CONTRIBUTI DI RICERCA CRENOS

A VECTOR MULTIPLICATIVE ERROR MODEL WITH SPILLOVER EFFECTS AND CO-MOVEMENTS

Edoardo Otranto

WORKING PAPERS

2024/04

arkadia

Centro Ricerche Economiche Nord Sud
(CRENoS)
Università di Cagliari
Università di Sassari

CRENOS was set up in 1993 with the purpose of organising the joint research effort of economists from the two Sardinian universities (Cagliari and Sassari) investigating dualism at the international and regional level. CRENoS' primary aim is to improve knowledge on the economic gap between areas and to provide useful information for policy intervention. Particular attention is paid to the role of institutions, technological progress and diffusion of innovation in the process of convergence or divergence between economic areas. To carry out its research, CRENoS collaborates with research centres and universities at both national and international level. The centre is also active in the field of scientific dissemination, organizing conferences and workshops along with other activities such as seminars and summer schools.
CRENoS creates and manages several databases of various socio-economic variables on Italy and Sardinia. At the local level, CRENoS promotes and participates to projects impacting on the most relevant issues in the Sardinian economy, such as tourism, environment, transports and macroeconomic forecasts.

www.crenos.unica.it
crenos@unica.it

CRENoS – Cagliari
Via San Giorgio 12, I-09124 Cagliari, Italia
tel. +39-070-6756397; fax +39-070- 6756402

CRENoS - Sassari
Via Muroni 25, I-07100 Sassari, Italia
tel. +39-079-213511

Title: A VECTOR MULTIPLICATIVE ERROR MODEL WITH SPILLOVER EFFECTS AND CO-MOVEMENTS

Prima Edizione: Febbraio 2024
ISBN: 978 88 68515 003

Viale Bonaria 98 - 09125 Cagliari
Tel. 070/6848663 - info@arkadiaeditore.it
www.arkadiaeditore.it

A Vector Multiplicative Error Model with Spillover Effects and Co-movements

Edoardo Otranto
University of Messina & CRENoS

Abstract

Modern approaches to financial time series aim to model in a multivariate framework the volatility of different indices or assets, which could influence each other, creating spillover effects. Furthermore, the integration of financial markets provides a similar dynamics (co-movement). We propose a new model for volatility vectors, belonging to the family of Multiplicative Error Models, which incorporates spillover and co-movement effects. By adopting an appropriate parameterization, it is possible to estimate this model even for high dimensional vectors of volatility. To reduce the number of unknown coefficients, we propose a 3-step model-based clustering procedure. The proposed model is applied to a set of seventeen world financial indices, providing a useful interpretation of spillover effects and co-movements. Furthermore, the proposed parameterization is compared with two alternatives, showing significantly better performance.

Keywords: high-dimensional time series, high-low range, model-based clustering, multiplicative factors, vector of volatility.
Jel Classification: C32, C38, C55, C58.

1 Introduction

The increasing globalization of financial markets and their mutual dependence have favored the development of multivariate models for vectors of volatility indices or assets. The early models extend the traditional GARCH (Bollerslev, 1986) approaches to multivariate cases for the analysis of covariance matrices (e.g. the BEKK model of Engle and Kroner, 1995) or correlation matrices (e.g. the DCC model of Engle, 2002a). When modeling covariance (correlation) matrices, a typical problem is to provide parameterizations involving estimated positive definite matrices, which are also parsimonious to avoid the so-called *curse of dimensionality* problem (Bauwens, Laurent, and Rombouts, 2006). The solutions adopted are generally based on scalar or diagonal specifications, excluding the dependence of each conditional variance on the other lagged variances; alternatively, the 2-step estimation procedure for DCC models considers independent univariate GARCH models for each conditional variance. In practice, the usual multivariate GARCH-type models preclude the possibility of including spillover effects in the model specification.

A more recent approach to volatility modeling is the Multiplicative Error Model (MEM) of Engle (2002b), where a model for positive time series was developed, based on two multiplicative factors representing the conditional mean of the series and the positive disturbance respectively. The success of this model and its extensions is due to the diffusion of more precise measures of volatility, based on the high-low daily range (HLR-Parkinson, 1980) and ultra-high frequency data, the so-called realized volatility (RV- Andersen and Bollerslev, 1998; Andersen, Bollerslev, Diebold, and Labys, 2003), with robust to microstructure noise specifications (Barndorff-Nielsen, Hansen, Lunde, and Shephard, 2008). Both measures use the intraday movements of the prices: the former is very simple to derive, being based on the difference between the maximum and the minimum value of the day; the latter is based on several equally spaced intradaily observations. Although RV is recognized as a more precise measure, the availability of intraday data is not frequent; the HLR, on the other hand, uses less information but is generally available for all indices and tickers. Both measures are robust to microstructure noises and their quality is superior to methods derived from the GARCH approach, based on squared or absolute returns (Chou, Chou, and Liu, 2015).

MEM is particularly suitable for modeling this type of time series. The extension to the multivariate case has inaugurated a recent new approach, called vector MEM (vMEM), proposed by Engle and Gallo (2006), where the dependent variable is represented by a vector of positive series. In their application, Engle and Gallo (2006) consider a 3-variate vector, containing three different measures of volatility (absolute return, daily range, and realized volatility); they introduce the strong hypothesis that the vector of disturbances is composed of uncorrelated elements to provide an equation-by-equation parameter estimation. A first improvement in vMEM estimation was made by Cipollini, Engle, and Gallo (2013), who developed a semiparametric model (without distributional hypotheses) and a GMM estimator, capable of estimating the entire model, including the interdependence effects. Cipollini, Engle, and Gallo (2017) propose a copula-based approach to link Gamma and log-Normal marginal distributions of the innovations; they show that these alternative specifications provide equivalent results to the semiparametric approach of Cipollini et al. (2013). A useful solution is that of Taylor and Xu (2017), who adopt a log-Normal distribution for innovations, modeling the logarithm of the conditional mean of volatility, with the advantage of not imposing restrictions on parameters to guarantee positiveness. They compare this method with the semiparametric approach of Cipollini et al. (2013) via simulations, obtaining a certain consistency of results. All these approaches are applied to 3-variate random vectors to avoid the curse of dimensionality problem (a full 3-variate VMEM involves estimating 24 parameters; they increase to 42 in the 4-variate case).

We propose a new log-vMEM, along the lines of Taylor and Xu (2017), to capture spillover effects in financial market volatility; furthermore, we include in the model a common variable that drives the long-run movements of all markets, representing the co-movement of the financial time series. The spillover effects are represented by the lagged interdependence parameters of the vMEM. The common features are preliminarily identified from the principal components of the panel of volatilities and then inserted into the model with separate loading coefficients for each volatility; this idea was used by Atak and Kapetanios (2013), adding them to the Heterogeneous AutoRegressive (HAR) model of Corsi (2009). In our specification, the co-movement effect does not directly affect the GARCH-type equation of the conditional mean, but we separate the long-run effect due to the common features of the volatilities from the idiosyncratic behavior of each index, also influenced by specific spillover effects. This can be achieved by decomposing the factor representing the conditional mean into two additive components; it can be considered an extension to the multivariate case of the Spillover Asymmetric MEM (SAMEM) of Otranto (2015), belonging to the class of Composite MEMs (Brownlees, Cipollini, and Gallo, 2012). We also address the problem of the high-dimensional vector of volatilities, proposing a reparameterization of the coefficients of the model, based on a model-based clustering algorithm. The new model is estimated on a data set composed of seventeen world financial indices and compared with two alternative parameterizations of the same basic model; we detect some evidence of improved performance of the proposed parameterization and useful interpretation of the results, despite the small number of coefficients used.

The paper is organized as follows: in the next section we present the new model: in subsection 2.1 we describe the fully parameterized model, while in subsection 2.2 we introduce the proposed parameterization for the high-dimensional case. Section 3 is devoted to the application in a 17-variate case, valorizing the output derived from the model estimation; this section describes in detail all the steps to build and validate the model, starting from reducing the number of unknown coefficients (subsection 3.1), then estimating the reparameterized model and interpreting the results (subsection 3.2), finally proposing a procedure to validate it and comparing the results with two alternative parameterizations, the simple scalar one and another less parsimonious one (subsection 3.3). Some final remarks will conclude the paper.

2 The model proposed

Let us consider n volatility indices collected in the vector $\boldsymbol{y}_t$. The proposed model is a vMEM (Cipollini et al., 2017), enriched by the presence of specific coefficients to consider spillover effects and by a common signal that drives all indices. For this purpose we adopt the logarithmic transformation of the variables, expressing the vMEM in the form proposed by Taylor and Xu (2017), modifying the conditional mean equation to include the co-movement effect. The log-specification for vMEM makes model estimation easier, allowing the adoption of the Normal distribution.

2.1 The fully parameterized model

Calling $\boldsymbol{x}_t$ the logarithm of $\boldsymbol{y}_t$, the model proposed, in the variance targeting version, is:

$$\begin{aligned} &\boldsymbol{y}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\varepsilon}_t \sim \ln N(\boldsymbol{m}, \boldsymbol{V}) \\ &\ln \boldsymbol{\mu}_t = \boldsymbol{\varsigma}_t + \boldsymbol{\delta}\xi_t \\ &\boldsymbol{\varsigma}_t = \bar{\boldsymbol{x}} + \boldsymbol{A}(\boldsymbol{x}_{t-1} - \bar{\boldsymbol{x}}) + \boldsymbol{B}(\boldsymbol{\varsigma}_{t-1} - \bar{\boldsymbol{x}}) \end{aligned} \tag{1}$$

where $\odot$ indicates the element-by-element Hadamard product. The vector of disturbances $\boldsymbol{\varepsilon}_t$ is a unit mean vector and $\boldsymbol{\mu}_t$ is the conditional mean of the volatility vector $\boldsymbol{y}_t$. Following Taylor and Xu (2017), $\boldsymbol{\varepsilon}_t$ is log-normally distributed with parameters $\boldsymbol{m} = (m_1, \ldots, m_n)'$ and the $n \times n$ matrix $\boldsymbol{V} = \{v_{ij}\}$, $(i, j = 1, \ldots, n)$. To satisfy the unit mean property, it is imposed the constraint $m_i = -v_{ii}/2$ $(i = 1, \ldots, n)$. As a consequence, $\boldsymbol{y}_t \sim \ln N(\ln \boldsymbol{\mu}_t + \boldsymbol{m}, \boldsymbol{V})$ and $\boldsymbol{x}_t \sim N(\ln \boldsymbol{\mu}_t + \boldsymbol{m}, \boldsymbol{V})$.

In (1) $\ln \boldsymbol{\mu}_t$ is decomposed into the sum of two unobservable components: the idiosyncratic log-volatility $\boldsymbol{\varsigma}_t$ and the common (weighted) signal ξ_t. The first represents the 'proper' dynamics of each volatility excluding the co-movement (in log terms), represented by ξ_t, which is loaded into each element of the full conditional log-volatility vector, $\ln \boldsymbol{\mu}_t$, with a different coefficient contained in the vector $\boldsymbol{\delta}$.

The common component ξ_t can be obtained as the first principal component of the n log-volatilities in $\boldsymbol{x}_t$. Of course, it is possible to consider more principal components to explain a high percentage of the entire variability, obtaining more common factors; however, due to the high degree of homogeneity of the volatility series, subject to a strong co-movement, very often (as in the application in Section 3) the percentage of variance explained is greater than 90% already with the first principal component.

The $(n \times 1)$ vector $\boldsymbol{\varsigma}_t$ follows a multivariate GARCH dynamics. The lagged effects of the volatility of each variable are represented by the elements on the diagonal of the matrix of coefficients $\boldsymbol{A} = \{\alpha_{ij}\}$; the off-diagonal elements are the coefficients of the spillover effects. More specifically, denoting with $x_{j,t}$ the log-volatility of the j-th series at time t (the j-th element of $\boldsymbol{x}_t$), the spillover effect from the j-th variable to the i-th variable at time t is $\alpha_{ij} x_{j,t}$.

We do not consider interactions between pairs of elements of $\boldsymbol{\varsigma}_{t-1}$, so that $\boldsymbol{B} = diag(\boldsymbol{\beta})$, where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)'$ is an $n-$dimensional vector containing the elements of the diagonal of $\boldsymbol{B}$.[1]

The variance targeting specification (Engle and Mezrich, 1996) of the third equation in (1) implies that the expectation of $\boldsymbol{\varsigma}_t$ is the mean of the log volatilities (indicated with $\bar{\boldsymbol{x}}$).

It is worth noting that, by adopting a logarithmic transformation of volatility, it is not necessary to impose positivity constraints on the parameters. The stationarity and invertibility conditions involve considering only the third equation in (1), so they can be

[1] Bauwens and Otranto (2023), modeling realized covariance matrices, show empirical evidence of non-interdependence between the elements of the lagged conditional covariance matrix, and also an equal coefficient, so that $\boldsymbol{\beta}$ is scalar.

easily derived from Appendix A of Taylor and Xu (2017). Indicating with $\boldsymbol{I}_n$ the $n \times n$ identity matrix, the process (1) is stationary if the modulus of the roots of

$$|\boldsymbol{I}_n - (\boldsymbol{A}z + \boldsymbol{B}z)| = 0$$

are greater than one. The process (1) is invertible if the modulus of the roots of

$$|\boldsymbol{I}_n - \boldsymbol{B}z| = \prod_{i=1}^{n}(1 - \beta_i z) = 0$$

are greater than one.

Parameter estimation is performed using the Quasi Maximum Likelihood Estimator (QMLE). Indicating with $\boldsymbol{\theta}$ the set of parameters to be estimated, under the hypothesis of log-normality of the disturbances, the log-likelihood is given by:

$$l(\boldsymbol{\theta}) = -\frac{Tn}{2}\ln 2\pi - \frac{T}{2}\ln|\boldsymbol{V}| - \sum_{t=1}^{T}\left[\sum_{i=1}^{n} x_{i,t} - \frac{1}{2}\left(\boldsymbol{x}_t - \ln\boldsymbol{\mu}_t - \boldsymbol{m}\right)'\boldsymbol{V}^{-1}\left(\boldsymbol{x}_t - \ln\boldsymbol{\mu}_t - \boldsymbol{m}\right)\right] \tag{2}$$

Robust standard errors can be obtained by applying a sandwich estimator of the covariance matrix (White, 1982). As suggested by Taylor and Xu (2017), consistency and asymptotic normality of QMLE, in this context, can be shown using the results of Nakatani and Teräsvirta (2009).

2.2 Parameterization for large datasets

The number of unknown coefficients in (1) is $n\left[(n+1)/2 + n + 2\right]$ and, if n is large, we run into the curse of dimensionality problem: some parameterization is needed to attain a parsimonious and feasible model. The reduction of the number of parameters can be obtained by identifying groups of volatilities that follow similar dynamics. This could be achieved with a simple agglomerative algorithm, composed of three steps.

Combining the second and third equations of model (1), the dynamics of the $i-th$ element of $\ln\boldsymbol{\mu}_t$ is given by:

$$\ln\mu_{i,t} = (1 - \alpha_{ii} - \beta_i)\bar{x}_i + \alpha_{ii}x_{i,t-1} + \beta_i\varsigma_{i,t-1} + \sum_{j\neq i}\alpha_{ij}x_{j,t-1} + \delta_i\xi_t \tag{3}$$

Setting $c_i = (1 - \alpha_{ii} - \beta_i)\bar{x}_i$, adding and subtracting $x_{i,t}$ and $\beta_i x_{i,t-1}$ in equation (3), and replacing $\varsigma_{i,t-1}$ with the corresponding expression derived from the second equation in (1), we can specify the model for $x_{i,t}$ as:

$$x_{i,t} = c_i + (\alpha_{ii}+\beta_i)x_{i,t-1} + (x_{i,t}-\mu_{i,t}) - \beta_i(x_{i,t-1}-\mu_{i,t-1}) + \sum_{j\neq i}\alpha_{ij}x_{j,t-1} + \delta_i\xi_t - \beta_i\delta_i\xi_{t-1} \tag{4}$$

In practice $x_{i,t}$ follows an ARMAX process, with $(x_{i,t} - \mu_{i,t})$ representing the disturbance of the ARMA(1,1) part with AR coefficient equal to $(\alpha_{ii} + \beta_i)$ and MA coefficient given by β_i; furthermore, it depends on the $n-1$ lagged variables $x_{j,t-1}$ and on the common signal at time t and $t-1$.

The reduction in the number of coefficients could be based on the idea of identifying groups of series that follow the same dynamics and assigning them the same parameters. To this end, first, we estimate n univariate MEMs as (3) and then we use the estimates of the parameters to group the series. We distinguish the ARMA dynamics from the spillover part and the common feature part. To this end, we adopt separate model-based clustering steps, one for each dynamic part of model (4).

Step 1: similar α_{ii} and β_i parameters. We form clusters of series following the same ARMA dynamics. A common way is to adopt a classical clustering algorithm based on the so-called AR distance proposed by Piccolo (1990). It is based on the Euclidean distance between the parameters of the AR(∞) representations of two invertible time series. For the ARMA(1,1) case illustrated in equation (4), we can derive a closed-form of this distance (*ARMA distance*) between series i and j as:

$$d_{ARMA} = \left[\frac{\alpha_{ii}^2}{1-\beta_i^2} + \frac{\alpha_{jj}^2}{1-\beta_j^2} - 2\frac{\alpha_{ii}\alpha_{jj}}{1-\beta_i\beta_j}\right]^{1/2} \tag{5}$$

The clustering algorithm will provide k_1 groups of pairs of coefficients (α, β); let us call the $2k_1$ coefficients $\alpha_1, \ldots, \alpha_{k_1}, \beta_1, \ldots, \beta_{k_1}$. The final Appendix shows how to derive this distance for the generic case of two ARMA(1,1) processes.

Step 2: similar α_{ij} $(i \neq j)$ parameters. We can also adopt a clustering algorithm for the coefficients representing the spillover effects. We consider all the $n(n-1)$ parameters α_{ij} $(i \neq j)$ estimated by the n univariate models (3), corresponding to the off-diagonal elements of matrix $\boldsymbol{A}$ in (1). We group these coefficients with the clustering algorithm based on the Euclidean distance between the coefficients representing the spillover from the series j to i and from the series s to r, given by:

$$d_{spill} = |\alpha_{ij} - \alpha_{rs}| \tag{6}$$

The clustering algorithm will provide k_2 groups of coefficients α_{ij} $(i \neq j)$; we call them $a_1, \ldots, a_{k_2}$.

Step 3: similar δ_i coefficients. Finally, we detect the similar loading coefficients of the co-movement represented by variable ξ_t. In equation (4) this part is characterized by the parameters δ_i and $-\beta_i\delta_i$; as a consequence, the corresponding Euclidean distance between series i and j is given by:

$$d_{com} = \left[(\delta_i - \delta_j)^2 + (\delta_j\beta_j - \delta_i\beta_i)^2\right]^{1/2} = \left[\delta_i^2(1+\beta_i^2) + \delta_j^2(1+\beta_j^2) - 2\delta_i\delta_j(1+\beta_i\beta_j)\right]^{1/2} \tag{7}$$

The clustering algorithm will provide k_3 groups of coefficients δ_i $(\delta_1, \ldots, \delta_{k_3})$. In this case the role of δ_i is dominant compared to β_i in the calculation of the distance d_{com}: it is much larger than β_i, as we will see in Section 3. For this reason, we only use this step to group the δ coefficients and not the β coefficients, identified in step 1.

The three steps of clustering will provide, in equation 1, a matrix $\boldsymbol{A}$ of coefficients with k_1 distinct elements on the diagonal and k_2 different off-diagonal values. It is worth noting that this parameterization provides a not symmetric $\boldsymbol{A}$; in principle, this is a desirable result because the spillover effect between two variables changes depending on the direction of causality. Similarly, the vector $\boldsymbol{\beta}$, containing the elements of the diagonal of $\boldsymbol{B}$, is composed of k_1 different elements, whereas the vector $\boldsymbol{\delta}$ of k_3 different elements.

Finally, the covariance matrix $\boldsymbol{V}$ also needs a feasible parameterization; we propose to rewrite it as a linear transformation of the sample covariance matrix of $(\boldsymbol{x}_t - \widehat{\ln \boldsymbol{\mu}_t})$ (call it $\bar{\boldsymbol{V}}$), where the hat indicates the estimated value. Formally:

$$\boldsymbol{V} = \vartheta \bar{\boldsymbol{V}}(\hat{\boldsymbol{\theta}}), \tag{8}$$

where ϑ is a scalar parameter. By adopting an iterative algorithm to maximize the log-likelihood (2), at the end of each iteration, the value $\hat{\boldsymbol{\theta}}$ is used to evaluate $\widehat{\ln \boldsymbol{\mu}_t}$ and $\bar{\boldsymbol{V}}(\hat{\boldsymbol{\theta}}) = COV(\boldsymbol{x}_t - \widehat{\ln \boldsymbol{\mu}_t})$; the last value of $\bar{\boldsymbol{V}}(\hat{\boldsymbol{\theta}})$ is used in the next iteration to maximize (2) until convergence.

In summary, the number of estimated coefficients for the reparameterized model is $p = 2k_1 + k_2 + k_3 + 1$. For example, if $n = 5$ and the clustering algorithm provides $k_1 = 2$, $k_2 = 2$, $k_3 = 3$ groups, the fully parameterized model requires the estimation of 50 coefficients, while the proposed reparameterization requires only 10.

Evaluating reparameterization for large datasets is not possible, because the benchmark fully parameterized model (1) is not feasible. In the next section, we will propose a procedure to compare the two models. Hereafter we denote with FPM the fully parameterized model, and RPM the reparameterized model.

3 An Example with Real Data

To empirically illustrate the proposed model, we consider the HLR proxy calculated on seventeen indices collected by the Yahoo Finance website for the period January 21, 2014-January 19, 2024 (2600 daily data). We selected the series that can be freely downloaded from the section *World Indices* of the Yahoo Finance website. Then we selected only the observations in correspondence of the dates common to all indices. The list of indices is: NASDAQ Composite (US market, label IXIC); NYSE Composite Dow Jones (US, NYA); NYSE AMEX Composite (US, XAX); Russell 2000 (US, RUT); ESTX 50 PR.EUR (Europe, STOXX50E); DAX Performance (Germany, GDAXI); Nikkei 225 (Japan, N225); BEL 20 (Belgium, BFX); Euronext 100 Index (Europe, N100); Hang Seng (China, HSI); SSE Composite (China, SS); Shenzhen Index (China, SZ); KOSPI Composite (South Korea, KS11); TSEC weighted index (Taiwan, TSII); S&P/TSX Composite (Canada, GSPTSE); IBOVESPA (Brazil, BVSP); IPC MEXICO (Mexico, MXX).

Following Parkinson (1980), the HLR proxy of volatility is obtained as the difference of the logarithms of the highest and the lowest value of the day, rescaled by the factor $(\pi/8)^{1/2}$.

3.1 Grouping the coefficients

The number of coefficients involved in model (1) is 476, so it is impossible to estimate it. The parameterization proposed in subsection 2.2 requires three separate clustering procedures to select groups of coefficients with similar values. It requires as a preliminary step the estimation of the common component ξ_t, as the first principal component of the set of seventeen time series. It explains 94.1% of the total variance, confirming that the series are subject to a strong co-movement. We then estimate seventeen univariate MEMs, as (3), one for each index, providing the values of the coefficients to cluster.

Table 1: Clusters of parameters in the diagonal of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ (labeled with 1, 2, 3; step 1) and in the vector $\boldsymbol{\delta}$ (labeled with 1, 2, 3, 4; step 3).

index	IXIC	NYA	XAX	RUT	STOXX50E	GDAXI	N225	BFX	N100
step 1	1	3	2	3	2	3	2	2	3
step 2	4	3	2	4	3	3	2	4	3
index	HSI	SS	SZ	KS11	TWII	GSPTSE	BVSP	MXX	
step 1	1	1	1	1	1	2	2	2	
step 2	1	1	1	1	1	4	1	2	

The three clustering procedures, performed employing an agglomerative hierarchical algorithm with average linkage criterion, choose the number of clusters that provides the largest vertical difference between nodes. In the first step, in which similar parameters are identified in the diagonal of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, the number of clusters obtained is $k_1 = 3$; in the second step the number of different off-diagonal parameters of $\boldsymbol{A}$ is $k_2 = 3$; in the third step, the number of different coefficients in the vector $\boldsymbol{\delta}$ is $k_3 = 4$. In Table 1 we indicate the clusters identified for the ARMA and co-movement parts (steps 1 and 3 respectively; step 2 involves 272 coefficients and is available on request). Interestingly, the Asian indices (excluding N225) belong to the same cluster for both classifications. The two most peculiar indices, IXIC, characterized by companies in the IT sector, and RUT,

Table 2: Estimation results for the reparameterized (Model A) and the constrained (Model B) vMEMs with spillover effects and co-movements; battery of tests on parameters with corresponding p-values.

	Model A		Model B		Test	
Parameter	Estimate	st. .	Estimate	st. err.	Hypothesis	p-value
α_1	0.1371	0.0072	0.1368	0.0072	$\alpha_1 = \alpha_2 = \alpha_3$	0.000
α_2	0.1136	0.0053	0.1121	0.0050	$\alpha_1 = \alpha_2$	0.000
α_3	0.1097	0.0051			$\alpha_1 = \alpha_3$	0.000
β_1	0.8190	0.0104	0.8187	0.0106	$\alpha_2 = \alpha_3$	0.176
β_2	0.8490	0.0079	0.8506	0.0076	$\beta_1 = \beta_2 = \beta_3$	0.000
β_3	0.8518	0.0076			$\beta_1 = \beta_2$	0.000
a_1	0.0007	0.0002	0.0008	0.0002	$\beta_1 = \beta_3$	0.000
a_2	0.0003	0.0004			$\beta_2 = \beta_3$	0.460
a_3	0.0007	0.0002			$a_1 = a_2 = a_3$	0.444
δ_1	5.5057	0.2636	5.4949	0.2279	$a_1 = a_2$	0.216
δ_2	6.4717	0.3084	6.4577	0.2589	$a_1 = a_3$	0.805
δ_3	8.7979	0.3921	8.7466	0.3077	$a_2 = a_3$	0.207
δ_4	7.2778	0.3394	7.2481	0.2723	$\delta_1 = \delta_2 = \delta_3 = \delta_4$	0.000
ϑ	0.8241	0.0127	0.8242	0.0127	$\delta_1 = \delta_2$	0.000
Log-Likelihood	-83442.6		-83445.3		$\delta_1 = \delta_3$	0.000
AIC	64.222		64.221		$\delta_1 = \delta_4$	0.000
BIC	64.254		64.244		$\delta_2 = \delta_3$	0.000
					$\delta_2 = \delta_4$	0.003
					$\delta_3 = \delta_4$	0.000

which represents small-cap companies, belong to two unique combinations of clusters.

3.2 Estimation of the vMEM with spillover effects and co-movement

Adopting the previous grouping, the RPM (Model A) contains 14 parameters; the estimation results are reported in the first columns of Table 2. At first glance, it is clear that the spillover parameters a_i ($i = 1, 2, 3$) are very small but significant (excluding a_2). The δ coefficients referring to the loading parameters of the co-movement effect are larger, with the highest value for the third cluster. It is appropriate to verify the equality of the three groups of coefficients, having the possibility of further reducing the number of parameters and dealing with more parsimonious models. In the last two columns of Table 2 we show the battery of tests to verify the equality (joint and pairwise) of the parameters. The results show that we can further merge clusters 2 and 3 of the ARMA coefficients and consider a single parameter for the spillover effects. The derived model (Model B) requires the estimation of only 10 coefficients, but in terms of AIC and BIC, it is better than Model A. For this reason, the rest of the analysis will be conducted using Model B.

For the sake of simplicity, we conduct a graphical analysis on three indices, XAX, N100, and HSI. Figure 1 shows the dynamics of the three indices: they are very similar, particularly in early 2020 when the COVID pandemic provided a common high spike in volatility. The 2015-16 period shows several peaks, particularly in the N100 and HSI series; this effect could be ascribed to the stock market sell-off that began in the USA, but above all to the Chinese stock market, which recorded a crash in June 2015, with several declines also in the following months, and a subsequent strong sell-off in January 2016. Other common movements can be noticed after September 2018 (cryptocurrency crash) and around the 2022 stock market decline. The proposed vMEM, despite the use of a reduced number of parameters, seems able to follow the profile of HLR proxies.

Model B provides a single parameter to represent the spillover effects, therefore, considering the variable from which the spillover starts, its dynamics does not depend on the variable towards which the effect is directed. Figure 2 illustrates the spillover effect of the three indices XAX, N100, and HSI; it is reported in exponential terms, therefore it is a multiplicative effect for the element in $\boldsymbol{\mu}_t$: a value greater than 1 indicates a spillover effect which increases the volatility of the target variable, less than 1 an attenuation of volatility. The crash due to the COVID pandemic is conveyed by XAX and N100; The HSI shows a stronger effect in 2015-16 (Chinese stock market crash), while the XAX in 2022 (US stock market decline), with no effect from N100. On the contrary, during 2021 and 2022, N100 appears to suffer the spillover effect of other markets; Figure 3 shows the entire spillover effect towards the three indices, given by (the exponential of) the sum of the spillover effects in the corresponding equation of (1). In 2021-22, N100 presents the highest peaks of the spillover effects while they are quite moderate for XAX and HSI over the entire period considered (multiplication factors between approximately 0.98 and 1.01).

Figure 4 shows the co-movement factors for the three series; the profile of the three factors is the same, differing only for the loading parameter δ_i (higher for N100, belonging to cluster 3, lower for HSI-cluster 1; see Table 1, corresponding to step 3 of clustering). Also in this case the strongest co-movements are observed in correspondence with the

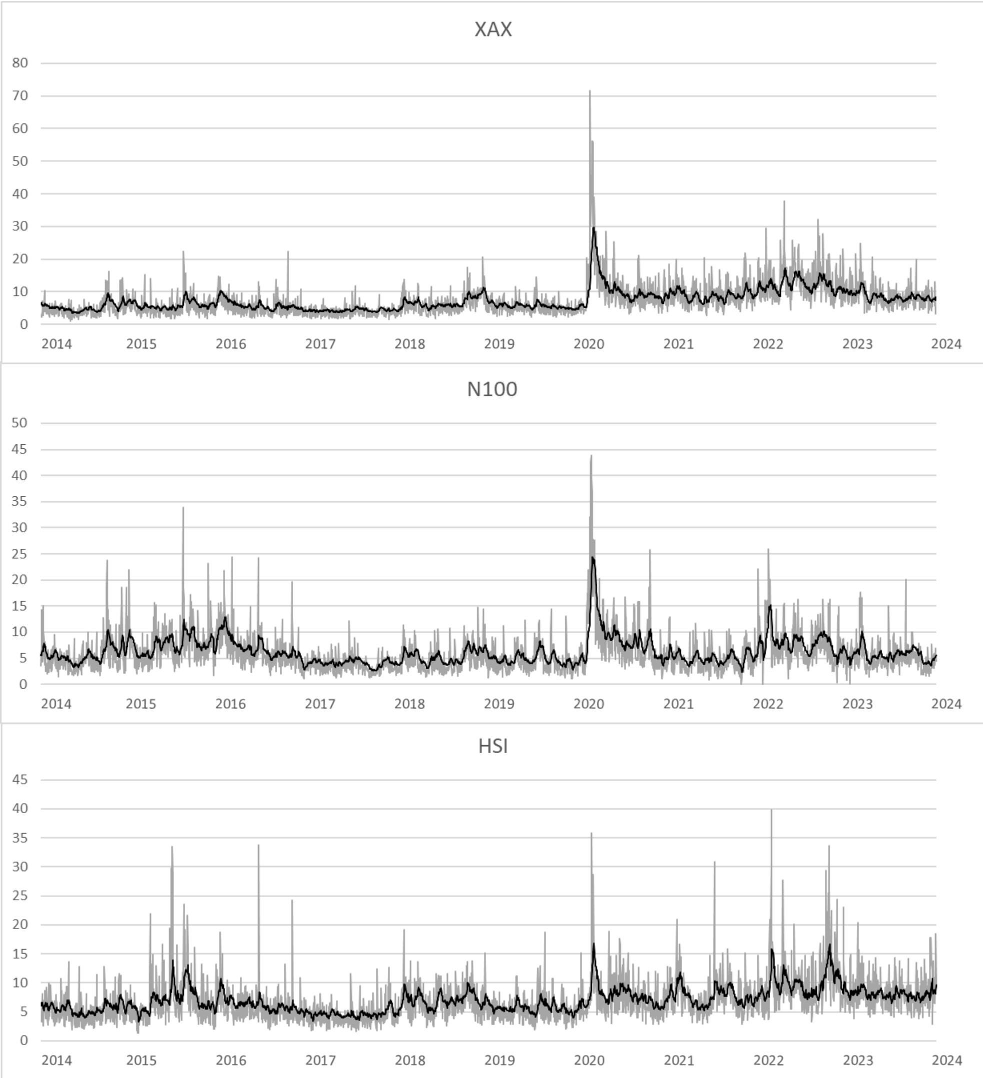

Figure 1: HLR of three indices (gray lines) and corresponding estimation of μ_t (black lines) by using Model B.

crises of 2015-16 and 2021-22, and the shock of 2020.

3.3 Model evaluation

The natural benchmark for evaluating the RPM is the FPM; as mentioned it involves 476 unknown coefficients, so it is not feasible. Since a direct comparison can only be made considering a few number of variables, a simple alternative to check whether the adopted parameterization is a valid approximation of the FPM is to select several small subsets of the seventeen series, estimate the FPM and the RPM, and check the differences between the $\boldsymbol{\mu}_t$ obtained with the two models. The steps of the proposed procedure can be summarized as follows:

1. divides the n series into randomly selected separate subgroups of k series; if n is not a multiple of k, some series can be randomly included in multiple subsets;

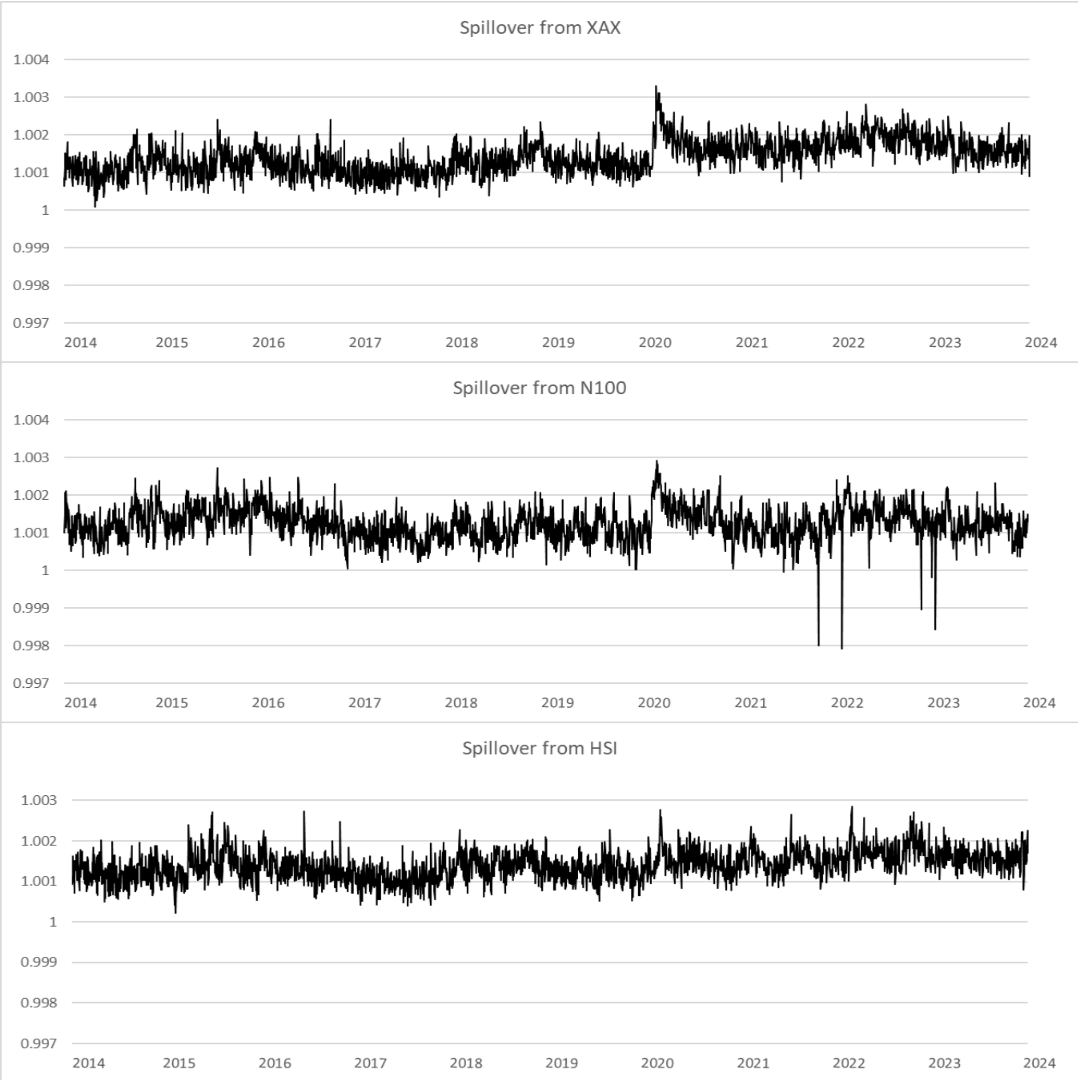

Figure 2: Multiplicative spillover effects from variables XAX, N100, and HSI .

2. for each subset estimate FPM and RPM;

3. compare the $k-$dimensional estimated $\boldsymbol{\mu}_t$ obtained with the two models with a Diebold-Mariano test in its multivariate version (mDM, Mariano and Preve, 2012).

We set $k = 3$, and then we obtain 6 subsets, as shown in the first column of Table 3; the last subset is integrated by another random draw to obtain the same k, so XAX is included in two subsets. The constraints on the RPM parameters are obtained from the three clustering procedures described in subsection 3.1. The number of coefficients p to estimate varies with the subsets and is reported in the second column of Table 3; the maximum p (11) is at the last subset, while the FPM requires 21 parameters for each subset. To estimate a positive definite $\boldsymbol{V}$ in the FPM, we estimate the elements of a lower triangular matrix $\boldsymbol{P}$ with positive diagonal entries (representing the Cholewsky factorization of $\boldsymbol{V}$) and therefore we set $\boldsymbol{V} = \boldsymbol{P}\boldsymbol{P}'$.

The mDM statistic is calculated by comparing the quadratic errors derived from the two models; the null hypothesis of equal performance of the two models is never rejected at

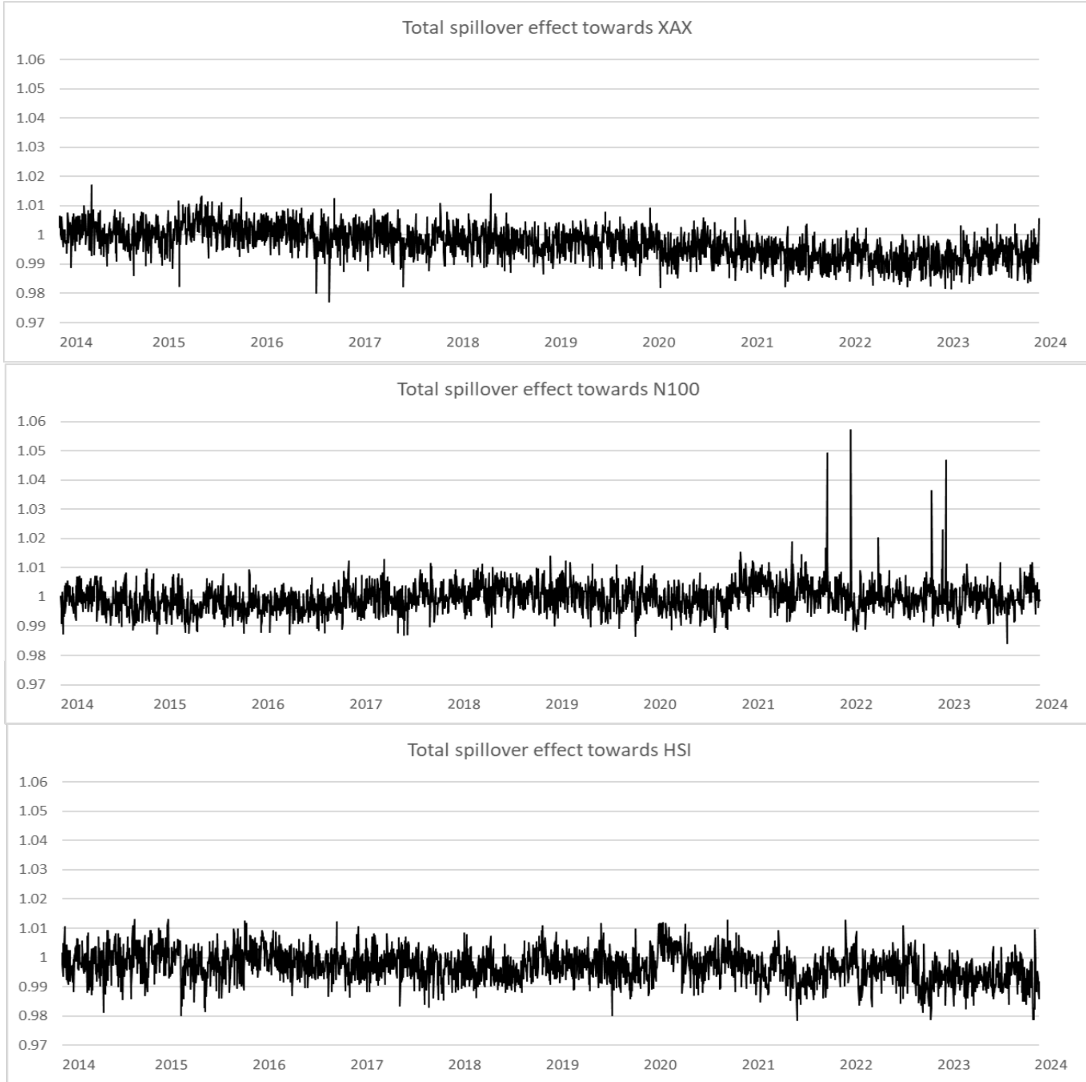

Figure 3: Multiplicative total spillover effects toward variables XAX, N100, and HSI .

Table 3: Multiple Diebold-Mariano statistic (mDM) and corresponding p-value to compare FPM (number of estimated coefficients equal to p) and RPM (number of estimated coefficients equal to 21) for six subsets of the HLR dataset.

Subset	p	mDM	p-value
HSI-KS11-SS	6	5.419	0.1436
GPTSE-TWII-XAX	10	4.968	0.1742
MXX-STOXX50E-N100	9	4.670	0.1976
NYA-N225-GDAXI	9	11.246	0.0105
BFX-BVSP-IXIC	10	5.824	0.1205
RUT-SZ-XAX	11	6.277	0.0989

a significance level of 1%; only the null hypothesis for the subset with NYA, N225, GDAXI is rejected at the 5% significance level. In conclusion, the adopted reparameterization seems to have a good performance.

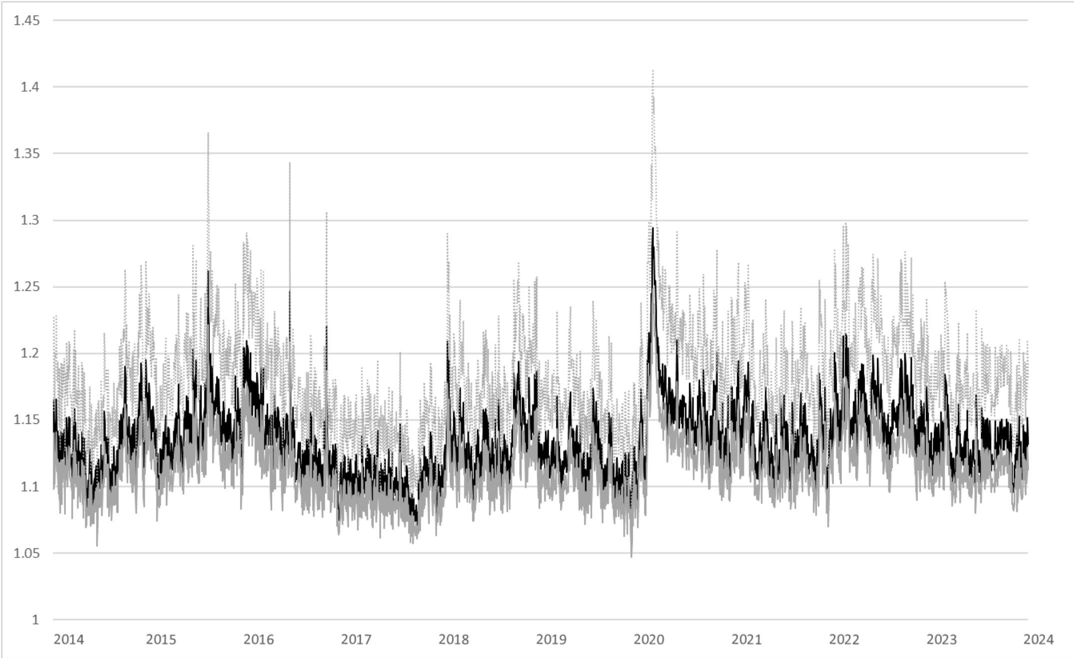

Figure 4: Co-movement factor for XAX (black line), N100 (gray dotted line), and HSI (gray continuous line).

Table 4: Comparison of different parameterizations of model (1) with different number of coefficients p: loss functions and mDM statistic to compare RPR with the alternative parameterizations.

Parameterization	p	AIC	BIC	MSE	mDM
RPR	10	64.221	64.244	175.282	
scalar	5	64.262	64.275	176.648	48.37
outer product	36	66.762	66.843	205.774	92.21

An alternative simple version of (1), maintaining the parameterization of $\boldsymbol{V}$ as in (8), could be represented by the scalar representation, where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{\delta}$ are scalars: this implies the estimation of only 5 unknown coefficients. A less parsimonious representation is obtained by parameterizing $\boldsymbol{A}$ as the outer product of a vector and its transpose and $\boldsymbol{B}$ as a scalar;[2] again maintaining the parameterization of $\boldsymbol{V}$ as in (8), this representation implies $2n + 2$ estimated parameters but imposes the constraint of considering symmetric spillover effects because $\boldsymbol{A}$ is symmetric, which, in principle, is a strong assumption.

Table 4 reports the values of AIC, BIC, and MSE to compare the three alternative parameterizations. RPR always outperforms the other two; the outer product parameterization shows poor performance, perhaps due to the constraint on autoregressive coefficients. Comparing RPR to the other two alternatives using the mDM test, we see high values of the statistic implying a p-value of zero and a significantly better performance of RPR than the other two.

[2]Bauwens and Otranto (2023), developing a multivariate model for matrices of covariance realized, empirically obtain a constant scalar coefficient of the lagged conditional covariance matrix.

4 Concluding Remarks

An important strand of statistical and econometric literature aims to include the dependence on movements of other markets in volatility models, generating various multivariate approaches, concerning, in particular, spillover and contagion effects (see, for example, Pericoli and Sbracia, 2003). Most approaches develop ad hoc models to capture these effects, in particular by inserting them into a VAR framework (Gallo and Otranto, 2008; Diebold and Yilmaz, 2012). The vMEM seems an excellent candidate to include spillover effects, adopting the flexible dynamics GARCH to model the volatility and, resorting to the composite MEM extension (Brownlees et al., 2012), having the possibility to specify several unobservable signals. We use this last property to divide the expected conditional variance as the sum of the GARCH part, including spillover effects and volatility persistence, and the co-movement part, which is a typical characteristic of financial markets (Forbes and Rigobon, 2002).

To provide the ability to handle a large set of volatility series, we propose a model-based clustering algorithm to dramatically reduce the number of unknown coefficients; in the proposed empirical exercise, the number of estimated coefficients is only 10, while the full model contains 476 parameters, with a reduction of 98%. Evaluating the reparameterized model would require estimating the benchmark (the fully parameterized model), which is not feasible, but we propose a procedure to evaluate the performance of the reduced model by comparisons on subsets of series, which are exhaustive of the complete data set.

The proposed model could be usefully used for forecasting purposes or to evaluate the direction of spillover effects, identifying groups of markets with different degrees of transmission of shocks (the so-called dominant markets; see Gallo and Otranto, 2008; Otranto and Gargano, 2014). It might also be interesting to investigate the possibility of including in this model alternative methods for grouping the parameters, such as the fuzzy clustering approach of Cerqueti, D'Urso, Mattera, and Vitale (2023), or of using the results for classification purposes.

Details on the results of the clustering procedures and the estimation of models with scalar and outer product parameterizations are available upon request. The GAUSS codes used for the application were written by the author.

Funding

The author acknowledges financial support from the Italian PRIN 2022 grant "Methodological and computational issues in large-scale time series models for economics and finance" (20223725WE).

Appendix

A The distance between ARMA(1,1) processes

Let us consider two ARMA(1,1) processes:

$$x_{i,t} = \varphi_i x_{i,t-1} + \eta_{i,t} - \psi_i \eta_{i,t-1} \qquad i = 1,2 \tag{9}$$

with $\eta_{i,t}$ representing a white noise process.

Using the lag operator L ($L^j x_{i,t} = x_{i,t-j}$), equation (9) can be written with lag polynomials:

$$(1 - \varphi_i L)x_{i,t} = (1 - \psi_i L)\eta_{i,t} \tag{10}$$

The AR distance proposed by Piccolo (1990) is the Euclidean distance between the infinite AR coefficients relating to two stochastic processes; calling $\pi_{i,j}$ the $j - th$ AR coefficient of the process i ($i = 1,2$), the AR distance is given by:

$$d_{AR} = \left[\sum_{j=1}^{\infty}(\pi_{1,j} - \pi_{2,j})^2\right]^{1/2} \tag{11}$$

This distance has been widely used to verify whether two models can be considered as reproducing the same dynamics (see, for example, Otranto, 2010; Otranto and Gargano, 2014). For the specific case ARMA(1,1), we can extend the AR distance along the following lines.

Under the assumptions of invertibility, we write (10) as:

$$(1 - \varphi_i L)(1 - \psi_i L)^{-1} x_{i,t} = \eta_{i,t} \tag{12}$$

and, under the invertibility constraint $|\psi_i| < 1$,

$$(1 - \varphi_i L)\sum_{j=1}^{\infty}(\psi_i L)^j x_{i,t} = \eta_{i,t} \tag{13}$$

Putting the AR coefficients on the right side of equation (13), the j-th AR coefficient is given by:

$$\pi_{i,j} = \varphi_i \psi_i^{j-1} - \psi_i^j \tag{14}$$

Consequently, by substituting (14) into (11), we obtain the following expression for the AR distance between two ARMA(1,1) processes:

$$\begin{aligned}&\left[\sum_{j=1}^{\infty}(\varphi_1\psi_1^{j-1} - \psi_1^j)^2 + (\varphi_2\psi_2^{j-1} - \psi_2^j)^2 - 2(\varphi_1\psi_1^{j-1} - \psi_1^j)(\varphi_2\psi_2^{j-1} - \psi_2^j)\right]^{1/2} = \\ &\left[\sum_{j=1}^{\infty}\left(\varphi_1^2\psi_1^{2(j-1)} + \psi_1^{2j} - 2\varphi_1\psi_1^{2j-1} + \varphi_2^2\psi_2^{2(j-1)} + \psi_2^{2j} - 2\varphi_2\psi_2^{2j-1} - 2\varphi_1\varphi_2(\psi_1\psi_2)^{j-1} + \right.\right. \\ &\left.\left. 2\varphi_2\psi_1(\psi_1\psi_2)^{j-1} + 2\varphi_1\psi_2(\psi_1\psi_2)^{j-1} - 2(\psi_1\psi_2)^j\right)\right]^{1/2}\end{aligned} \tag{15}$$

For invertibility, $\psi_1^2 < 1$, $\psi_2^2 < 1$, $\psi_1\psi_2 < 1$; labeling with κ one of the three previous functions of ψ_1 and/or ψ_2, each $\sum_{j=1}^{\infty}\kappa^{j-1} = \frac{1}{1-\kappa}$ and $\sum_{j=1}^{\infty}\kappa^j = \frac{\kappa}{1-\kappa}$. Therefore, (15) is equal to:

$$\left[\frac{\varphi_1^2}{1-\psi_1^2} + \frac{\psi_1^2}{1-\psi_1^2} - 2\frac{\varphi_1\psi_1}{1-\psi_1^2} + \frac{\varphi_2^2}{1-\psi_2^2} + \frac{\psi_2^2}{1-\psi_2^2} - 2\frac{\varphi_2\psi_2}{1-\psi_2^2} - 2\frac{\varphi_1\varphi_2}{1-\psi_1\psi_2} + 2\frac{\varphi_2\psi_1}{1-\psi_1\psi_2} + 2\frac{\varphi_1\psi_2}{1-\psi_1\psi_2} - 2\frac{\psi_1\psi_2}{1-\psi_1\psi_2}\right]^{1/2}$$

Finally, we can express the distance between two ARMA(1,1) processes as:

$$d_{ARMA} = \left[\frac{(\varphi_1 - \psi_1)^2}{1 - \psi_1^2} + \frac{(\varphi_2 - \psi_2)^2}{1 - \psi_2^2} - 2\frac{(\varphi_1 - \psi_1)(\varphi_2 - \psi_2)}{1 - \psi_1\psi_2} \right]^{1/2} \tag{16}$$

Considering the parameterization in equation (4), the ARMA distance (16) takes the form in (5). A similar distance was derived in Otranto (2010) for DCC models.

References

Andersen, T. and T. Bollerslev (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review 39*, 885–905.

Andersen, T., T. Bollerslev, F. Diebold, and P. Labys (2003). Modeling and forecasting realized volatility. *Econometrica 71*, 579–625.

Atak, A. and G. Kapetanios (2013). A factor approach to realized volatility forecasting in the presence of finite jumps and cross-sectional correlation in pricing errors. *Economics Letters 120*, 224–228.

Barndorff-Nielsen, O., P. Hansen, A. Lunde, and N. Shephard (2008). Designing realised kernels to measure the ex-post variation of equity prices in the presence of noise. *Econometrica 76*, 1481–1536.

Bauwens, L., S. Laurent, and J. Rombouts (2006). Multivariate GARCH models: A survey. *Journal of Applied Econometrics 21*, 79–109.

Bauwens, L. and E. Otranto (2023). Modeling realized covariance matrices: A class of Hadamard exponential models. *Journal of Financial Econometrics 21*, 1376–1401.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*, 307–327.

Brownlees, C. T., F. Cipollini, and G. M. Gallo (2012). Multiplicative error models. In L. Bauwens, C. Hafner, and S. Laurent (Eds.), *Handbook of Volatility Models and Their Applications*, pp. 223–247. Wiley.

Cerqueti, R., P. D'Urso, R. Mattera, and V. Vitale (2023). Fuzzy clustering of financial time series based on volatility spillovers. *Annals of Operations Research*.

Chou, R. Y., H. Chou, and N. Liu (2015). *Range Volatility: A Review of Models and Empirical Studies*, pp. 2029–2050. New York, NY: Springer New York.

Cipollini, F., R. F. Engle, and G. M. Gallo (2013). Semiparametric vector MEM. *Journal of Applied Econometrics 28*, 1067–1086.

Cipollini, F., R. F. Engle, and G. M. Gallo (2017). Copula–based vMEM specifications versus alternatives: The case of trading activity. *Econometrics 5*(2).

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics 7*, 174–196.

Diebold, F. X. and K. Yilmaz (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers. *International Journal of Forecasting 28*, 57–66.

Engle, R. F. (2002a). Dynamic conditional correlation - a simple class of multivariate GARCH models. *Journal of Business & Economic Statistics 20*, 339–350.

Engle, R. F. (2002b). New frontiers for ARCH models. *Journal of Applied Econometrics 17*, 425–446.

Engle, R. F. and G. M. Gallo (2006). A multiple indicators model for volatility using intra-daily data. *Journal of Econometrics 131*, 3–27.

Engle, R. F. and F. K. Kroner (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory 11*, 122–150.

Engle, R. F. and J. Mezrich (1996). GARCH for groups. *Risk 9*, 36–40.

Forbes, J. K. and R. Rigobon (2002). No contagion, only interdependence: Measuring stock market comovement. *Journal of Finance 57*, 2223–2261.

Gallo, G. M. and E. Otranto (2008). Volatility spillovers, interdependence and comovements: A Markov Switching approach. *Computational Statistics & Data Analysis 52*, 3011–3026.

Mariano, R. and D. Preve (2012). Statistical tests for multiple forecast comparison. *Journal of Econometrics 169*, 123–130.

Nakatani, T. and T. Teräsvirta (2009). Testing for volatility interactions in the constant conditional correlation GARCH model. *The Econometrics Journal 12*, 147–163.

Otranto, E. (2010). Identifying financial time series with similar dynamic conditional correlation. *Computational Statistics & Data Analysis 54*, 1–15.

Otranto, E. (2015). Capturing the spillover effect with multiplicative error models. *Communications in Statistics-Theory and Methods 44*, 3173–3191.

Otranto, E. and R. Gargano (2014). Financial clustering in presence of dominant markets. *Advances in Data Analysis and Classification 9*, 315–339.

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. *The Journal of Business 53*, 61–65.

Pericoli, M. and M. Sbracia (2003). A primer on financial contagion. *Journal of Economic Surveys 17*, 571–608.

Piccolo, D. (1990). A distance measure for classifying ARIMA models. *Journal of Time Series Analysis 11*, 153–164.

Taylor, N. and Y. Xu (2017). The logarithmic vector multiplicative error model: an application to high frequency NYSE stock data. *Quantitative Finance 17*, 1021–1035.

White, H. (1982). Maximum likelihood estimation of misspecified models. *Econometrica 50*, 1–25.

Ultimi Contributi di Ricerca CRENoS

I Paper sono disponibili in: http://www.crenos.unica.it

24/03 *Luca Serafini, Emanuela Marrocu, Raffaele Paci,* "Smart Strategies, Smarter Performance: the Impact of S3 and Industry 4.0 on Firms' Outcomes"
24/02 *Hugues Champeaux, Elsa Gautrain, Karine Marazyan,* "Men's premarital migration and marriage payments: Evidence from Indonesia"
24/01 *Pablo Álvarez-Aragón, Hugues Champeaux,* "Measuring Norms and Enumerator Effects: Survey Method Matters"
23/15 *Fabio Cerina, Xavier Raurich,* "Saving Behaviour and the Intergenerational Allocation of Leisure Time"
23/14 *Leonardo Vargiu, Bianca Biagi, Maria Giovanna Brandano, Paolo Postiglione,* "Research Infrastructures and Regional Growth: the case of Europe"
23/13 *Alberto Tidu,* "Dissecting inequality: conceptual problems, trends and drivers"
23/12 *Luciano Mauro, Francesco Pigliaru,* "Italy's National Recovery and Resilient Plan: Will it Narrow the North-South Productivity Gap?"
23/11 *Fabrizio Antolini, Samuele Cesarini, Giorgio Garau,* "The economic impact of the tourism sector on the overall Italian economy: An Input-Output Approach"
23/10 *Giorgio Garau, Andrea Karim El Meligi,* "The Impact of the Pandemic and War on Surplus Redistribution Mechanisms: A Sectoral Analysis of France and Italy"
23/09 *Maria Giovanna Brandano, Alessandra Faggian, Adriana C. Pinate,* "The impact of COVID-19 on the tourism sector in Italy: a regional spatial perspective"
23/08 *Fabrizio Cipollini, Giampiero M. Gallo, Alessandro Palandri,* "Modeling and evaluating conditional quantile dynamics in VaR forecasts"
23/07 *Ugo M. Gragnolati, Luigi Moretti, Roberto Ricciuti,* "Early railways and industrial development: Local evidence from Sardinia in 1871–1911"
23/06 *Giampiero M. Gallo, Demetrio Lacava, Edoardo Otranto* "Volatility jumps and the classification of monetary policy announcements"
23/05 *Diego Dessì, Raffaele Paci,* "The impact of Global Value Chains participation on countries' productivity"
23/04 *Edoardo Otranto, Luca Scaffidi Domianello,* "On the relationship between Markov Switching inference and Fuzzy Clustering: A Monte Carlo evidence"
23/03 *Mario Agostino Maggioni, Emanuela Marrocu, Teodora Erika Uberti, Stefano Usai,* "The role of localised, recombinant and exogenous technological change in European regions"
23/02 *Bianca Biagi, Laura Ciucci, Claudio Detotto, Manuela Pulina,* "University study programmes and students dynamics"
23/01 *Giovanni B. Concu, Claudio Detotto, Marco Vannini,* "Drivers of intentions and drivers of actions: willingness to participate versus actual participation in fire management in Sardinia, Italy"
22/06 *Alberto Tidu, Stefano Usai, Frederick Guy,* Measuring spatial dispersion: an experimental test on the M-index
22/05 *Luca Scaffidi Domianello, Giampiero M. Gallo, Edoardo Otranto,* "Smooth and Abrupt Dynamics in Financial Volatility: the MS-MEM-MIDAS"
22/04 *Claudio Detotto, Riccardo Marselli, Bryan C. McCannon, Marco Vannini,* "Experts and Arbitration Outcomes: Insights from Public Procurement Contract Disputes"
22/03 *Gerardo Marletto, Cécile Sillig,* "Global influence and national diversity in socio-technical transitions: a sectoral taxonomy"
22/02 *Federico Aresu, Emanuela Marrocu, Raffaele Paci,* "Public capital and institutions' quality in the Italian regions"

www.crenos.unica.it

ISBN 9788868515003