows a reduction by more than two percentage points in the incidence of deprivation. The no crossing of the TID curves indicates also a reduction in the two other dimensions of deprivation, intensity and inequality. Formal dominance test confirms these results (lower panel of Figure 4). The TID dominance is assessed by considering M=500 replications. The estimated difference $\widehat{\Delta}(u_j)$ between 2019 and 2022 are always

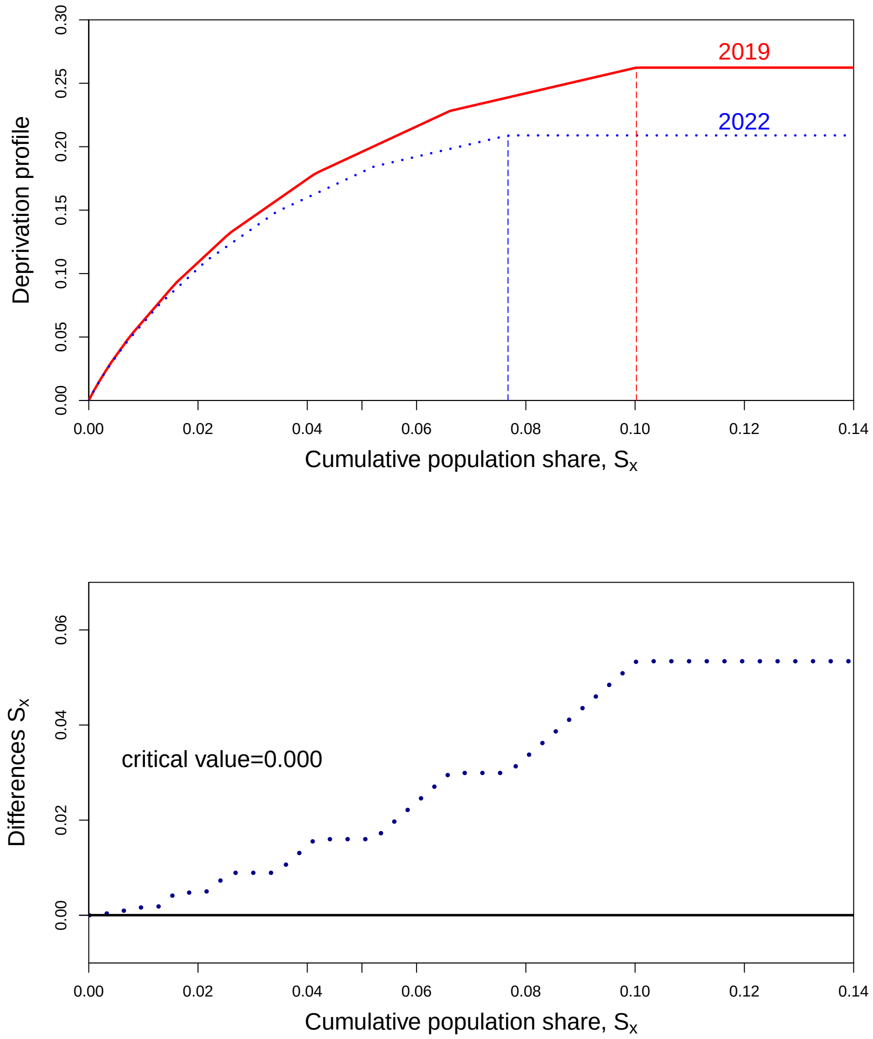
[8] The set of the 13 items is available from 2014. Previously, material deprivation was measured considering only nine items.

[9] Eu-Silc2004-2022_v.2 (release Autumn 2023).

[10] Austria (AT), Belgium (BE), Bulgaria (BG), Czechia (CZ), Denmark (DK), Estonia (EE), Finland (FI), Ireland (IE), Italy (IT), Greece (EL), Latvia (LV), Lithuania (LT), Hungary (HU), Netherlands (NL), Poland (PL), Portugal (PT), Romania (RO), Slovakia (SK), Slovenia (SI).

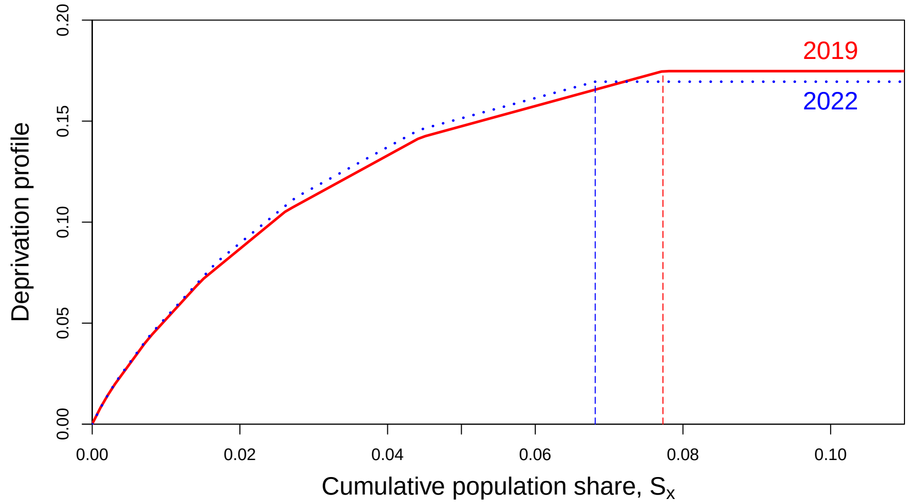
above the critical value $d_{\mathcal{U},1-\alpha} = 0.00011$ with $\alpha = 0.05$. Therefore the hypothesis that the year 2022 dominates 2019 cannot be rejected.

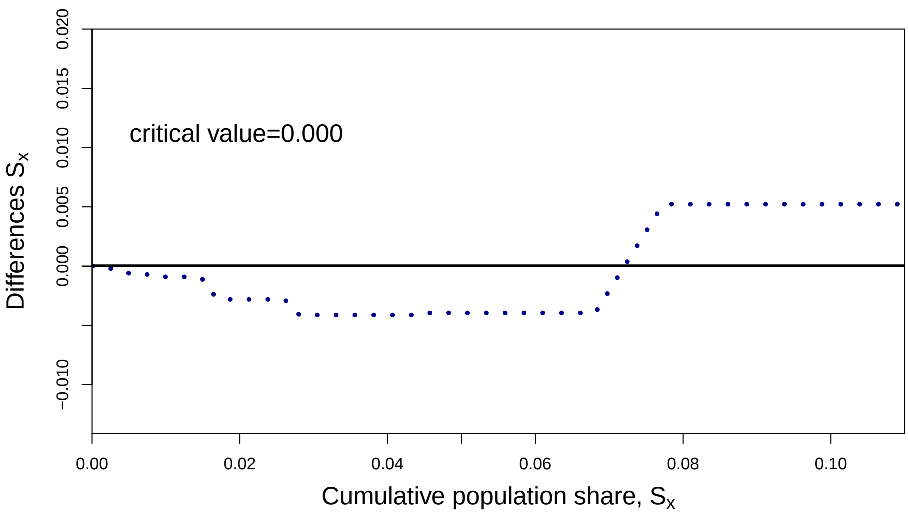
Figure 4: Poland: Estimated TID curves in 2019 and 2022 (upper panel); Estimated differences between the TID curves along with the critical value $d_{\mathcal{U},0,95}$ (lower panel).



In a few countries which experience a reduction in the incidence of deprivation, their deprivation curves intersect. These countries are: Latvia, Estonia, Slovakia. Figure 5 shows the estimated deprivation profiles for Estonia in 2019 (red) and in 2022 (blue). The sample sizes are $n_{19} = 12,395$ and $n_{22} = 9,733$, respectively. The vertical lines indicate the point at which the TID curves flatten, and they cross the horizontal axis at the value of the incidence of deprivation. The significant decrease of the deprivation rate, equal to one percentage point, between 2019 and 2022 is also accompanied by a reduction of intensity of deprivation, varying from 0.175 to 0.170. However the 2019 curve intersects the 2022 TID curve at around 7 percent of cumulative population share (see the upper panel in Figure 5, and, more evidently, the lower panel). This crossing shows that the intensity of deprivation in 2019 was higher than in 2022 but only from this point onwards, indicating that the poorest Estonian citizens were more deprived in 2022 than in 2019.

Figure 5: Estonia: Estimated TID curves in 2019 and 2022 (upper panel); Estimated differences between the TID curves along with the critical value $d_{\mathcal{U},0,95}$ (lower panel).



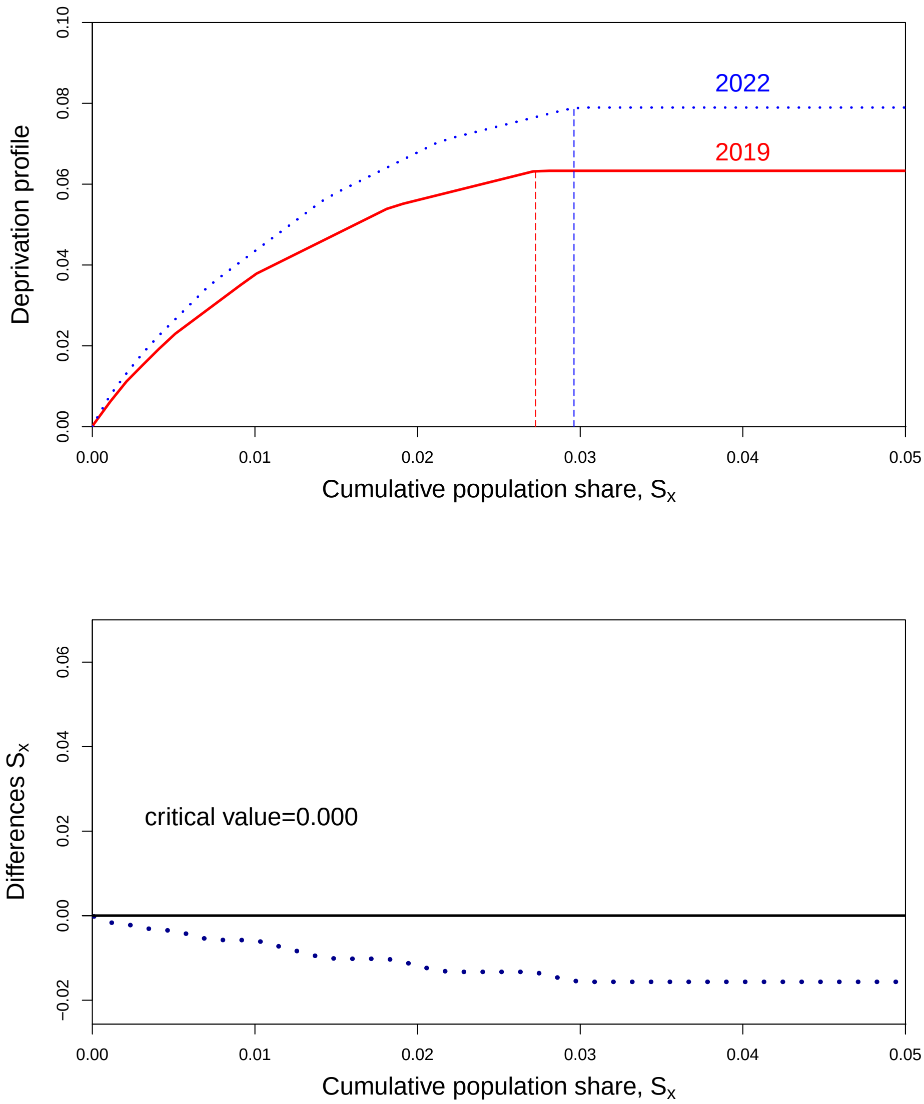


In a few countries the incidence of deprivation instead increased.[11] For Germany, France and Luxembourg significant methodological changes were introduced in the structure of the EU-SILC survey (Eurostat, 2023), therefore temporal comparison should be taken with caution. For this reason we illustrate the deprivation profiles

[11] France (FR), Germany (DE), Luxembourg (LU), Sweden (SE), Spain (ES).

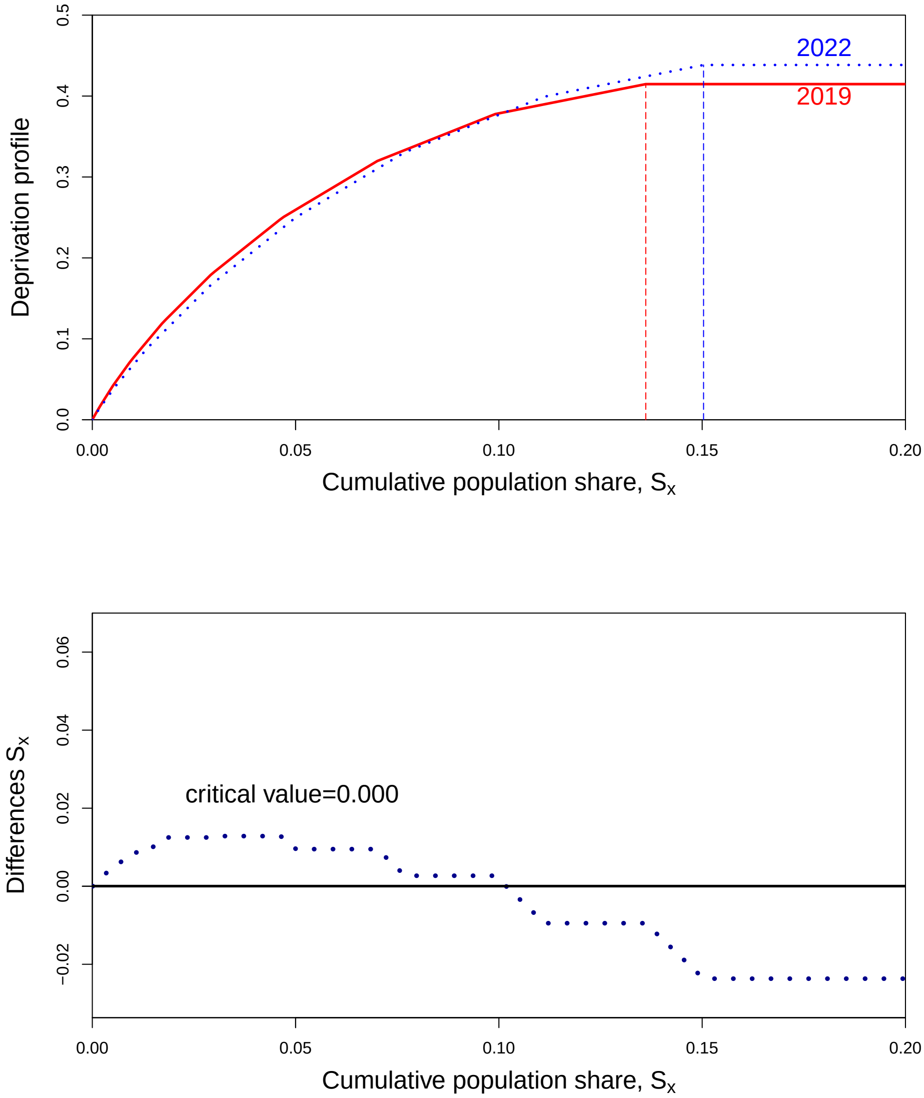
only for Sweden and Spain. For Sweden the estimated TID 2019 curve is below the estimated 2022 curve in each point of the grid (upper panel of Figure 6), an indication of an upsurge of the other dimensions of poverty, intensity and inequality. This evidence is confirmed by the stochastic dominance test (lower panel of Figure 6). The sample sizes are $n_{19} = 10,125$ and $n_{22} = 14,856$, respectively.

Figure 6: Sweden: Estimated TID curves in 2019 and 2022 (upper panel); Estimated differences between the TID curves along with the critical value $d_{\mathcal{U},0,95}$ (lower panel).



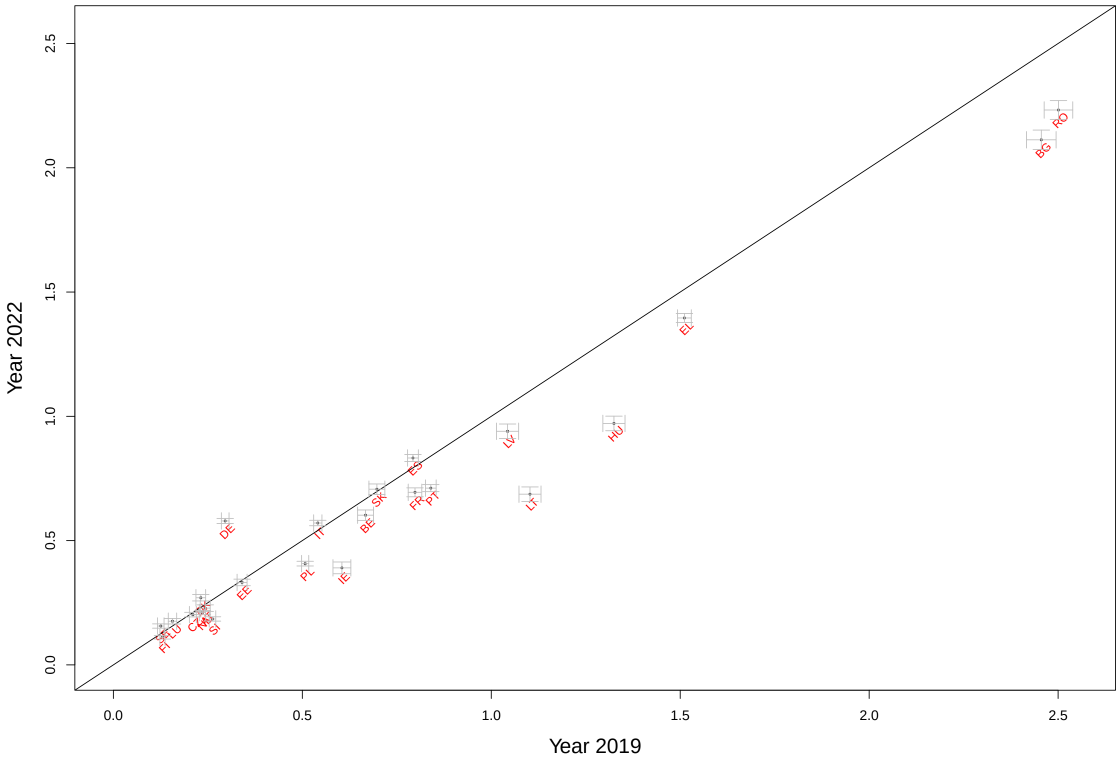
The case of Spain is less straightforward (see Figure 7), because a reduction of the incidence of deprivation by 1.4% occurred, but stochastic dominance does not apply. The sample sizes are $n_{19} = 32,779$ and $n_{22} = 49,050$, respectively. The crossing of the curves at around 10 percent of cumulative share of the population reveals that the poorest decile of individuals suffered from deprivation more in 2019 than in 2022. Deprivations are more equally distributed among the poor in 2022 than in 2019, as confirmed by a significant reduction of the Gini inequality among the deprived, from 0.349 in 2019 to 0.325 in 2022. Changes in the intensity and inequality dimensions of aggregate deprivation offset changes in incidence in deprivation, and this dynamic is clear only with the TID curves.

Figure 7: Spain: Estimated TID curves in 2019 and 2022 (upper panel); Estimated differences between the TID curves along with the critical value $d_{\mathcal{U},0,95}$ (lower panel).



Finally, Figure 8 compares the MDI indexes of the European countries between 2019 and 2022. Countries below the 45-degree line indicates a reduction in deprivation.

Figure 8: Estimates and 95% error bars of Multiple Deprivation Indices in European countries, years 2019 and 2022.



7 Conclusions

This study is motivated by the raising interest in using categorical variables to complement the traditional evaluation of monetary poverty by a full-fledged analysis of several deprivations suffered by individuals. We introduced a formal representation of a deprivation profile when deprivations are measured by binary variables, and individuals by a total score generated by summing the individual deprivation affirmative responses. With individuals ranked in descending order of total score, the profile is a piecewise linear curve that plots the cumulative per capita deprivation gap against the corresponding cumulative share of individuals.

The deprivation profile takes into account the incidence, the intensity and the inequality of deprivation, and, being an extension of the 'Three I's of Poverty' (TIP) curve of Jenkins and Lambert (1997), we named it the 'Three I's of Deprivation' (TID) curve. In complementing statistical inference for TIP curves already proposed in the literature, we provided confidence bounds for TID curves, and consistent tests for deprivation stochastic dominance relations. TID dominance implies the same ranking of a class of deprivation counting measures satisfying a set of axioms. One of these measures is the extension in a binary setting of the Sen-Shorrocks poverty measure.

These curves can be particularly appropriate to shed lights on different aspects of deprivation. Comparing TID curves over time, we drew an empirical portrait of the evolution of material and social deprivation in Europe that goes beyond the mere analysis of head-count ratios. In most countries the reduction of incidence before and immediately after the pandemic was also accompanied by a reduction in intensity and inequality of deprivation. In these cases, formal tests confirmed the dominance of the 2022 TID curve over the 2019 curve. Similarly, a few countries experienced an increase of deprivation in all its aspects. The 2019 dominated the 2022 and the deprivation incidence increased. However, in some cases deprivation ordering was ambiguous since decreasing (increasing) deprivation rates were associated with a decrease (increase) in incidence and inequality. These situations were in fact characterized by intersection of the TID curves.

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Appendix

Proof Proposition 1. Statement 1 is an immediate consequence of the multidimensional Central limit Theorem. Statement 2 is an immediate consequence of Statement 1 and the relationship $\mathcal{W}_n = \mathrm{L}\mathcal{T}_n$, L being the lower triangular matrix

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}. \tag{A.1}$$

Finally, Statement 3 is a consequence of Statement 2 and the relationship $\mathcal{V}_n = \mathrm{B}\mathcal{W}_n$, where B is the lower triangular matrix

$$\mathbf{B} = \begin{bmatrix} D^* & 0 & 0 & \cdots & 0 \\ 1 & D^*-1 & 0 & \cdots & 0 \\ 1 & 1 & D^*-2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}. \tag{A.2}$$

Proof Proposition 2. As a consequence of eqns. (12), (13), it is easy to verify that, as $n \to \infty$, the following relationships asymptotically hold:

$$\Pr\left(\widehat{F_y} - \frac{d_{\mathcal{W},1-\alpha}}{\sqrt{n}} \leq F_y \leq \widehat{F_y} + \frac{d_{\mathcal{W},1-\alpha}}{\sqrt{n}}\right) = 1-\alpha, \ \forall y, \tag{A.3}$$

$$\Pr\left(\widehat{H_y} - \frac{d_{\mathcal{V},1-\alpha}}{\sqrt{n}} \leq H_y \leq \widehat{H_y} + \frac{d_{\mathcal{V},1-\alpha}}{\sqrt{n}}\right) = 1-\alpha, \ \forall y. \tag{A.4}$$

Hence, using the Bonferroni correction, simultaneous confidence intervals for each pair (F_y, H_y) can be defined as:

$$\left(\widehat{F_y} - \frac{d_{\mathcal{W},1-\alpha/2}}{\sqrt{n}} \leq F_y \leq \widehat{F_y} + \frac{d_{\mathcal{W},1-\alpha/2}}{\sqrt{n}} \bigwedge \widehat{H_y} - \frac{d_{\mathcal{V},1-\alpha/2}}{\sqrt{n}} \leq H_y \leq \widehat{H_y} + \frac{d_{\mathcal{V},1-\alpha/2}}{\sqrt{n}}\right) \tag{A.5}$$

where the overall confidence level of the D^* regions $(A.5)$ is $\geq 1-\alpha$.

Proof Proposition 3. First of all, using the Delta method, it can be seen that, as n_A, n_B tend to infinity:

$$\mathcal{U}_{n_A,n_B}(\mathbf{u}) = \sqrt{\frac{n_A n_B}{n_A+n_B}}\left(\widehat{\Delta}(\mathbf{p}) - \Delta(\mathbf{p})\right) \xrightarrow{d} \mathcal{U}(\mathbf{u}) \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_{\mathcal{U}}), \tag{A.6}$$

where the covariance matrix $\mathbf{\Sigma}_{\mathcal{U}}$ is possibly of not full rank (but this does not affect the subsequent developments). As a consequence of $(A.6)$, from the continuous mapping theorem it follows that

$$\max_j \mathcal{U}_{n_A,n_B}(u_j) \xrightarrow{d} \max_j \mathcal{U}(\mathbf{u_j}) = \mathcal{U}^{max} \ \text{ as } n_a,\, n_B \to \infty. \tag{A.7}$$

Moreover, from well-known properties of the Multinormal distribution, the r.v. $\mathcal{U}^{max}$ possesses absolutely continuous distribution, so that there exists a unique percentile $d_{\mathcal{U},1-\alpha}$ for which $P(\mathcal{U}^{max} > d_{\mathcal{U},1-\alpha}) = 1-\alpha$. This proves statement 1.

To prove statement 2, it is enough to observe that, as a consequence of the above arguments, the following chain of relationships holds:

$$\begin{aligned} 1-\alpha &= \Pr\left(\max_{1\leq j\leq J}\mathcal{U}(u_j) \leq d_{\mathcal{U},1-\alpha}\right) \\ &\simeq \Pr\left(\sqrt{\frac{n_A n_B}{n_A+n_B}}(\widehat{\Delta}(u_j)-\Delta(u_j)) \leq d_{\mathcal{U},1-\alpha}, \forall j=1,\cdots J\right) \\ &= \Pr\left(\Delta(u_j) \geq \widehat{\Delta}(u_j) - \sqrt{\frac{n_A+n_B}{n_A n_B}}d_{\mathcal{U},1-\alpha}, \forall j=1,\cdots J\right). \end{aligned} \tag{A.8}$$

On the basis of eqn. $(A.8)$, the half-lines

$$\left[\widehat{\Delta}(u_j) - \sqrt{\frac{n_A+n_B}{n_A n_B}}d_{\mathcal{U},1-\alpha},\ +\infty\right),\quad j=1,\ \ldots,\ J$$

are simultaneous confidence regions of overall asymptotic level $1-\alpha$.

Finally, statement 3 is an immediate consequence of 2.

Proof Proposition 4. In the first place by standard arguments, it is easy to see that

$$\sup_{x\in\mathbb{R}}|\widehat{R}_{n_A,n_B,M}(x) - P(\mathcal{U}^{\max} \leq x)| \xrightarrow{\text{a.s.}} 0 \text{ as } n_A, n_B, M \to \infty.$$

The continuity of the d.f. $P(\mathcal{U}^{\max} \leq x)$, again by standard arguments, implies then the convergence of quantile: $\widehat{d}_{\mathcal{U},1-\alpha} \stackrel{a.s.}{\to} d_{\mathcal{U},1-\alpha}$ as n_A, n_B, M increase.

Proof Proposition 5. By elementary algebra, $\sqrt{n}(\widehat{MDI}-MDI)$ can be decomposed as follows:

$$\sqrt{n}(\widehat{MDI}-MDI) = \sqrt{n}(A_{1,n}+A_{2,n}-B_n) \tag{A.9}$$

where

$$\begin{aligned} A_{1,n} &= 2\sum_{j=0}^{D^*} j(\widehat{\Pi}_j - \frac{\widehat{\pi}_j}{2})(\widehat{\pi}_j - \pi_j); \\ A_{2,n} &= 2\sum_{j=0}^{D^*} j\pi_j(\widehat{\Pi}_j - \Pi_j); \\ B_n &= \sum_{j=0}^{D^*} j\pi_j(\widehat{\pi}_j - \pi_j). \end{aligned}$$

Denote by $o_P(1)$ a term tending to 0 in probability as n goes to infinity. Using the Law of Large Numbers, the term $\sqrt{n}A_{1,n}$ is equal to

$$\begin{aligned}\sqrt{n}A_{1,n} &= 2\sum_{j=0}^{D^*} j(\Pi_j - \frac{\pi_j}{2})\sqrt{n}(\widehat{\pi}_j - \pi_j) + o_P(1) \\ &= 2\boldsymbol{a}_1^T\sqrt{n}(\widehat{\boldsymbol{\pi}} - \boldsymbol{\pi}) + o_P(1). \qquad \text{(A.10)}\end{aligned}$$

Next, from the relationship $\widehat{\boldsymbol{\Pi}} - \boldsymbol{\Pi} = \boldsymbol{L}(\widehat{\boldsymbol{\pi}} - \boldsymbol{\pi})$, it follows that the term $\sqrt{n}A_{2,n}$ can be written as:

$$\begin{aligned}\sqrt{n}A_{2,n} &= 2\boldsymbol{a}_2^T\sqrt{n}(\widehat{\boldsymbol{\Pi}} - \boldsymbol{\Pi}) \\ &= 2\boldsymbol{a}_2^T\boldsymbol{L}\sqrt{n}(\widehat{\boldsymbol{\pi}} - \boldsymbol{\pi}). \qquad \text{(A.11)}\end{aligned}$$

The term $\sqrt{n}B_n$ can be expressed, in its turn, as

$$\sqrt{n}B_n = \boldsymbol{a}_2^T\sqrt{n}(\widehat{\boldsymbol{\pi}} - \boldsymbol{\pi}). \qquad \text{(A.12)}$$

Finally, from $(A.10)$-$(A.12)$, the following equality can be immediately obtained:

$$\sqrt{n}(\widehat{MDI} - MDI) = 2\boldsymbol{c}^T\sqrt{n}(\widehat{\boldsymbol{\pi}} - \boldsymbol{\pi}) + o_P(1)$$

from which the proposition follows.

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