CONTRIBUTI DI RICERCA CRENOS

EFFECTS OF MIGRATION WITH ENDOGENOUS LABOR SUPPLY AND HETEROGENEOUS SKILLS

Gianfranco Atzeni
Luca G. Deidda
Marco Delogu
Dimitri Paolini

WORKING PAPERS

2021/11

arkadia

CENTRO RICERCHE ECONOMICHE NORD SUD
(CRENoS)
UNIVERSITÀ DI CAGLIARI
UNIVERSITÀ DI SASSARI

CRENOS was set up in 1993 with the purpose of organising the joint research effort of economists from the two Sardinian universities (Cagliari and Sassari) investigating dualism at the international and regional level. CRENoS' primary aim is to improve knowledge on the economic gap between areas and to provide useful information for policy intervention. Particular attention is paid to the role of institutions, technological progress and diffusion of innovation in the process of convergence or divergence between economic areas. To carry out its research, CRENoS collaborates with research centres and universities at both national and international level. The centre is also active in the field of scientific dissemination, organizing conferences and workshops along with other activities such as seminars and summer schools.
CRENoS creates and manages several databases of various socio-economic variables on Italy and Sardinia. At the local level, CRENoS promotes and participates to projects impacting on the most relevant issues in the Sardinian economy, such as tourism, environment, transports and macroeconomic forecasts.

www.crenos.unica.it
crenos@unica.it

CRENoS – Cagliari
Via San Giorgio 12, I-09124 Cagliari, Italia
tel. +39-070-6756397; fax +39-070- 6756402

CRENoS - Sassari
Via MURONI 25, I-07100 Sassari, Italia
tel. +39-079-213511

Title: EFFECTS OF MIGRATION WITH ENDOGENOUS LABOR SUPPLY AND HETEROGENEOUS SKILLS

Prima Edizione: Dicembre 2021

ISBN: 978 88 68513 801

Viale Bonaria 98 - 09125 Cagliari
Tel. 070/6848663 - info@arkadiaeditore.it
www.arkadiaeditore.it

Effects of migration with endogenous labor supply and heterogeneous skills*

Gianfranco Atzeni
University of Sassari and CRENoS
Luca Deidda
University of Sassari and CRENoS
Marco Delogu
University of Sassari, CRENoS, DEM, University of Luxembourg
Dimitri Paolini
University of Sassari, CRENoS, CORE, Université catholique de Louvain

Abstract

We analyze the effects of migration allowing for endogenous labor supply in a standard two-region model with monopolistically competitive producers and love for variety. We find that the welfare effects of migration depend on firms' market power in the final good markets. If market power is sufficiently high, migration of low-skill individuals positively affects the welfare of native high skill individuals in the destination region, while low skill individuals are unaffected. Natives of the origin region are always better off, irrespective of their skills. Differently, if market power is sufficiently low, low skill migration makes both high and low individuals native of the destination region better off.

Keywords: Monopolistic Competition- Labor Supply-Migration-Welfare Analysis
Jel Classification: F12, F22, F62, J20.

* We wish to thank Cristopher Deuster for precious suggestions. We would like to acknowledge the participants to the SasCa conference (2021, Alghero). The authors gratefully acknowledge financial support by Regione Autonoma della Sardegna (Legge n.7, "Dynamics of Human Capital Accumulation and Skill Biased Technological Change" (codice RAS RASSR89213) finanziato a valere sul Bando "Invito a presentare progetti di ricerca di base - Annualità 2017" L.R.7/2007 e con le risorse del Fondo di Sviluppo e Coesione (FSC) 2014-2020") and by Università degli studi di Sassari (fondo di Ateneo per la ricerca 2019 and fondo di Ateneo per la ricerca 2020). All errors remain our own.

1 Introduction

In this paper we study how the endogeneity of individual labor decisions shapes the welfare effects of migration in a model with love for variety and endogenous migration flows. We find that the welfare effects of unskilled and skilled labor depend on the level of market power of final good producers. If market power is sufficiently high, only high skill workers strictly benefit from migration, while welfare of unskilled workers stays unchanged. These theoretical findings help rationalizing the evidence about the global tendency toward more restrictive immigration policies, which goes hand in hand with the observed increase of market power over time as documented in figure 1, which portrays the average trends of market power and the leniency of immigration policies, worldwide. The descriptive evidence reported in figure 1 is corroborated by the positive and significant correlation between the tightening of immigration policies and the level of market power reported in table 1.[1]

[1] Table 1 reports the estimates obtained regressing changes in immigration policies on markup estimates. We have data for 38 developed countries and 16 years. In our regression we do not include year fixed effect as the markup shows an upward trend overtime. Our data suggest that higher levels of market power tend to be associated with more stringent immigration policies.

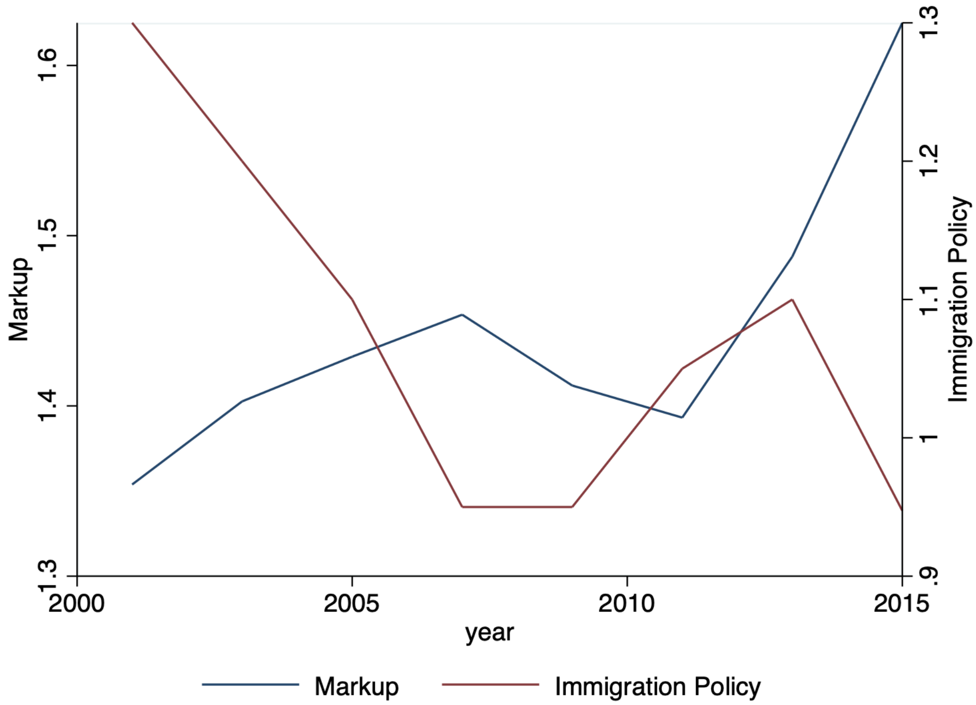

Figure 1: Data Source: De Loecker et al. (2020), United Nations.

Table 1: Markup and Immigration Policies

	(1)	(2)	(3)
	Immigration Policy	Immigration Policy	Immigration Policy
markup	0.676*	0.650*	0.533
	(2.10)	(2.03)	(1.51)
immigrants		-0.000000174	0.000000505
		(-0.86)	(0.52)
c.markupc.immigrants			0.000000470
			(0.71)
constant	1.681**	1.684**	1.513**
	(3.79)	(3.80)	(2.94)
R^2	0.526	0.528	0.530
N	150	150	150

t statistics in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.001)
Data on markup are taken from De Loecker et al. (2020), data on migration policies from UN and the definition of the stringency of the policy follows Deuster (2021). Immigration Policies is a discrete variable that take value equal to 2 when the government opted for less stringent policies, 1 with no change and 0 if immigration policies became more restrictive. The markup values are multiplied by -1 to facilitate interpretation.

According to our theory, more stringent immigration policy could emerge in economies characterized by higher levels of market power because when market power is sufficiently high, only high skill workers benefit from migration, which provides more fertile grounds for policies against migration.[2]

We model a one period stylized two-countries economy populated by monopolistically competitive producers

[2]This interpretation is in line with the empirical findings suggesting that low-educated voters are more likely to vote for far-right candidates as a result of higher immigration. See also Edo et al. (2019) and related references.

of final goods, and individuals who consume different varieties of final goods, are heterogeneous with respect to work-related skills, and have an elastic labour supply. We characterize the general equilibrium in the short run, where the number of firms is given, and in the long run, which we interpret in a structural sense, allowing for free entry in the market of final goods. By imposing cross-country differences in the composition of workforce, we study endogenous migration flows and their long run macroeconomic consequences with and without international trade of final goods. In particular, we analyze the effects of migration flows on production and, therefore, welfare.

By showing that once we allow for endogenous labor supply, the welfare effects of endogenous migration flows depend on firms' market power, we contribute to three strands of literature. First, there is an extensive literature on the real effects of migration on GDP, income distribution and welfare. Most of the contributions employ a standard neoclassical framework characterized by a perfectly competitive market for the homogeneous final good. A general result within this setup is that workers who, due to migration inflows, experience greater competition are worse off, at least in the short run. Yet, the overall welfare effect of migration considering both origin and destination countries is generally positive, giving rise to the standard trade-off between efficiency and distributive effects.[3] However, the above result is not robust if one allows for consumers' love for variety and monopolistic competition in the market for consumption goods. Specifically, Iranzo and Peri (2009) show that migration could benefit workers in the destination country, irrespective of whether they face more competition in the labor market. Using a similar setup, Di Giovanni et al. (2015) and Aubry et al. (2016) find that migration makes native workers of the destination country better off by inducing a greater variety of consumption goods.[4] We contribute to this strand of literature by endogenizing migration flows and studying the interplay between migration and labor supply decisions, which uncovers the importance of competition in goods market as a key determinant of migration flows and of their welfare consequences.

Some papers investigated how the welfare gains from trade change with endogenous labors supply. Arkolakis and Esposito (2014) show that accounting for endogenous labor supply amplifies the gains from trade, disregarding migration and with market power being sufficiently low. Ago et al. (2017) develop an international trade model with

[3]See Clemens (2011) and Borjas (2014).

[4] *"On average, the market-size effect increases the welfare of all workers by* 1.0% *in the OECD, whereas the average fiscal effect equals* 0.4%*, and the average labor market effect equals* 0.1% *for college graduates and* 0.2% *for the less educated"*, see Aubry et al. (2016) page 3.

preferences that generate a labor supply curve with a U-shaped relationship with technological progress to account for increased working hours in the first stage of development and decreasing in the second state. According to those papers, endogenous labor supply leads to higher welfare gains than models with constant labor supply.

The second main strand of literature to which we contribute is the one investing the determinants of agglomeration. The seminal contributions of Krugman (1991) and Forslid and Ottaviano (2003) develop a two regions model on which agglomeration occurs when transportation costs are sufficiently low. Differently, we disregard from trade to show that forces driving to agglomeration can be generated through the interaction between elastic labor supply and market power. Recently Ago et al. (2018) develop a two regions model featuring trade, immobile labor, and mobile capital. They define agglomeration in terms of capital, their mobile factor, showing that it occurs when labor supply is sufficiently elastic. Differently, our model allows the mobility of workers between countries. We show that when only low skill workers can migrate, agglomeration occurs when good markets are sufficiently competitive, i.e. agglomeration requires firms' market power in the goods market to bee sufficiently low. When high skill workers are allowed to migrate – which we interpret as entrepreneurs' mobility, we find that agglomeration takes place if and only if market power is neither too low or too high, while polarization occurs otherwise.

Our analysis also contributes to the growing literature on market power and macroeconomic outcomes. De Loecker et al. (2020) document, the rise in firms' market power in the US economy since 1955. They show that such trend is correlated with several regularities including declining labor and capital income shares and, related to our work, inter-state migration flows.

Finally, this paper contributes to recent contributions investigating the theoretical properties of the standard *Dixit Stiglitz* framework extended to feature endogenous labor supply, see Kushnir et al. (2021).

The paper is organized as follows. Section 2 describes the model and characterizes the short and long run equilibrium of the closed economy. Section 3 analyzes endogenous migration. Section 4 concludes.

2 Closed economy model

We model an economy populated by a continuum of size L of individuals and an endogenously determined continuum of size N of incumbent firms that produce final goods. Individuals own the firms, have a time endowment $\bar{t}$

each, which they can use to supply labor to the firms in exchange of a salary, and consume final goods. As workers, individuals have heterogenous skills. A mass L_l of individuals is unskilled and a mass L_h is skilled, so that

$$L = L_h + L_l \tag{1}$$

2.1 Individuals' behavior

Individual preferences are defined over consumption and labour and are described by the following CES utility function

$$u(c,s) = c - \left(\frac{\theta}{1+\theta}\right) s^{\frac{\theta+1}{\theta}} \tag{2}$$

where c is utility from consumption, s is individual labor, and $\theta > 0$ measures the elasticity of labor supply with respect to wages. We model love for varieties by assuming

$$c = \left[\int_0^N c\left(i\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where $c(i)$ is consumption of final good of variety i, and σ, with $\sigma > 1$, measures the elasticity of substitution across varieties. According to equation (3), c is defined as a CES consumption aggregator.[5] The income of an individual is given by the salary, Ws, and firms' dividends per capita, D. We assume that all individuals own the same share of all firms, so that

$$D = \frac{\int_0^N \Pi(i)di}{L} \tag{4}$$

where $\Pi(i)$ is firm i's profit and L is given by equation (1). Each individual decides the level of consumption, c, its composition in terms of varieties of final goods, $\{c(i) : i \in [0,N]\}$, and how much to work, s, in order to maximize her utility, given by equation (2), subject to the budget constraint. Formally, each individual solves the following maximization problem

$$\max_{\{c(i):i\in[0,N]\},s} u(c,s) = c - \left(\frac{\theta}{1+\theta}\right) s^{\frac{\theta+1}{\theta}} \tag{5}$$

$$s.to. \int_0^N P\left(i\right) c\left(i\right) di \leq Ws + D \tag{6}$$

$$s \leq \bar{t} \tag{7}$$

[5]We highlight that this formulation prevents the arising of income effects.

taking the price of each variety i, $P(i)$, for $i \in [0, N]$, the wage W, the number of firms, N, and firm-profits, $\Pi(i)$, as given. The solution of the above maximization problem yields the individual Marshallian demand for variety i

$$c(i) = \left(\frac{P(i)}{P}\right)^{-\sigma} (ws + d) \tag{8}$$

and the individual supply of labor

$$s = \min(\bar{t}, w^{\theta}) \tag{9}$$

where $s = \bar{t}$ is the corner solution and $s = w^{\theta}$ is the interior solution.

$$P = \left[\int^{N} P(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \tag{10}$$

is the general price index, $w = W/P$ is the real wage and $d = D/P$ are the real dividends per capita.[6]

We note that, when $s = w^{\theta}$ holds, individual labor supply, s, depends positively on the real wage, and for given P on the nominal wage, W. In general, a salary increase has substitution and an income effects. In line with Ago et al. (2018) we disregard our analysis disregards from income effects.

Finally, by substituting in equation (2) using equations, (3), (8) and (9), we find the following expression for the individual indirect utility function associated with the optimal consumption-labor decision,

$$v = \begin{cases} \left(\frac{W}{P}\right)^{1+\theta}\left(\frac{\theta}{1+\theta}\right) = w^{1+\theta}\left(\frac{\theta}{1+\theta}\right) + d & \text{if } s = w^{\theta} \\ \left(\bar{t}\frac{W}{P}\right) - \frac{\theta}{1+\theta}\bar{t}^{\frac{\theta}{1+\theta}} + d & \text{if } s = \bar{t} \end{cases} \tag{11}$$

2.2 Firms behavior

The mass N of incumbent firms operate in a monopolistically competitive market for final goods. Each firm i, with $i \in [0, N]$, produces a different variety i of final good. Firms produce using high and low skill labor. Following Forslid and Ottaviano (2003), we assume that high-skill labor is the input required to setup a firm, while low-skill labor is the only factor of production required to actually produce. More precisely we assume that in order to set up a firm, a quantity F of high-skill labor is required, while to produce a quantity $y(i)$ of final good firm i uses βL_l

[6] The full solution of the maximization problem 5 along with the derivation of the price index, P, are reported in the appendix.

units of low-skill labor, with $\beta > 0$. The total cost of production associated with such production technology is the following:

$$TC(i) = W_h F + W_l \beta y(i), \tag{12}$$

where W_h is the nominal wage paid to high-skill labor, and W_l is the nominal wage paid to low-skill labor.

The aggregate demand of variety i resulting from the aggregation of individual demand (8) over consumers is

$$y^d(i) = \left(\frac{P(i)}{P}\right)^{-\sigma} \left[w_l s_l L_l + w_h s_h L_H + \int_0^N \pi\left(j\right) dj\right] \tag{13}$$

where $\pi\left(j\right)$ is firm's j real profits. Each firm i sets the price, $P(i)$, in order to maximizes its profits given the demand function (13), taking P, W_h, and W_l as given. Formally,

$$\max_{P(i)} \quad \Pi(i) \quad = P(i)y(i) - W_h F - \beta W_l y\left(i\right) \tag{14}$$

$$s.to \quad y(i) = y^d(i), \tag{15}$$

where y^d is given by equation (13). Substituting for $y(i) = y^d(i)$ in the objective function using equation (13), the first order condition associated with the above problem is

$$\frac{d\Pi\left(i\right)}{dP\left(i\right)} = c(i) + \left[P\left(i\right) - W_l \beta\right] \frac{dc\left(i\right)}{d\left(P(i)\right)} = 0. \tag{16}$$

Solving for $P(i)$ to obtain

$$P\left(i\right) = \frac{\sigma \beta W_l}{\sigma - 1} \Rightarrow p(i) = \frac{\sigma \beta w_l}{\sigma - 1} \qquad \forall i \in [0, N]. \tag{17}$$

The final good price of each variety is a markup over the marginal cost where the size of markup depends on the inverse of the elasticity of substitution across varieties, σ. Following the standard interpretation, we take $\frac{1}{\sigma}$ as a measure of market power. According to this interpretation, as σ increases, the market power of a firm gets smaller, due to the fact that consumers are more willing to substitute a variety of consumption good with another one, and as a result the optimal price that the firm charges is lower.

In a symmetric equilibrium, $P(i) = P(j)$ holds for $i \neq j$, with $i, j \in [0, N]$. Therefore, given equation (10), we have

$$P = N^{\frac{1}{1-\sigma}} P(i) \qquad \forall i \in [0, N], \tag{18}$$

dividing through by P we finally find

$$p(i) = N^{\frac{1}{\sigma-1}} \qquad i \in [0, N] \tag{19}$$

where $p(i) = P(i)/P$ is the equilibrium relative price (real price) of variety i, for given N. Substituting in the demand function (13) by means of equation (19), we find the following expression for the level of production of variety i

$$y(i) = N^{-\frac{\sigma}{\sigma-1}} \left(w_l s_l L_l + w_h s_h L_h + \int_0^N \pi(j)\, dj \right), \tag{20}$$

in a symmetric equilibrium, for given N. Using the above expression and equation (17) to substitute in the objective function (14) we find the implied equilibrium expression of profits of firm i

$$\pi(i) = \sigma^{-\sigma} \left(\frac{\sigma \beta w_l}{\sigma - 1} \right)^{1-\sigma} \left(w_l s_l L_l + w_h s_h L_h + \int_0^N \pi(j)\, dj \right) - w_h F. \tag{21}$$

2.3 Equilibrium analysis

In this section, we characterize the general equilibrium of the model. We first solve for the short run general equilibrium, when the number of firms, N, is given. Then, we turn to the long run general equilibrium by imposing free entry.

2.3.1 Short run

Here we characterize the short run general equilibrium of the economy We determine prices, wages, production, profits, consumption and indirect utility.

Definition 1. *A short run equilibrium of our economy is a vector* $\{c_l(i), c_h(i), y(i), s_h, s_l, W_l, W_h, P(i), ; i \in [0, N]\}$ *of prices and quantities such that, given the number of operating firms* N*:*

1. *Individuals choose s_l, s_h, $c_l(i)$, $c_h(i)$, with $i \in [0, N]$, to maximize utility;*

2. *Firms choose prices $P(i)$, with $i \in [0, N]$ in order to maximize profits;*

3. *Markets for final goods and labor clear.*

Labor markets. In equilibrium, the demand of high-skill and low-skill labor must be equal to the supply. That is,

$$\beta y\,(i)\,N = L_l s_l, \tag{22}$$

$$FN = L_h s_h \tag{23}$$

where equations (22) and (23) refer to the market for low-skill and high-skill labor, respectively. Substituting for s_h in equation (23) using (9) we find the equilibrium level of real wages in the high-skill labor market,

$$w_h = \begin{cases} \left(\frac{FN}{L_h}\right)^{\frac{1}{\theta}} & \text{if } N < \frac{\bar{t}L_h}{F} \\ \frac{L_l}{\sigma\beta L_h \bar{t}}\left(\frac{\sigma-1}{\beta\sigma}\right)^{\theta}\left(\frac{L_h\bar{t}}{F}\right)^{\frac{1+\theta}{\sigma-1}} & \text{if } N = \frac{\bar{t}L_h}{F} \end{cases} \tag{24}$$

Note that $N < \bar{t}L_h/F$ is associated with an equilibrium value of individual supply of skilled labor, s_h, satisfying $s_h < \bar{t}$, while $s_h = \bar{t}$ holds, if $N = \frac{\bar{t}L_h}{F}$.

Similarly, combining equations (17) and (19) we find the equilibrium level of real wages low-skill labor market

$$w_l = \frac{\sigma - 1}{\beta\sigma}N^{\frac{1}{\sigma-1}} \tag{25}$$

Note that the expression of w_l stays the same irrespective of the value of N.

Finally, the equilibrium individual unskilled labor supply is as follows:

$$s_l = \begin{cases} \left(\frac{((\sigma-1))N^{\frac{1}{\sigma-1}}}{(\beta\sigma)}\right)^{\theta} & \text{if } N < \left[\frac{\beta\sigma\bar{t}^{\frac{1}{\theta}}}{\sigma-1}\right]^{\sigma-1} \\ \bar{t} & \text{if } N \geq \left[\frac{\beta\sigma\bar{t}^{\frac{1}{\theta}}}{\sigma-1}\right]^{\sigma-1} \end{cases} \tag{26}$$

With no loss of generality, we make the following assumption

Assumption 1.

$$L_h > \bar{t}^{\frac{1-\theta(\sigma-1)}{\theta(\sigma-1)}} F \left(\frac{\beta\sigma}{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{27}$$

The above assumption implies that the individual labor supply of unskilled workers is never constrained.

Final good production, profits and dividends. Substituting for s_l and w_l, in equation (22) using equations (9) and (25), we find the short run equilibrium level of production of each variety i of final good

$$y(i) = \frac{L_l}{\beta^{\theta+1}}\left\{\frac{\sigma-1}{\sigma}\right\}^{\theta} N^{\frac{\theta-\sigma+1}{\sigma-1}} \tag{28}$$

Given the expressions of the equilibrium price, $p(i)$, $y(i)$ and w_h, given by equations (19), (28), and (24), and imposing market clearing in the labor market for high skill labor, we can write the firm-level profits as

$$\pi = \begin{cases} L_l \left(\frac{1}{\beta\sigma}\right)^{1+\theta} N^{\frac{2+\theta-\sigma}{\sigma-1}} (\sigma-1)^{\theta} - \left(\frac{N}{L_h}\right)^{\frac{1}{\theta}} F^{\frac{1+\theta}{\theta}} & \text{if } N < \frac{\bar{t}L_h}{F} \\ 0 & \text{if } N = \frac{\bar{t}L_h}{F} \end{cases} \tag{29}$$

where we dropped the index i since profits are equal across firms. Accordingly, the equilibrium value of real dividends per capita is

$$d = \frac{\pi}{L} \tag{30}$$

We note that an increase in N has two effects opposing on profits. Firms face a stronger competition which reduces their market share as well as firm's profits (*business stealing effect*, see Mankiw and Whinston (1986). However, as N increases the real wages of individuals get larger boosting the demand of final goods, which increases firm's profits (*demand effect*).[7] The following result holds:

Lemma 1. *Short run profits are decreasing in N if $\sigma > 2+\theta$ or $1+\theta < \sigma < 2+\theta$ and $N \geq \widehat{N}$. Short run profits are increasing in N as long as $\sigma < 1+\theta$ and $N \geq \widehat{N}$, where*

[7] Corsetti et al. (2007) considering a model with elastic labor supply puts condition on parameters to have the *business stealing effect* always larger than the demand effect. However, they consider a model with income effects and perfectly elastic labor supply. In our benchmark model the Frish elasticity is equal to θ and we do not make assumption to compress the size of the *demand effect*.

$$\widehat{N}=\left(\frac{\sigma\beta}{\theta L_l}\left(\frac{F}{L_h}\right)F\left(\frac{\sigma-1}{\beta\sigma}\right)^{-\theta}\frac{\theta+2-\sigma}{\sigma-1}\right)^{\frac{(\sigma-1)\theta}{(\theta+1)(\theta+1-\sigma)}} \tag{31}$$

Short run profits are otherwise decreasing in N.

Proof. See appendix.

According to lemma 1, assuming that N is larger than both $\hat{N}$ profits are increasing with the number of competitors as long as $\sigma < 1+\theta$. Under this condition, the demand effect always offsets the business stealing effect. The contrary happens when $\sigma > 1+\theta$, where as long as N is larger than $\hat{N}$ profits increases with the number of competitors.

Short run consumption and welfare. Here we report the short run equlibrium value of consumption and welfare for the case in which N is such that, both for unskilled and skilled workers, the time constraint is not binding.

Using equations (9), (19), (24), (25), (29), and (30) to substitute in equation (8), we find

$$c_l(i) = \left(\frac{\sigma-1}{\beta\sigma}\right)^{1+\theta}N^{\frac{1+\theta-\sigma}{\sigma-1}}+\frac{1}{L}\left[L_l\left(\frac{1}{\beta\sigma}\right)^{1+\theta}N^{\frac{2+\theta-2\sigma}{\sigma-1}}(\sigma-1)^{\theta}-(L_h)^{-\frac{1}{\theta}}N^{\frac{-\sigma\theta+\sigma-1}{\theta(\sigma-1)}}F^{\frac{1+\theta}{\theta}}\right] \tag{32}$$

$$c_h(i) = \left(\frac{F}{L_h}\right)^{\frac{1+\theta}{\theta}}N^{\frac{\sigma-(1+\theta)}{\sigma-1}}+\frac{1}{L}\left[L_l\left(\frac{1}{\beta\sigma}\right)^{1+\theta}N^{\frac{2+\theta-2\sigma}{\sigma-1}}(\sigma-1)^{\theta}-(L_h)^{-\frac{1}{\theta}}N^{\frac{-\sigma\theta+\sigma-1}{\theta(\sigma-1)}}F^{\frac{1+\theta}{\theta}}\right]. \tag{33}$$

Correspondingly, the equilibrium values of the consumption bundles are

$$c_l = \left(\left(\frac{\sigma-1}{\beta\sigma}\right)^{1+\theta}N^{\frac{1+\theta}{\sigma-1}}+\frac{1}{L}\left(L_l\left(\frac{1}{\beta\sigma}\right)^{1+\theta}N^{\frac{2+\theta-\sigma}{\sigma-1}}(\sigma-1)^{\theta}-(L_h)^{-\frac{1}{\theta}}N^{\frac{\sigma-1}{\theta(\sigma-1)}}F^{\frac{1+\theta}{\theta}}\right)\right), \tag{34}$$

$$c_h = \left(\left(\frac{F}{L_h}\right)^{\frac{1+\theta}{\theta}}N^{\frac{1+\theta}{\theta}}+\frac{1}{L}\left(L_l\left(\frac{1}{\beta\sigma}\right)^{1+\theta}N^{\frac{2+\theta-\sigma}{\sigma-1}}(\sigma-1)^{\theta}-(L_h)^{-\frac{1}{\theta}}N^{\frac{1}{\theta}}F^{\frac{1+\theta}{\theta}}\right)\right). \tag{35}$$

Similarly, using equations (24), (25) and (30) to substitute in equation (11) we find the short run equilibrium

equilibrium values of the indirect utility of low-skill and high-skill workers,

$$v_l = \left(\frac{\theta}{1+\theta}\right)\left(\frac{\sigma-1}{\beta\sigma}\right)^{1+\theta} N^{\frac{1+\theta}{\sigma-1}} + \frac{L_l\left(\frac{1}{\beta\sigma}\right)^{1+\theta} N^{\frac{2+\theta-\sigma}{\sigma-1}}(\sigma-1)^{\theta} - \left(\frac{N}{L_h}\right)^{\frac{1}{\theta}} F^{\frac{1+\theta}{\theta}}}{L}, \tag{36}$$

$$v_h = \left(\frac{\theta}{1+\theta}\right)\left(\frac{FN}{L_h}\right)^{\frac{1+\theta}{\theta}} + \frac{L_l\left(\frac{1}{\beta\sigma}\right)^{1+\theta} N^{\frac{2+\theta-\sigma}{\sigma-1}}(\sigma-1)^{\theta} - \left(\frac{N}{L_h}\right)^{\frac{1}{\theta}} F^{\frac{1+\theta}{\theta}}}{L}. \tag{37}$$

2.4 Long run equilibrium

In this section, we characterize the long run equilibrium, which we define as follows.

Definition 2. *A long-run equilibrium of the economy is a vector* $\{c_l, c_h, y(i), s_h, s_l, P(i), W_l, W_h; i \in [0, N^e]\}$ *of prices and quantities, and a mass of operating firms,* N^e*, such that:*

- *Necessary and sufficient conditions for a short run general equilibrium are satisfied;*
- *The mass firms,* N^e*, is such that* $\pi_{N^e} = 0$ *and*[8]

$$\pi\Big|_{N=N_e} = 0 \tag{38}$$

$$\frac{d\pi}{dN}\Big|_{N=N_e} \leq 0 \tag{39}$$

Lemma 1 is useful also to determine the long run equilibrium level of operating firms N as well as the equilibrium level of individual labor supply of high skill workers. The following result holds,

Corollary 1. *If* $1+\theta < \sigma < 2+\theta$ *and* $N < \widehat{N}$ *or* $\sigma < 1+\theta$ *and* $N \geq \widehat{N}$*, in any long run equilibrium,* $s_h = \bar{t}$ *and* $N_e = \frac{L_h \bar{t}}{F}$ *hold.*

More clearly, as long as profits are increasing with the number of firms competition on high skill labor to open new firms will lead such workers to choose the maximum amount of hours, $s_h = \bar{t}$. If the conditions of corollary 1 are not met, the equilibrium value of s_h can be either constrained or not depending on the value of L_h, other things equal.

[8]These conditions are equivalent to those put forward by Mankiw and Whinston (1986).

Following corollary 1, the long run equilibrium mass of operating firms satisfies

$$N_e = \begin{cases} \left(\frac{1}{F} \left(\beta\sigma\right)^{-\theta} L_l^{\frac{\theta}{1+\theta}} L_h^{\frac{1}{1+\theta}} \left(\sigma - 1\right)^{\frac{\theta^2}{1+\theta}} \right)^{\frac{\sigma-1}{\sigma-(\theta+1)}} . & \text{if } s_h < \bar{t} \\ \frac{L_h \bar{t}}{F} & \text{if } s_h = \bar{t} \end{cases} \tag{40}$$

In the following discussion, we characterize the long run equilibrium in in the case in which s_h is not constrained.[9]

Final good prices, production, and wages The long run equilibrium real price, $p(i)$, of each variety of final good is found using equation (40) to substitute for N in equation (19),

$$p(i) = \left(\frac{1}{F} \left(\beta\sigma\right)^{-\theta} L_l^{\frac{\theta}{1+\theta}} L_h^{\frac{1}{1+\theta}} \left(\sigma - 1\right)^{\frac{\theta^2}{1+\theta}} \right)^{\frac{1}{\sigma-(\theta+1)}} . \tag{41}$$

Similarly, using equation (40) to substitute for N in equations (24) and (25), we find the long run equilibrium values of real wages

$$w_h = \left(\frac{F}{L_h} \right)^{\frac{1}{\theta}} \left(\frac{1}{F} \left(\beta\sigma\right)^{-\theta} L_l^{\frac{\theta}{1+\theta}} L_h^{\frac{1}{1+\theta}} \left(\sigma - 1\right)^{\frac{\theta^2}{1+\theta}} \right)^{\frac{\sigma-1}{[\sigma-(\theta+1)]\theta}} , \tag{42}$$

$$w_l = \frac{\sigma - 1}{\beta\sigma} \left(\frac{1}{F} \left(\beta\sigma\right)^{-\theta} L_l^{\frac{\theta}{1+\theta}} L_h^{\frac{1}{1+\theta}} \left(\sigma - 1\right)^{\frac{\theta^2}{1+\theta}} \right)^{\frac{1}{\sigma-(\theta+1)}} . \tag{43}$$

Using equation (40) to substitute for N in equation (28), we find the long run equilibrium level of production of each variety i of final good

$$y(i) = \frac{L_l}{\beta^{\theta+1}} \left\{ \frac{\sigma - 1}{\sigma} \right\}^{\theta} \left(\frac{1}{F} \left(\beta\sigma\right)^{-\theta} L_l^{\frac{\theta}{1+\theta}} L_h^{\frac{1}{1+\theta}} \left(\sigma - 1\right)^{\frac{\theta^2}{1+\theta}} \right)^{-1} . \tag{44}$$

Finally, aggregate production is found multiplying individual production by the number of operating firms, which yields

$$Y = \frac{L_l}{\beta^{\theta+1}} \left\{ \frac{\sigma - 1}{\sigma} \right\}^{\theta} \left(\frac{1}{F} \left(\beta\sigma\right)^{-\theta} L_l^{\frac{\theta}{1+\theta}} L_h^{\frac{1}{1+\theta}} \left(\sigma - 1\right)^{\frac{\theta^2}{1+\theta}} \right)^{\frac{\theta}{\sigma-(\theta+1)}} . \tag{45}$$

[9] The characterization of the long run equilibrium for the case in which $s_h = \bar{t}$ would be the same as the one we provided for the short run equilibrium.

Long run consumption and welfare We compute the equilibrium values of the indirect individual utility of low and high-skill individuals, and of the aggregate consumption index $C = L_h c_h + L_l c_l$, where c_h and c_l are the equilibrium values of the individual consumption indexes of high and low-skill individuals, respectively.

Substituting for the equilibrium values of real wages, supply of labor and relative price of variety i in equation (8), we find the equilibrium value of individual consumption of variety i for low-skill and high-skill individuals

$$c_{l(i)} = F L_l^{-\frac{\theta}{1+\theta}} L_h^{-\frac{1}{1+\theta}} \frac{1}{\beta\sigma} (\sigma-1)^{\frac{1+\theta-\theta^2}{1+\theta}} \tag{46}$$

$$c_h(i) = F L_l^{\frac{1}{1+\theta}} (\beta\sigma)^{-1} (\sigma-1)^{\frac{\theta}{1+\theta}} L_h^{\frac{(\theta-2)}{(1+\theta)}} \tag{47}$$

In a symmetric equilibrium, $P(i) = P(j)$, for any $i \neq j$, which implies $c(i) = c(j)$, so that

$$c = N^{\frac{\sigma}{\sigma-1}} c(i) \tag{48}$$

which combined with equations (46) and (47) leads to

$$c_h = L_h^{\frac{(\sigma-1)(\sigma-\theta)+2}{(\sigma-(1+\theta))(1+\theta)}} L_l^{\frac{\sigma-1}{\sigma-(1+\theta)}} F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}} (\sigma-1)^{\frac{\theta(\sigma-1)}{(1+\theta)(\sigma-(1+\theta))}} (\beta\sigma)^{\frac{-\theta^2(1+\sigma)-\theta}{(1+\theta)(\sigma-(1+\theta))}}, \tag{49}$$

$$c_l = F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}} L_l^{\frac{\theta}{\sigma-(1+\theta)}} L_h^{\frac{1}{\sigma-(1+\theta)}} (\sigma-1)^{\frac{(\sigma-\theta)(1+\theta)}{\sigma-(1+\theta)}} (\beta\sigma)^{\frac{-(1+\theta)(\sigma-1)}{\sigma-(1+\theta)}}. \tag{50}$$

We now turn to the indirect utility. With zero profits and substituting the long run equilibrium value of N we get

$$v_l = \left(\frac{1}{1+\theta}\right) (\sigma-1)^{\frac{\sigma-(1+\theta)+\theta(\sigma-1)}{\sigma-(1+\theta)}} (\beta\sigma)^{\frac{(1+\theta)(1-\sigma)}{\sigma-(1+\theta)}} F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}} L_l^{\frac{\theta}{\sigma-(1+\theta)}} L_h^{\frac{1}{\sigma-(1+\theta)}}, \tag{51}$$

$$v_h = \left(\frac{1}{1+\theta}\right) (\sigma-1)^{\frac{\theta(\sigma-1)}{\sigma-(1+\theta)}} (\beta\sigma)^{\frac{(1+\theta)(1-\sigma)}{\sigma-(1+\theta)}} F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}} L_l^{\frac{\sigma-1}{\sigma-(1+\theta)}} L_h^{\frac{2+\theta-\sigma}{(\sigma-1-\theta)\theta}}. \tag{52}$$

2.5 Market power and welfare effects of exogenous inflows of workers

In this section, we study the long run general equilibrium effects of an exogenous change in the population of either low or high-skill individuals. This analysis is preliminary to the study of endogenous migration flows, with and without trade, which we tackle in sections 3 and 4. The following result holds

Proposition 1 (Welfare effects of exogenous changes in the population)**.** *The effects of am exogenous increase in the size of the workers' population are as follows:*

- *if* $\sigma < 1+\theta$ *an increase in the stock of low skilled workers makes high skilled better off and low skilled unaffected.*
- *if* $\sigma > 1+\theta$ *an increase in the stock of low skilled workers makes high skilled and low skilled better off when high skilled labor choice is unconstrained*
- *if* $\sigma < 1+\theta < 2$ *an increase in the stock of high skilled workers makes better of both low and high skilled*
- *if* $\sigma < 1+\theta$ *an increase in the stock of high skilled makes better of both types of individuals.*
- $1+\theta < \sigma < 2+\theta$ *an increase in the stock of high skilled workers makes better of both low and high skilled*
- *If* $\sigma > 2+\theta$ *an increase in the stock of high skilled workers makes low skilled better off and high skilled worse off.*

Proof. See appendix.

In the next section, we explore endogenous migration, which allows as to evaluate under which conditions population differences across countries lead to migration flows that imply the welfare effects that we have preliminary identified in proposition 1.

3 Endogenous Migration

In this section, we study the effects of migration considering two identical economies described by the model previously defined. We label the two economies a and b. We first assess the effects of high-skill migration and then, we address the possibility of low-skill migration. The utility differential drives the migration desire, and we account for migration costs. Specifically, we assume that migrants incur a migration cost measured as a fraction $x \in [0, 1]$, of the indirect utility, v, at destination, so that the net indirect utility ad destination is, $v(1-x)$. We analyse the effects of migration allowing for migration of high and low skill individuals, separately.

3.1 High Skilled Migration

First, we consider the case of two regions that differ only in the the stock of low-skill labor. Without loss of generality, we assume that the supply of low-skill is larger in the region a, $L_{a,l} > L_{b,l}$.

Lemma 2 (High-skill migration, with $L_{a,h} = L_{b,h} = L_h$). *Migration only takes place if migration costs are sufficiently low. That is, migration from b to a could occur only if*

$$x < \frac{v_b}{v_a} - 1 \equiv \widehat{x}_{b,a} \tag{53}$$

while migration from a to b could occurs only if

$$x < \frac{v_a}{v_b} - 1 \equiv \widehat{x}_{a,b} \tag{54}$$

1. *If $\sigma > 1 + \theta$ and $x < \widehat{x}_{b,a}$ high-skill individuals migrate from b to a. Under this case, if $\sigma \in [1 + \theta, 2 + \theta]$, full agglomeration of high-skill workers occur, that is, $M = L_{h,b}$. Otherwise, if $\sigma > 2 + \theta$, migration takes place from a to b*

$$M = L_h \frac{1 - \left(\frac{L_{la}}{L_{lb}}\right)^{\frac{\sigma-1}{2+\theta-\sigma}} (1-x)^{\frac{\sigma-1-\theta}{2+\theta-\sigma}}}{1 + \left(\frac{L_{la}}{L_{lb}}\right)^{\frac{\sigma-1}{2+\theta-\sigma}} (1-x)^{\frac{\sigma-1-\theta}{2+\theta-\sigma}}} \tag{55}$$

2 *If $\sigma < 1 + \theta$ and $x < \widehat{x}_{a,b}$ $s_h = \bar{t}$. High skilled move from b to a and there is a tendency towards full agglomeration as long as $\sigma < 2$ and there is a tendency towards full agglomeration that is $M = L_{h,b}$. On the contrary when $\sigma > 2$, indirect utility of high skilled workers decreases with L_h, migrants move from a towards b and the number of migrants is obtained equal to*

$$M = \frac{L_h \left(\frac{(L_{l_b})}{(L_{l_a})(1-x)} - 1\right)}{\left(\left(\frac{(L_{l_b})}{(L_{l_a})(1-x)}\right)^{\frac{\sigma-1}{2-\sigma}} + 1\right)} \tag{56}$$

Proof. See appendix.

Consider now the case of two regions a and b endowed with the same stock of low-skill individuals, such that the stock of high-skill individuals is larger in b than a, $L_{a,h} > L_{b,h}$. Then, the following result holds

Lemma 3 (High-skill migration, with $L_{a,l} = L_{b,l}$)**.** *Migration only takes place if migration costs are sufficiently low. That is, migration from b to a could occur only if*

$$x < \frac{v_b}{v_a} - 1 \equiv \widehat{x}_{b,a} \tag{57}$$

while migration from a to b could occur only if

$$x < \frac{v_a}{v_b} - 1 \equiv \widehat{x}_{a,b} \tag{58}$$

1. *If $\sigma > 1+\theta$ and $x < \widehat{x}_{b,a}$ high-skill individuals migrate from b to a. Under this case, if $\sigma \in [1+\theta, 2+\theta]$, full agglomeration of high-skill workers occur, that is, $M = L_{h,b}$. Otherwise, if $\sigma > 2+\theta$, high-skill individuals migrate from a to b, and*

$$M = \frac{L_{a,h} - L_{b,h}\,(1-x)^{\frac{(\sigma-(1+\theta))}{(2+\theta-\sigma)}}}{1 + (1-x)^{\frac{(\sigma-(1+\theta))}{(2+\theta-\sigma)}}}. \tag{59}$$

   with $M < L_{a,h}$;

2 *If $\sigma < 1+\theta$ and $s_h = \bar{t}$ and $s_l = \frac{W}{P}$ $x < \widehat{x}_{a,b}$ and $\sigma < 2+\theta$ high-skill individuals migrate from b to a, and full agglomeration occurs. When $\sigma > 2$ high skilled are better off in the less populated region and there is a flow of people from the more populated toward the less populated region. The number of migrants is given by:*

$$M = \frac{L_{b,h}\left((1-x)^{\frac{\sigma-1}{2-\sigma}}\right) + L_{a,h}}{1 + (1-x)^{\frac{\sigma-1}{2-\sigma}}} \tag{60}$$

Proof. See appendix.

Notice that utility, for the high skilled is larger in location a only if $\sigma \in [1+\theta, \sigma < 2+\theta]$. When this condition is met, migration amplifies the utility differential so that, as long as migration costs are small enough, there is a tendency towards full agglomeration. Low and high-skill individuals in the destination region are better off, while low-skill individuals at origin are worse off. When $\sigma > 2+\theta$, high skilled attain a larger utility in the smaller region, region a, and migration flows from region b, the more populated, to region a. Native high-skill individuals in the destination region are worse off, due to the increased competition in the labor market, which is more than offsets the love for variety effect. Low-skill individuals in the destination region are better off since they enjoy consuming more varieties, while low-skill in the origin country are worse off due to the reduction in the varieties. High-skill workers of the origin country are better off. When $\sigma < 1 + theta$ utility is always higher in the more populated region and the utility differential is amplified by migration flows. Accordingly, there is a tendency towards full agglomeration.

3.2 Low skilled Migration

Consider now the case in which only low-skill individuals can migrate. Assume that a is more populated than b, and either $L_{a,h} = L_{b,h}$ or $L_{a,l} = L_{b,l}$. The following result holds,

Lemma 4. *Migration only takes place if migration costs are sufficiently low. That is, migration from b to a could occur only if*

$$x < \frac{v_b}{v_a} - 1 \equiv \widehat{x}_{b,a} \tag{61}$$

while migration from a to b could occur only if

$$x < \frac{v_a}{v_b} - 1 \equiv \widehat{x}_{a,b} \tag{62}$$

1. *If $\sigma > 1 + \theta$, and $x < \widehat{x}_{a,b}$, migration takes place from country b to a and full agglomeration takes place, $M = L_{b,l}$;*

2. *If $\sigma < 1 + \theta$, and $x < \widehat{x}_{b,a}$ and $L_{ha} > L_{hb}$ migration takes place from country b to a and full agglomeration occurs. If $L_{ha} = L_{hb}$ migration does not take place*

As long as $\sigma > 1 + \theta$, low-skill utility increases with L_l so that full agglomeration takes place, if migration costs are not too high. Individuals in the destination region are better off as well as low-skill natives of the region of origin, while high-skill in the origin region are worse off. amplify the differential utility. A similar effect is predicted when $\sigma < 1 + \theta$ On the contrary if the two countries are identical in terms of high skill labor migration never occurs

3.3 Market power, migration flows and welfare effects

The following proposition summarizes our results for the economy with endogenous migration flows.

Proposition 2.

- *high skilled migration flows makes better both low and high skilled if* $\sigma < 2 + \theta$
- *high skilled migration flows makes better of low skilled and worse off high skilled if* $\sigma > 2 + \theta$. *In this case there is no tendency towards full agglomeration for high skilled.*
- *high skilled migration flows makes better of both low and high skilled at destination if* $\sigma < 1 + \theta < 2$
- *low skilled migration flows makes always better of low skilled and high skilled if* $\sigma > 1 + \theta$.
- *low skilled migration flows makes better of high skilled when* $\sigma < 1 + \theta$ *and leave unskilled individuals at destination unaffected.*

Proof. The proof follows directly from lemmata 1-3. □

4 Conclusion

We analyzed endogenous migration a two-region model with endogenous individual labor supply, monopolistically competitive producers of final goods and love for variety. Our analysis shows that the welfare effects of migration depend upon the market power of firms producing final goods. If market power is sufficiently high, migration of low-skill workers affects positive the welfare of native high-skill individuals in the destination region, while unskilled individuals are unaffected. natives of the origin region are always better off, irrespective of their skills. Differently, if market power is sufficiently low, low-skill migration makes individuals native of the destination region better off irrespectively of skills.

References

Ago, T., Morita, T., Tabuchi, T., and Yamamoto, K. (2017). Endogenous labor supply and international trade. *International Journal of Economic Theory*, 13(1):73–94.

Ago, T., Morita, T., Tabuchi, T., and Yamamoto, K. (2018). Elastic labor supply and agglomeration. *Journal of Regional Science*, 58(2):350–362.

Arkolakis, C. and Esposito, F. (2014). Endogenous labor supply and the gains from international trade. *Processed, Yale University*.

Aubry, A., Burzyński, M., and Docquier, F. (2016). The welfare impact of global migration in oecd countries. *Journal of International Economics*, 101:1–21.

Borjas, G. J. (2014). *Immigration economics*. Harvard University Press.

Clemens, M. A. (2011). Economics and emigration: Trillion-dollar bills on the sidewalk? *Journal of Economic perspectives*, 25(3):83–106.

Corsetti, G., Martin, P., and Pesenti, P. (2007). Productivity, terms of trade and the 'home market effect'. *Journal of International economics*, 73(1):99–127.

De Loecker, J., Eekchout, J., and Unger, G. (2020). The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics*, 135(2):561–644.

Deuster, C. (2021). international migration and government policies: Immigration vs emigration restrictions. Technical report, Mimeo.

Di Giovanni, J., Levchenko, A. A., and Ortega, F. (2015). A global view of cross-border migration. *Journal of the European Economic Association*, 13(1):168–202.

Edo, A., Giesing, Y., Öztunc, J., and Poutvaara, P. (2019). Immigration and electoral support for the far-left and the far-right. *European Economic Review*, 115(C):99–143.

Forslid, R. and Ottaviano, G. I. (2003). An analytically solvable core-periphery model. *Journal of Economic Geography*, 3(3):229–240.

Fujita, M., Krugman, P. R., and Venables, A. J. (2001). *The spatial economy: Cities, regions, and international trade*. MIT press.

Iranzo, S. and Peri, G. (2009). Migration and trade: Theory with an application to the eastern–western european integration. *Journal of International Economics*, 79(1):1–19.

Krugman, P. (1991). Increasing returns and economic geography. *Journal of political economy*, 99(3):483–499.

Kushnir, A., Tarasov, A., and Zubrickas, R. (2021). On equilibrium in monopolistic competition with endogenous labor. *Economics Letters*, 201:109774.

Mankiw, N. G. and Whinston, M. D. (1986). Free entry and social inefficiency. *The RAND Journal of Economics*, pages 48–58.

A Individuals' maximization problem

For the sake of completeness the appendix reports the full solution of the model. We solve the individual's problem following the standard two-step procedure, outlined in Fujita et al. (2001).

1. Taking as given the total expenditure in the Consumption good c we determine the amount demanded of each variety A.1.

2. We determine the optimal allocation of time among work and leisure. By doing so we retrieve the individual's income which is fully spent in c see Section A.2.

A.1 Optimal demand of variety j

The consumer optimally chooses the amount of any variety j taking into account both the price of the variety, $P(j)$, and its budget, which we label X. Given that the solution procedure does not change along the skill type of the individuals this section does not differentiate for the individual's skill type. In our setup X coincides with labor income: the product of the nominal wage and the hours actually worked. Each individual maximize the consumption aggregate, see equation 3, for each variety j subject to the constraint below

$$X = \int_0^n P(j)c(j)dj,$$

taking derivative with respect any two varieties i and j and considering their ratio:

$$(c(i)) = \left(\frac{P(i)}{P(j)}\right)^{-\sigma} c(j) \tag{63}$$

and multiplying both sides of the equation above by $P(i)$ we have

$$(P(i))(c(i)) = P(j)^{\sigma} P(i)^{1-\sigma} c(j). \tag{64}$$

Integrate both sides of the equation above with respect to variety i

$$\int (P(i))(c(i)) = P(j)^{\sigma} c(j) \int P(i)^{1-\sigma} \tag{65}$$

Notice that the LHS of the above equation is equal to Ws. It follows that the demand of a variety j is equal to:

$$c(j) = \frac{Ws}{P(j)^{\sigma} \int P(i)^{1-\sigma}} \tag{66}$$

The price index of our economy is already defined in the main text, see equation 10.

A.2 Labor supply

Once we know the optimal demand of any variety i given prices and a certain level of income we can determine the optimal labor supply. We setup the relevant Lagrangian exploiting equation 2 and taking into account the individual's budget constraint

$$\mathcal{L} = c - \frac{\theta s^{\frac{\theta+1}{\theta}}}{1+\theta} + \mu\left[Ws - PC\right] + \lambda\left(s - \bar{t}\right) \tag{67}$$

where μ is the Lagrangian multiplier relative to the budget constraint and λ is the Lagrangian multiplier relative to the time constraint. Deriving the Lagrangian for both c and s we obtain:

$$1 = \mu P, \tag{68}$$

$$s^{\frac{1}{\theta}} = \mu W - \lambda \tag{69}$$

When the time constraint is not binding, by taking the ratio of equation 69 and 68 and solving for s we obtain

$$s = \left(\frac{W}{P}\right)^{\theta} = w^{\theta}. \tag{70}$$

Equation 9 shows labor supply is an increasing function of the real wage, our preferences prevents the arising of income effects. Otherwise, when the time constraint is binding is easy to notice that the optimal solution coincides with $\bar{t}$

The real wage depends positively on the nominal wage, see the numerator, and negatively on the price index. However, the price index is a decreasing function of the number of varieties available in our economy. Conversely, an increase in the price index, P, decreases the the real income and it eventually reduces the individuals' labor supply, when the choice is unconstrained.

Substituting the optimal value of s, defined by equation 9, into equation 8 we obtain the Marshallian demand of any given variety.

$$c(j) = \left(\frac{P(j)}{P}\right)^{-\sigma} \left(\frac{W}{P}\right)^{1+\theta} \tag{71}$$

B Proof of Lemma 1

Taking the derivative of equation (29) with respect to N we obtain:

$$\frac{d\pi(i)}{dN} = \left(\frac{\theta+2-\sigma}{\sigma-1}\right)\frac{L_l}{\beta}\left(\frac{\sigma-1}{\beta\sigma}\right)^{\theta}\left(N^{\frac{\theta+3-2\sigma}{\sigma-1}}\right)\left(\frac{1}{\sigma}\right) - \frac{1}{\theta}\left(\frac{F}{L_h}\right)FN^{\frac{1-\theta}{\theta}} \tag{72}$$

From which it follows that $\frac{d\pi(i)}{dN} > 0$ if and only if

$$\left(N^{\frac{(\theta+1)^2-\sigma(1+\theta)}{(\sigma-1)\theta}}\right) > \frac{\sigma\beta}{\theta L_l}\left(\frac{F}{L_h}\right)F\left(\frac{\sigma-1}{\beta\sigma}\right)^{-\theta}\frac{\theta+2-\sigma}{\sigma-1} \tag{73}$$

Given the above equation and the definition of $\widehat{N}$, see equation 31 the results of lemma 1 follows immediately.□

C Proof of Proposition 1

Indirect utility of low and high skilled individuals when high skilled workers labor choice is unconstrained are given by equations 52. Notice that v_l increases with L_l and L_h as long as the condition $\sigma > 1+\theta$ holds, which is a necessary condition to have high skilled labor choice unconstrained. Taking derivatives for both L_l and L_h we obtain

$$\frac{dv_l}{dL_l} = \left(\frac{1}{1+\theta}\right)\left(\frac{\theta}{\sigma-(1+\theta)}\right)(\sigma-1)^{\frac{\sigma-(1+\theta)+\theta(\sigma-1)}{\sigma-(1+\theta)}}(\beta\sigma)^{\frac{(1+\theta)(1-\sigma)}{\sigma-(1+\theta)}}F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}}L_l^{\frac{2\theta+1-\sigma}{\sigma-(1+\theta)}}L_h^{\frac{1}{\sigma-(1+\theta)}} \tag{74}$$

$$\frac{dv_l}{dL_h} = \left(\frac{1}{1+\theta}\right)\left(\frac{1}{\sigma-(1+\theta)}\right)(\sigma-1)^{\frac{\sigma-(1+\theta)+\theta(\sigma-1)}{\sigma-(1+\theta)}}(\beta\sigma)^{\frac{(1+\theta)(1-\sigma)}{\sigma-(1+\theta)}} F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}} L_l^{\frac{\theta}{\sigma-(1+\theta)}} L_h^{\frac{2+\theta-\sigma}{\sigma-(1+\theta)}} \tag{75}$$

The equation below shows that v_h increases with L_l as long as $\sigma > 1+\theta$:

$$\frac{dv_h}{dL_l} = \left(\frac{1}{1+\theta}\right)\left(\frac{\sigma-1}{\sigma-(1+\theta)}\right)(\sigma-1)^{\frac{\theta(\sigma-1)}{\sigma-(1+\theta)}}(\beta\sigma)^{\frac{(1+\theta)(1-\sigma)}{\sigma-(1+\theta)}} F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}} L_l^{\frac{\theta}{\sigma-(1+\theta)}} L_h^{\frac{2\theta+1-\sigma}{\sigma-(1+\theta)}} \tag{76}$$

On the contrary the effect of an inflow of L_h on v_h is positive if $\sigma < 2+\theta$

$$\frac{dv_h}{dL_h} = \left(\frac{1}{1+\theta}\right)\left(\frac{(\theta+2-\sigma)}{\sigma-(1+\theta)}\right)(\sigma-1)^{\frac{\theta(\sigma-1)}{\sigma-(1+\theta)}}(\beta\sigma)^{\frac{(1+\theta)(1-\sigma)}{\sigma-(1+\theta)}} F^{\frac{-(1+\theta)}{\sigma-(1+\theta)}} L_l^{\frac{\sigma-1}{\sigma-(1+\theta)(-\theta\sigma-1-\theta)}} L_h^{\frac{-\theta\sigma-1-\theta}{(1+\theta)}} \tag{77}$$

When high skilled labor is constrained we have that:

$$v_h = \left(\frac{\bar{t}}{F}\right)^{\frac{1}{\sigma-1}} L_h^{\frac{2-\sigma}{\sigma-1}} \left(\frac{L_l}{\beta\sigma}\right) - \frac{\theta}{1+\theta}\bar{t}^{\frac{1+\theta}{\theta}}$$

Assumption 1 prevents low skilled labor choice to be constrained. In this case v_l is given by:

$$v_l = \left(\frac{\sigma-1}{\beta\sigma}\frac{\bar{t}L_h}{F}\right)^{\frac{\sigma-1}{\beta\sigma}}$$

Which shows that when high skilled labor choice is constrained the low skilled utility does not increase with low skilled workers. Hence the effect is positive □

D Proof of lemma 2

A high-skill individual migrates from b to a if

$$v_a(1-x) < v_b \tag{78}$$

Solving for x we find that migration from a to be occurs if

$$x < \frac{v_b}{v_a} - 1 \equiv \widehat{x}_{b,a} \tag{79}$$

Case 1. Given, $\sigma > 1 + \theta$, it is evident from equation 52 that the indirect utility of high-skill individuals, v_h, is higher in region a so long as $L_{a,l} > L_{b,l}$ and $L_{a,h} = L_{b,h}$. Accordingly, if and only if $x < \widehat{x}_{b,a}$, no migration is not an equilibrium as high-skill workers would have an incentive to migrate. Furthermore, it is immediate to verify that as long as $1 + \theta < \sigma < 2 + \theta$ migration amplifies the utility differential, $v_{a,h}(1 - x) - v_{b,h}$, and hence the economy reaches a full agglomeration equilibrium with all high-skill workers located in region a. On the other hand, it follows directly from equation (52) that if $\sigma > 2 + \theta$, v_h is decreasing in L_h, so that the $v_{a,h}(1 - x) - v_{b,h}$ is decreasing with migration. Accordingly, the economy reaches an equilibrium characterized by a mass of migrants such that $v_{a,h}(1 - x) - v_{b,h} = 0$. Solving for the value of M that satisfies the above equation yields (55).

Case 2. When $\sigma < 1 + \theta$ the high skilled labor choice is constrained in both regions. In this case the indirect utility of high-skill individuals, v_h, is higher in region a so long as $L_{a,l} > L_{b,l}$ and $L_{a,h} = L_{b,h}$. Furthermore, v_h is increasing in L_h if $\sigma < 2$. In this case there is tendency towards full agglomeration. Otherwise, if $\sigma > 2$ the utility decreases with the flow of migrants and the flow of workers from b to a is determined by a standard arbitrage argument □

E Proof of lemma 3

The proof of this lemma follows the same line of reasoning outlined in lemma 2. □

Ultimi Contributi di Ricerca CRENoS

I Paper sono disponibili in: http://www.crenos.unica.it

21/10 *Alberto Tidu, Stefano Usai, Frederick Guy,* "Agglomeration in manufacturing and services: an experimental application of a distance-based measure to Sardinia"
21/09 *Andrea Caria, Fabio Cerina, Marco Nieddu,* "Political Selection and Monetary Incentives in Local Parliamentary Systems"
21/08 *Giorgio Garau, Alessio Tola, Maria Veronica Camerada, Salvatore Lampreu, Silvia Carrus,* "Economic and social polarization dynamics in the EU"
21/07 *Emanuela Marrocu, Raffaele Paci, Stefano Usai,* "Direct and indirect effects of universities on European regional productivity"
21/06 *Pasqualina Arca, Gianfranco Atzeni, Luca Deidda,* "The Signalling Role of Trade Credit on Loan Contracts: Evidence from a Counterfactual Analysis"
21/05 *Grazia Sveva Ascione, Laura Ciucci, Claudio Detotto, Valerio Sterzi,* Do universities look like patent trolls? An Empirical Study of University Patent Infringement Litigation in the United States
21/04 *Michele Battisti, Massimo Del Gatto, Antonio Francesco Gravina, Christopher F. Parmeter,* "Robots versus labor skills: a complementarity / substitutability analysis"
21/03 *William Addessi, Marco Delogu,* "Infrastructure Accumulation in Developing Countries: the Role of the Informal Sector"
21/02 *Luca De Benedictis, Vania Licio, Anna Maria Pinna,* "From the historical Roman road network to modern infrastructure in Italy"
21/01 *Silvia Balia, Rinaldo Brau, Marco G. Nieddu,* "Depowering Risk: Vehicle Power Restriction and Teen Driver Accidents in Italy"
20/08 *Giampiero M. Gallo, Demetrio Lacava, Edoardo Otranto,* "On Classifying the Effects of Policy Announcements on Volatility"
20/07 *Luc Bauwens, Edoardo Otranto,* "Modelling Realized Covariance Matrices: a Class of Hadamard Exponential Models"
20/06 *Demetrio Lacava, Giampiero M. Gallo, Edoardo Otranto,* "Measuring the Effects of Unconventional Policies on Stock Market Volatility"
20/05 *Gianfranco Atzeni, Luca G. Deidda, Marco Delogu, Dimitri Paolini,* "Drop-out decisions in a cohort of Italian university students"
20/04 *Emanuela Marrocu, Raffele Paci, David Rigby, Stefano Usai,* "Smart Specialization Strategy: any relatedness between theory and practice?
20/03 *Giorgio Garau, Stefano Deriu,* "Total Factor Productivity and Relative Prices: the case of Italy"
20/02 *Fabio Cerina, Alessio Moro, Michelle Rendall,* "A Note on Employment and Wage Polarization in the U.S."
20/01 *Elias Carroni, Dimitri Paolini,* "Business models for streaming platforms: content acquisition, advertising and users"
19/16 *Daniela Sonedda*, "Regional variation in apprenticeship and permanent employment rates: which causes?"
19/15 *Daniela Sonedda,* "Regional disparities in the functioning of the labour markets"
19/14 *Bianca Biagi, Barbara Dettori, Raffaele Paci, Stefano Usai,* "Economic development in Sardinia: overcoming the insularity gap"
19/13 *Miguel Casares, Luca Deidda, Jose E. Galdon-Sanchez,* "On financial frictions and firm market power"
19/12 *Massimiliano Bratti, Maurizio Conti, Giovanni Sulis,* "Employment Protection and Firm-provided Training: Quasi-experimental Evidence from a Labour Market Reform"
19/11 *Jessica Goldberg, Mario Macis, Pradeep Chintagunta*, "Incentivized Peer Referrals for Tuberculosis Screening: Evidence from India"

www.crenos.unica.it

ISBN 9788868513801

9 788868 513801 >