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MEASURING THE EFFECTS OF UNCONVENTIONAL POLICIES ON STOCK MARKET VOLATILITY

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Measuring the Effects of Unconventional Policies on Stock Market Volatility

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Abstract

As a response to the Great Recession, many central banks resorted to unconventional monetary policies, in the form of a balance sheet expansion. Our research aims at analyzing the impact of the ECB policies on stock market volatility in four Eurozone countries (France, Germany, Italy and Spain) within the Multiplicative Error Model framework. We propose a model which allows us to quantify the part of market volatility depending directly on unconventional policies by distinguishing between the announcement the implementation effects. While we observe an increase in volatility on announcement days, we find a negative implementation effect, which causes a remarkable reduction in volatility in the long term. A Model Confidence Set approach finds how the forecasting power of the proxy improves significantly after the policy announcement; a multi–step ahead forecasting exercise estimates the duration of the effect, and, by shocking the policy variable, we are able to quantify the reduction in volatility which is more marked for debt–troubled countries.

Keywords: Unconventional monetary policy, Financial market, Realized Volatility, Multiplicative Error Model, Model Confidence Set.

Jel Classification: C32, C58, E44, E52, E58, G17.

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1 Introduction

During the Great Recession, with interest rate close to the zero lower bound, many central banks resorted to unconventional monetary policies in order to stimulate the real economy. Unconventional monetary policies consist of the central bank balance sheet expansion – generally through asset purchase programmes – which affects the real economy by modifying inflation rate expectations during periods in which the liquidity trap makes the conventional policy no longer effective.

Following other central banks, such as the Federal Reserve (Fed) and the Bank of England (BoE), the European Central Bank (ECB) established different unconventional monetary measures during the period 2009-2019. Even though the main objective pursued by the ECB through the unconventional policies was to give new stimulus to the real economy, they had unintended effects on financial markets that have been largely studied in the recent literature. Among these effects, the positive influence that quantitative easing (QE) had on market uncertainty is crucial. Thus, while most authors analyse the effect of unconventional policies on bond market (Boeckx et al., 2014; De Santis, 2020; Joyce et al., 2011; Krishnamurthy et al., 2018), some others focus on stock market (Ciarlone and Colabella, 2016; Georgiadis and Gräb, 2016) emphasizing the role played by the portfolio-rebalancing channel in transmitting monetary policy decisions (Breedon et al., 2012).

Surprisingly, there exists a narrow literature concerning the impact of quantitative easing on volatility as a key research objective, modelling volatility mainly through the GARCH family models (Engle, 1982; Bollerslev, 1986). For example, Shogbuyi and Steeley (2017) find no significant effect of QE programmes by FED in reducing volatility in the US market through a multivariate GARCH model. Despite the increase in market volatility on specific days of QE operations by BoE, QE programmes successfully reduce volatility in the UK market, on the one hand, and increase the covariance between the UK and the US markets, on the other.

A significant effect on the US market volatility emerges in Tan and Kohli (2011) in which the VIX index fell significantly during the QE programme and increased when the programme ended. In a similar way, using realized volatility as a proxy for market uncertainty, Converse (2015) finds that, during the first year of the program, the FED QE3 program increased bond market volatility, while reducing equity volatility. A GARCH model is also estimated in Apostolou and Beirne (2017) to investigate the volatility spillovers due to unconventional policies by the FED and the ECB – measured as the change in their balance sheet size – in many emerging economies, in which they record positive volatility spillovers in the bond market and negative ones in the stock market.

Similarly, Ciarlone and Colabella (2018) by means of a DCC-MGARCH analyse the effect of the ECB's unconventional policies on CESEE (Central, Eastern and South Eastern European) economies. They proxy for Asset Purchase Programmes (APPs) through three different variables, and in particular the ECB's holding of securities for monetary policy purpose, finding a sort of spillover effects into these economies, which decrease stock market and foreign exchange market volatility, while there is a no significant effect for what concerns bond market volatility.

Beetsma et al. (2014) focus on the Eurozone market finding a no significant impact of monetary policy common news, which becomes significant considering country–specific news. Moreover, the considered news - regardless of whether they are common or country–specific - decrease correlation between distressed economies and Germany, whereas they increase that between distressed countries. Finally, Balatti et al. (2016) find an inverted V shaped effect: initially the impact of US and UK QE programmes on volatility is positive and becomes negative after five months, on average. According to them, this indicates a spike in market volatility on days immediately following the announcement, while in the long run there would be a quiet period probably because of lower price movements deriving by the QE implementation.

Despite the effectiveness of GARCH models, the new frontier in analysing volatility is represented by the Multiplicative Error Model, MEM (Engle, 2002),Engle:Gallo:2006, in which volatility is the product of a time-varying factor (following a GARCH process) and a positive random variable ensuring positiveness without resorting to logs. Within the MEM class, Brownlees et al. (2012) propose a model - the Composite AMEM (ACM) - in which the conditional variance is the sum of a short–run and a long–run component; similarly, Otranto (2015) proposes a new model to capture spillovers effects in financial markets, by decomposing the mean equation as the sum of two components, both evolving according to GARCH models. These models could be considered as a general framework where inserting the effect of QE as an unobservable factor, providing its estimate and its weight on the level of volatility. For this purpose, in our specification, the first equation composing the mean equation evolves as a GARCH–type dynamics (capturing the pure volatility mechanism) while the second one follows an autoregressive process with exogenous variables, to capture both the announcement and the implementation effects of unconventional measures on volatility.

Our research aims at analyzing the impact of unconventional monetary policies by ECB on stock market volatility taking four Eurozone countries (France, Germany, Italy and Spain) as our leading case, considering the latter two as representative of debt–burdened markets. We proxy for unconventional policies by using the ratio between the securities purchased by the ECB for unconventional policy purposes and the ECB total asset (d'Amico et al., 2012; Voutsinas and Werner, 2011). In carrying out our analysis we employ a realized volatility measure based on high frequency data, which should remove endogeneity arising when monetary policy decisions coincide with a stock price reduction, as argued by Ghysels et al. (2017).

The paper is organized as follow. Section 2 describes data as well as the stylized facts deriving from the unconventional policies implementation, while Section 3 analyses the high frequency methodology employed in our empirical analysis. Section 4 presents the empirical results, discussing model estimation and inference (sub–section 4.1), model comparisons including an analysis of which models enter Model Confidence Set in various subsamples (4.2) and a multi–step forecasting exercise to determine the estimated duration of the effects and the volatility response to a shock to the policy variable (4.3). Finally, Section 5 concludes with some remarks.

2 Empirical evidence from unconventional monetary policies

In investigating the impact of unconventional monetary policies by ECB we consider two different variables, which refer to announcement and implementation effects on volatility, respectively. The former is measured by means of a dummy variable taking value of 1 on days in which ECB releases communications regarding a monetary policy decision[1]. Finally, to proxy for the implementation effect we use the amount of securities held by ECB as a fraction of total asset, named UMP/TA[2] (similarly to d'Amico et al. (2012); Voutsinas and Werner (2011)).

Our analysis is based on a dataset consisting of 2686 daily observations of annualized realized kernel volatility[3] (RV hereafter) which is a robust estimator of the volatility, in particular with respect to microstructure noise of the markets (Barndorff-Nielsen et al., 2008). The analysis concerns four Eurozone market indexes (CAC40 for France, DAX30 for Germany, FTSE MIB for Italy and IBEX35 for Spain – referred to by country in what follows) for the periods between June 1, 2009 and December 31, 2019.

[1]Information is available at `https://www.ecb.europa.eu/press/pr/activities/mopo/html/index.en.html`

[2]Data are taken from the ECB website and Datastream.

[3]Data are provided by the Oxford Man Institute(`https://realized.oxford-man.ox.ac.uk/data/download`)

Figure 1 shows the evolution of our series. It emerges clearly a similar behaviour in all the series, with a period of low volatility in the first year of the sample. However, whereas RV series seem not to respond to the unconventional monetary policies established by ECB at the begin of the crisis (i.e. the 12-month Longer Term Refinancing Operations programme - the LTRO - and the Covered Bond Purchase Programme - the CBPP - which aimed to contain the liquidity crisis and the consequent credit crunch the Eurozone was experiencing), starting from May 2010, in all the series, one can notice how RV reacted to some important events (reported as vertical lines in Figure 1), such as:

- SMP (Security Market Programme) announcement on May 10th, 2010. By means of purchases of government bond in the secondary market, the ECB aimed to manage the spread increase and to restore the proper functioning of monetary policy transmission channel. While RV jumped when the SMP was announced, it emerges clearly the depressive effect that the implementation of this programme had on volatility.

- "Whatever it takes" declaration by Mario Draghi on July 26th, 2012, which served to reassure investors regarding the emerging denomination risk. Through this declaration, the ECB announced the Outright Monetary Transaction (OMT), which replaced the SMP successfully, depressing volatility until the end of 2014.

- EAPP (Expanded Asset Purchase Programme) announcement on January 22nd, 2015. It was established mainly to improve monetary policy transmission mechanisms as well as to adjust the inflation rate toward the target level of 2%. It refers to a series of unconventional measures such as the Assed Backed Securities Purchase Programme (ABSPP), the CBPPs and the Corporate and Public Sector Purchase Programme (CSPP and PSPP, respectively), through which ECB conducted monthly securities purchases.

- March 10th, 2016. The amount of securities purchased within the EAPP passed from the initial level of € 60 billion to € 80 billion per month, causing a downward trend in the RV series.

- October 26th, 2017. Volatility increases after the announcement through which ECB communicated the cut in the monthly purchases, which were reduced to € 15 billion. In contrast to the previous announcement, it caused an increasing trend in all the considered markets.

A simple visual inspection shows the decrease of volatility in correspondence of these events; moreover, what emerges is an effect also caused by the amount purchased by ECB. We aim to quantify this effect and its weight on the level of volatility.

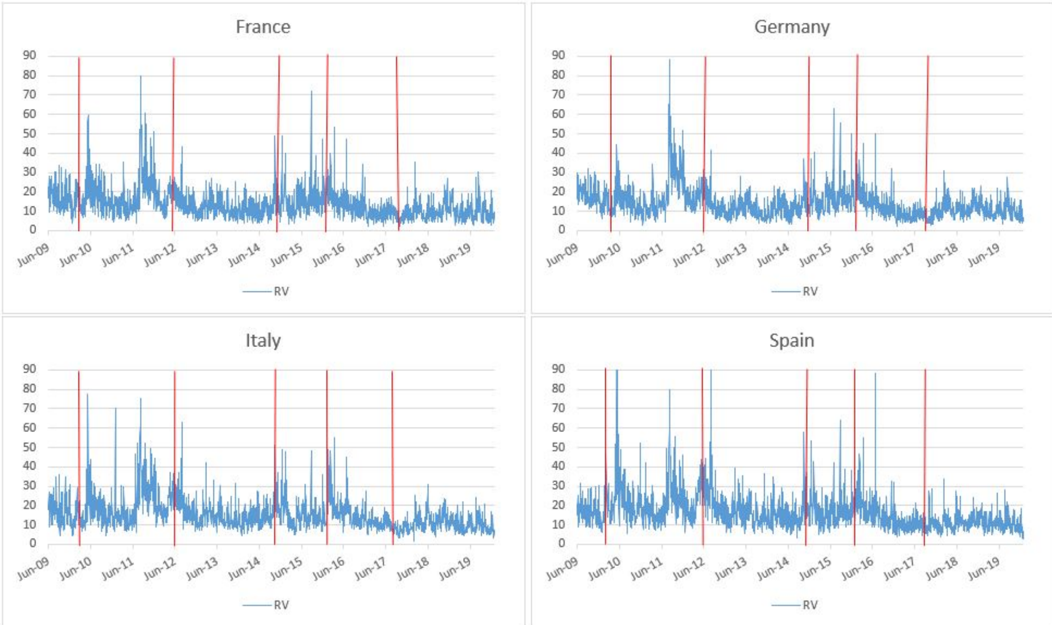

Figure 1: France, Germany, Italy and Spain RV. Sample period: June 1, 2009 to December 31, 2019. Number of observations: 2686. The vertical lines represent relevant events for policy actions (see text).

3 Volatility components for unconventional monetary policy effects

Volatility is generally investigated in the GARCH framework (Engle, 1982),Bollerslev:1986, even though in the last two decades the new frontier in modelling market volatility has shifted toward the Multiplicative Error Model (MEM) as defined by Engle (2002) and successively revised by Engle and Gallo (2006) to allow for asymmetric effects (AMEM).

Let us call RV_t the realized volatility of a certain asset (index) at time t. Since volatility is the evolution of a non-negative process, Engle (2002) and Engle and Gallo (2006) propose to model it as the product of a time-varying factor μ_t, representing the conditional expectation of the volatility, and a positive random variable, ϵ_t.

$$RV_t = \mu_t \epsilon_t, \quad \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta})$$

where Ψ_t is the information set available at time t. Following Engle and Gallo (2006) we consider the Gamma distribution for error term, ϵ_t, which depends only on a parameter, ϑ. From this assumption, considering the conditional distribution of RV_t, we get:

$$E\left(RV_t|\Psi_{t-1}\right) = \mu_t \qquad Var\left(RV_t|\Psi_{t-1}\right) = \mu_t^2/\vartheta.$$

This last property shows that the AMEM possesses a very flexible structure, implying not only a time–varying conditional mean, but also a time–varying conditional variance (volatility of volatility), with the possibility to capture possible clustering in volatility.

Basing on the process behind μ_t, it is possible to specify different models within the AMEM framework. Indeed, the AMEM represents the basis for the Composite AMEM (ACM) developed by Brownlees et al. (2012) in which the conditional variance is the sum of a short– and a long–run component. In a similar way, Otranto (2015) (through the Spillover AMEM, SAMEM) captures volatility spillovers among markets in a univariate framework, by decomposing the mean equation μ_t as the sum of two components, both evolving according to a GARCH model. Actually, the SAMEM and the ACM could be considered similar each other, in the sense that the SAMEM could be seen as a particular specification of the ACM, suitable for the analysis of spillover effects. Starting from these specifications, we develop it to insert the effect of unconventional policies as a latent factor, which affects the dynamics of the volatility. More precisely, our model is based on the decomposition of the volatility level in the sum of two unknown components, representing the base volatility component (ς) and the effect of the unconventional policies (ξ), respectively. The great advantage of this representation consists in the possibility to quantify this last component and to verify its effect and its weight on the global level of volatility.

Composite AMEM (ACM). The model we propose (following Brownlees et al. (2012), we call it Composite AMEM–ACM) consists of four equations:

$$\begin{aligned}
&RV_t = \mu_t \epsilon_t, \quad \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta}) \\
&\mu_t = \varsigma_t + \xi_t \\
&\varsigma_t = \omega + \alpha RV_{t-1} + \beta \varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\
&\xi_t = \delta(E\left(x_t|\Psi_{t-1}\right) - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi \xi_{t-1}
\end{aligned} \tag{1}$$

In model (1), D_{t-1} represents a dummy variable taking value of 1 if the return of the asset (index) at time $t-1$ is negative, 0 otherwise; x_t and Δ_t are exogenous variables, representing the implementation and announcement effect

respectively; finally $x^\star$ is a no time-dependent value, i.e. the unconditional mean or rather the initial value (if x_t is a random walk process). Similar comments are valid for $\Delta^\star$ and Δ_t; this structure, under the hypothesis of stationarity, provides a zero unconditional mean for ξ_t.
For the stationarity of the ACM model, it is required that both components are stationary in covariance, that is $(\alpha + \beta + \frac{\gamma}{2}) < 1$ and $\psi < 1$; positiveness, instead, requires that $(\varsigma_t + \xi_t > 0)$ for each t, which could be ensured even though δ is negative, as we expect.

In this model, ς_t represents the base volatility component, due to its intrinsic dynamics, which evolves as the third equation in model (1); ξ_t represents the effect due to the unconventional policies and follows an AR(1) process with exogenous variables x_t and Δ_t. The timing of these exogenous variables deserves particular attention. Because of the Efficient Market Hypothesis and since monetary policy announcements are regularly scheduled, we can consider the current value of the announcement variable, Δ_t. For what concerns the proxy representing the implementation effect, since at time t we cannot know its current value, we have to use market expectations. From a preliminary analysis, it emerges how the proxy follows a random walk process, which allows us to measure market expectations on this variable by using its own lagged value. Thereby - by also replacing $x^\star$ and $\Delta^\star$ with the respective sample means $\bar{x}$ and $\bar{\Delta}$ - the last equation in model (1) can be written as: $\xi_t = \delta(x_{t-1} - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi\xi_{t-1}$. Finally, as argued by Engle and Lee (1999)), the coefficient ψ in ξ_t process is required to be less than β to ensure the identification of the model. Since one of the main aim pursued by ECB by means of unconventional policies is to stabilize financial markets in the short run, we expect an immediate effect of this kind of policy in reducing stock market volatility. In other words, in our model, the part of volatility depending directly on unconventional policies represents the short run component of realized volatility as well as the proper volatility dynamics represents the long–run component. Basing on this assumption, following Engle and Lee (1999) we expect the long–run component has an higher persistence than the short one, that is $0 < \psi < \beta < 1$. It follows that this condition identifies the model since if it is not the case, the two components would be interchangeable (Engle and Lee, 1999).

It is important to underline that ξ_t is an unobservable signal, with a proper dynamics, which represents the part of the conditional mean of realized volatility due to the unconventional policies. After estimation we will obtain an inference on this signal, so it will be possible to quantify and plot the effect of the unconventional ECB actions on the volatility RV_t. Moreover, we can also measure the weight of the volatility depending directly on unconventional policies with respect to the general level of volatility: this weight is simply given by $1 - \frac{\varsigma_t}{\mu_t} = \frac{\xi_t}{\mu_t}$.

The estimation procedure is based on the quasi maximum likelihood estimator, so that the estimators of the unknown coefficients in model (1) are consistent and asymptotically normal, as shown by Engle (2002) for the MEM case. As discussed by Engle and Gallo (2006), even if the Gamma distribution is not appropriate for ϵ_t, this procedure gives us consistent and efficient estimators (given the Quasi-Maximum likelihood interpretation); if θ is unknown (as usual), robust standard errors will shield against the shape of the Gamma distribution.

Importantly, since ξ_t is an unobservable signal, it is impossible to say for sure whether its impact on the general level of volatility should be considered in an additive way, as in model (1), rather than in a multiplicative way, therefore following a multiplicative specification, as developed by Brownlees et al. (2011). If the latter is the case, the identification of the model requires also that the unconditional mean of ξ_t is equal to one: in what follows we discuss two different specifications of the Multiplicative ACM which ensure the compliance with this constraint.

Logistic ACM (L-ACM). In the first specification we allow ξ_t to impact on RV_t through a logistic function - and by means of the delta method. The model, called Logistic-ACM (L-ACM), is specified as in model (2)

$$\begin{aligned} &RV_t = \mu_t \epsilon_t, \quad \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta}) \\ &\mu_t = 2\varsigma_t(\tfrac{exp(\xi_t)}{1+exp(\xi_t)}) \\ &\varsigma_t = \omega + \alpha RV_{t-1} + \beta\varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\ &\xi_t = \delta(E\left(x_t|\Psi_{t-1}\right) - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1} \end{aligned} \tag{2}$$

Linear ACM (Li-ACM). Alternatively, we ensure the compliance with the unit mean constraint of ξ_t by adding a constant term in its equation. Within this specification, named Linear-ACM (Li-ACM), our model is given by:

$$\begin{aligned} &RV_t = \mu_t \epsilon_t, \quad \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta}) \\ &\mu_t = \varsigma_t \xi_t \\ &\varsigma_t = \omega + \alpha RV_{t-1} + \beta\varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\ &\xi_t = (1-\psi) + \delta(E\left(x_t|\Psi_{t-1}\right) - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1} \end{aligned} \tag{3}$$

AMEMX. Starting from the μ_t expression in the ACM (Expression 1), and setting $\psi = \beta$, we get the AMEMX specification for μ_t (Engle and Gallo, 2006):

$$\mu_t = \omega + \alpha RV_{t-1} + \beta\mu_{t-1} + \gamma D_{t-1} RV_{t-1} + \delta(E\left(x_t|\Psi_{t-1}\right) - x^\star) + \varphi(\Delta_t - \Delta^\star), \tag{4}$$

which, in turn, nests the AMEM upon imposing $\delta = \varphi = 0$.
In the AMEM framework, the usual GARCH constraints for positiveness and stationarity are imposed: $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0, \gamma \geq 0$ and $(\alpha + \beta + \frac{\gamma}{2}) < 1$. In addition, given the assumptions, the unconditional mean turns out to be

$$\mu = \frac{\omega}{1-\alpha-\beta-\frac{\gamma}{2}}$$

A way to summarize the difference in dynamics with respect to the policy actions across model specifications is to derive the marginal effects of a change in x_{t-1} (respectively, Δ_t) on $\mu_{t+\tau}$ (τ-steps), as done in Table 1 (cf. Appendix A.1 for the derivation).

Table 1: Marginal effects of policy variables on $\mu_{t+\tau}$.

Model	Marginal effect on $\mu_{t+\tau}$
AMEMX	$\kappa\beta^{\tau}$
ACM	$\kappa\psi^{\tau}$
L-ACM	$2\varsigma_{t+\tau}\kappa\psi^{\tau}\frac{exp(\xi_{t+\tau})}{(1+exp(\xi_{t+\tau}))^2}$
Li-ACM	$\kappa\psi^{\tau}\varsigma_{t+\tau}$

Note: $\kappa = \delta$ for the marginal effects of the implementation variable x_{t-1};
$\kappa = \varphi$ for the marginal effects of the announcement variable Δ_t.

4 Empirical application

On the realized volatility series of the four indices in Subsection 4.1 we show parameter estimation of the different models, providing diagnostics on the residual autocorrelation (Ljung–Box tests), estimates of the average marginal effects of the policy variables and their graphical evolution. Goal of this section is to show how different models take the policy variables into consideration and how the effects are exerted on the level of volatility in different specifications.

Subsection 4.2 provides a synthesis of overall performance via the information criteria and the in–sample MSE and QLike and a detailed analysis of the composition of the Model Confidence Set (Hansen et al., 2011) across sub–samples (moving ahead one year at the time) for one–step ahead forecasting realized volatility out–of–sample. In this context, we show how the better performance of our models emerges particularly during periods in which the ECB was more active in pushing its nonconventional policy measures.

Subsection 4.3 analyzes the dynamic effects of the policy actions on volatility, by graphically appraising the multi–step forecasts by model, as well as the profile of the impulse responses to a one standard deviation shock to the policy variable. For either case, we can evaluate the persistence of the effects and the time taken by different models in converging to the unconditional level of volatility.

4.1 Estimation results

Estimation results are shown in Tables 2-5. Coefficients are highly significant in all cases with a volatility persistence ($\alpha+\beta+\frac{\gamma}{2}$) which decreases by about 2% passing from the AMEM to our more complex models accommodating unconventional policy effects. Moreover, the impact of the more recent observed values, measured by the α and γ coefficients, is generally higher in the AMEM. This indicates a lower influence of current shocks on volatility projections, in line with the expected calming effect that the unconventional monetary policies have on market volatility.

Ljung-Box statistics on standardized residuals for lags 1, 5 and 10 (at the bottom of the tables) shows how the AMEM is able to capture the persistence in the volatilities (fail to reject the null of no serial correlation at 1% significance level, with the exception of Germany at the highest lag, as well as Spain at lag 5). However, our additive model (the ACM) seems to have a better statistical performance in this respect, especially in the case of Spain, where we never reject the null of autocorrelation at 1% level.

For what concerns the unconventional policy proxies, coefficients are significant at 1% level and they enter the model with the expected sign. According to other researches in literature (see for example Bomfim (2003); Chan and Gray (2018) and Shogbuyi and Steeley (2017)) the coefficient φ of the dummy variable is positive, meaning that there is an immediate reaction in the market on announcement days. Conversely, δ is negative, therefore the unconventional policies implementation successfully reduces stock market volatility, as expected.

This implementation effect can also be seen in Figure 2, which plots the evolution of the two volatility components (the blue line for the base component, left axis, and the red dotted-line, for the policy component, right axis[4]). This effect is more evident starting from October 2014 - when the

[4]The line representing the ξ process is defined in three different levels: the lowest depends on the characteristic path of the ξ component; the highest represents, instead,

ECB implicitly communicated to the market that it would purchase corporate bonds as well (next to government bonds) - coming in the form of a change in the slope of ξ equation, lasting for the entire period of the program. This is one of the most interesting result, as it implies that the effectiveness of these policies rests on the total amount of securities held for monetary policy purposes, relative to ECB total asset. Moreover, an increase in this volatility component is observed in April 2017, which coincides with the reduction of the amount of securities purchased by ECB, set to € 60 billion per month from the previous level of € 80 billion.

Results remain by and large the same, when we consider the multiplicative specifications. Once again, the proxies enter the models with the expected sign and with the highest level of significance. In both cases, coefficients are fairly lower, in view of the multiplicative nature of these specifications: in other words, whereas in the ACM generally ξ_t is negative, within the multiplicative versions it is always positive, therefore unconventional policies depress volatility if the ξ_t process is less than 1: this requires lower proxies' coefficients, and this is achieved without imposing any kind of constraints.

In order to compare economic effects across models, it is instructive to compute the corresponding marginal effects of the policy variables (x_{t-1} and Δ_t) on μ_t: while in the additive specification they are constant and equal to the estimated coefficients, in the multiplicative specifications marginal effects are time varying. Looking at the average values[5] (Table 6), the higher marginal effect of the implementation proxy is associated to the LI-ACM. More specifically, a marginal increase in the proxy leads to a range of reductions in realized volatility comprised between -1.577 (Germany) and -3.382 (Spain); conversely, on announcement days volatility marginally increases, on average, by 2.603 and 4.146 points, in Germany and Spain, respectively. Overall, while unconventional policies had a higher impact on debt–troubled countries, the effect is present also for the others. The evolution of the marginal effects associated to the two multiplicative specifications is shown in Figures 3 and 4: for both models and for all the markets, marginal effects have a specular behaviour, to a certain extent mirroring the behaviour of the realized volatility measure. In particular, the marginal effects of Δ_t have peaks in correspondence of volatility spikes, whereas the period of lower marginal effects of the proxy corresponds to periods of low volatility in the market. We take this to be a result which validates the dynamics in our

volatility spikes due to the announcement effect; finally, the intermediate line, represents the level of volatility on days after a monetary policy announcement, when volatility comes back to its previous level.

[5]For what concerns Δ_t, the marginal effects are considered only with respect to announcement days, and, as such, the average relates just to such days.

models with the policy variables we adopted.

In the case of the ACM, the impact of unconventional policies on stock market volatility can also be measured as the ratio of the policy–related component ξ_t to the general level of volatility μ_t. The average of such a ratio signals a reduction in volatility associated with the ECB unconventional policies between -0.6% and -1.1%, when measured for Germany and France, which is more marked for Italy (-1.24%) and Spain (-1.38%).

As far as the dynamics of the ξ_t component is concerned, the persistence effects, driven by the coefficient ψ, are fairly weak: such a coefficient is significant at 5% only for France across models and only marginally so for Germany for the L-ACM model. We interpret this as evidence that the policies were particularly effective when they signaled a change (further expansion of the program and surprise announcements), more than being able to rely on a progressive diffusion of the effects through time.

Table 2: Model Estimation Results France: June 1, 2009 - December 31, 2019.

	AMEM	**AMEMX**	**ACM**	**L-ACM**	**Li-ACM**
	Coefficient estimates (robust s.e. in parentheses)				
ω	0.857	1.136	1.056	1.011	1.025
	(0.046)	(0.121)	(0.065)	(0.007)	(0.053)
α	0.171	0.165	0.154	0.151	0.153
	(0.017)	(0.02)	(0.015)	(0.015)	(0.017)
β	0.708	0.689	0.707	0.712	0.709
	(0.019)	(0.029)	(0.015)	(0.014)	(0.018)
γ	0.113	0.12	0.117	0.119	0.119
	(0.008)	(0.009)	(0.011)	(0.01)	(0.012)
δ	-	-0.636	-1.836	-0.297	-0.161
		(0.075)	(0.326)	(0.057)	(0.029)
φ	-	1.297	2.817	0.464	0.231
		(0.381)	(0.539)	(0.093)	(0.045)
ψ	-	-	0.111	0.194	0.134
			(0.06)	(0.081)	(0.083)
θ	7.559	7.728	7.817	7.827	7.82
	(0.222)	(0.228)	(0.231)	(0.23)	(0.231)
	p–values for Ljung-Box statistics				
lag 1	0.155	0.258	0.132	0.108	0.125
lag 5	0.111	0.167	0.215	0.192	0.207
lag 10	0.115	0.137	0.21	0.201	0.201

Table 3: Model Estimation Results Germany: June 1, 2009 - December 31, 2019.

	AMEM	**AMEMX**	**ACM**	**L-ACM**	**Li-ACM**
	Coefficient estimates (robust s.e. in parentheses)				
ω	0.957	1.081	1.034	1.026	1.034
	(0.017)	(0.052)	(0.016)	(0.059)	(0.519)
α	0.193	0.191	0.183	0.182	0.183
	(0.01)	(0.011)	(0.014)	(0.012)	(0.055)
β	0.692	0.684	0.696	0.696	0.694
	(0.011)	(0.016)	(0.013)	(0.014)	(0.094)
γ	0.087	0.09	0.089	0.09	0.09
	(0.006)	(0.011)	(0.008)	(0.008)	(0.008)
δ	-	-0.454	-1.328	-0.219	-0.115
		(0.035)	(0.283)	(0.052)	(0.057)
φ	-	1.067	2.317	0.378	0.188
		(0.311)	(0.47)	(0.073)	(0.073)
ψ	-	-	0.098	0.175	0.138
			(0.077)	(0.097)	(0.204)
θ	9.46	9.61	9.7	9.719	9.714
	(0.29)	(0.298)	(0.301)	(0.301)	(0.301)
	p–values for Ljung-Box statistics				
lag 1	0.171	0.262	0.184	0.173	0.194
lag 5	0.028	0.063	0.056	0.051	0.054
lag 10	0.002	0.004	0.003	0.003	0.003

Table 4: Model Estimation Results Italy: June 1, 2009 - December 31, 2019.

	AMEM	**AMEMX**	**ACM**	**L-ACM**	**Li-ACM**
	Coefficient estimates (robust s.e. in parentheses)				
ω	1.198	1.708	1.533	1.48	1.49
	(0.073)	(0.221)	(0.26)	(0.239)	(0.146)
α	0.268	0.274	0.249	0.248	0.25
	(0.018)	(0.032)	(0.033)	(0.042)	(0.022)
β	0.608	0.568	0.604	0.605	0.603
	(0.023)	(0.043)	(0.043)	(0.049)	(0.03)
γ	0.084	0.086	0.086	0.089	0.088
	(0.005)	(0.011)	(0.019)	(0.022)	(0.008)
δ	-	-1.074	-2.354	-0.343	-0.178
		(0.161)	(0.519)	(0.048)	(0.031)
φ	-	2.059	3.448	0.518	0.254
		(0.471)	(0.569)	(0.084)	(0.038)
ψ	-	-	0.051	0.089	0.058
			(0.083)	(0.055)	(0.073)
θ	10.593	11.03	11.183	11.185	11.181
	(0.423)	(0.427)	(0.436)	(0.437)	(0.436)
	p–values for Ljung-Box statistics				
lag 1	0.299	0.691	0.3	0.287	0.324
lag 5	0.497	0.721	0.8	0.805	0.824
lag 10	0.336	0.472	0.647	0.625	0.624

Table 5: Model Estimation Results Spain: June 1, 2009 - December 31, 2019.

	AMEM	**AMEMX**	**ACM**	**L-ACM**	**Li-ACM**
	Coefficient estimates (robust s.e. in parentheses)				
ω	1.055	1.533	1.438	1.41	1.426
	(0.075)	(0.11)	(0.135)	(0.621)	(0.132)
α	0.224	0.205	0.197	0.193	0.195
	(0.013)	(0.014)	(0.024)	(0.06)	(0.023)
β	0.671	0.656	0.671	0.674	0.671
	(0.007)	(0.02)	(0.027)	(0.091)	(0.028)
γ	0.078	0.087	0.085	0.086	0.086
	(0.022)	(0.007)	(0.015)	(0.017)	(0.009)
δ	-	-1.025	-2.823	-0.389	-0.207
		(0.086)	(0.528)	(0.133)	(0.037)
φ	-	1.964	3.428	0.501	0.247
		(0.361)	(0.671)	(0.087)	(0.042)
ψ	-	-	0.058	0.123	0.072
			(0.09)	(0.298)	(0.112)
θ	9.149	9.487	9.58	9.626	9.623
	(0.317)	(0.338)	(0.349)	(0.355)	(0.352)
	p–values for Ljung-Box statistics				
lag 1	0.01	0.025	0.011	0.006	0.008
lag 5	0.009	0.015	0.019	0.012	0.015
lag 10	0.047	0.086	0.119	0.089	0.1

Table 6: Average marginal effects of policy variables on μ_t – Sample period: June 1, 2009 - December 31, 2019 (parameter estimates as in Tables 2–5).

	Marginal effect of x_{t-1} on μ_t			Marginal effect of Δ_t on μ_t		
	ACM	L-ACM	Li-ACM	ACM	L-ACM	Li-ACM
France	-1.836	-1.571	-2.172	2.817	2.236	3.177
Germany	-1.328	-1.158	-1.577	2.317	1.821	2.603
Italy	-2.354	-2.033	-2.693	3.448	2.785	3.953
Spain	-2.823	-2.489	-3.382	3.428	2.912	4.146

Note: the average marginal effect of Δ_t refers to the announcement days (see text).

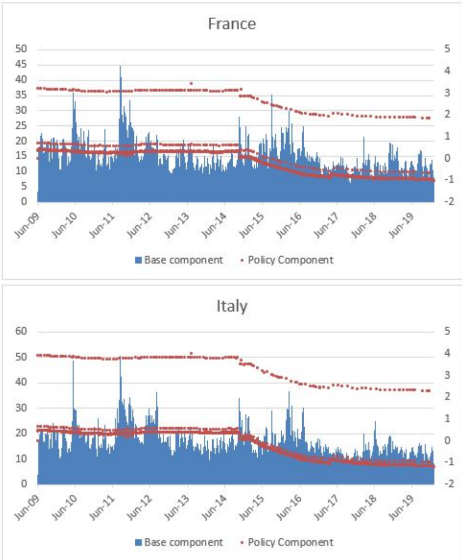

Figure 2: France, Germany, Italy and Spain Base (left axis) and Policy (right axis) component of volatility from the ACM. Sample period: June 1, 2009 to December 31, 2019

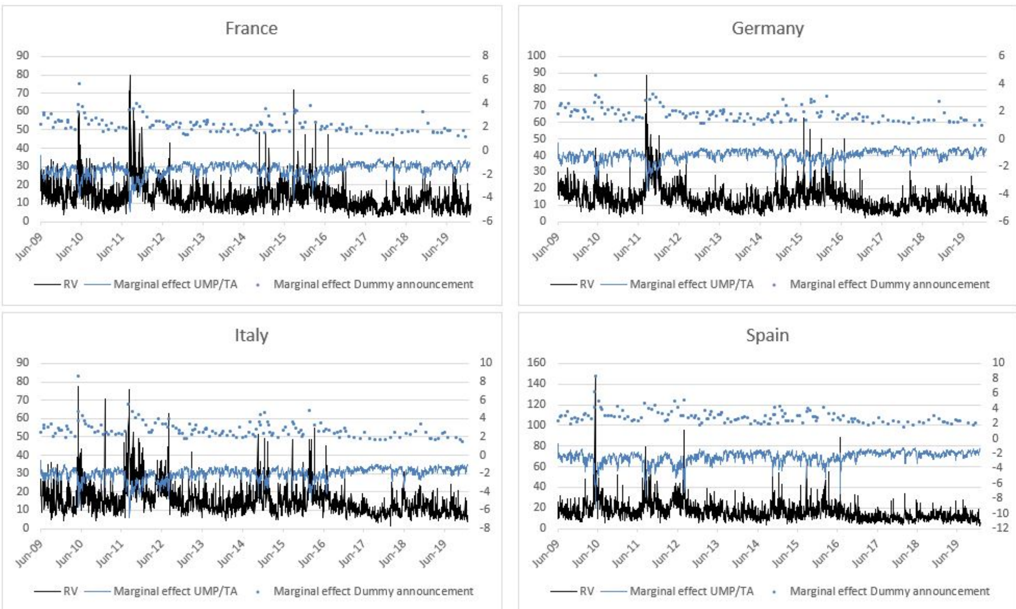

Figure 3: France, Germany, Italy and Spain: marginal effects from the L-ACM. Sample period: June 1, 2009 to December 31, 2019

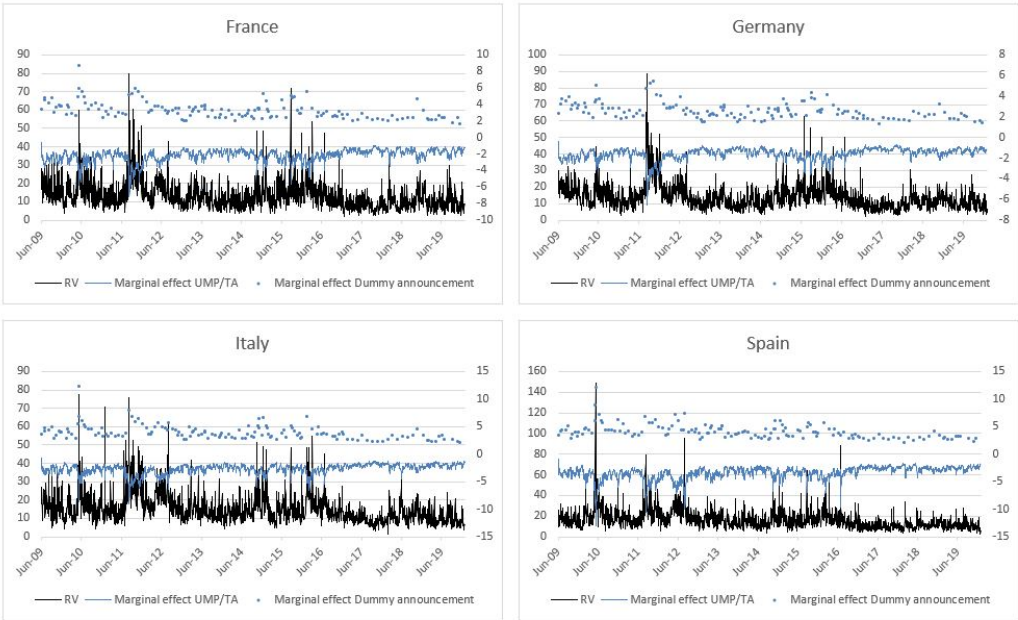

Figure 4: France, Germany, Italy and Spain: marginal effects from the Li-ACM. Sample period: June 1, 2009 to December 31, 2019

4.2 Model Comparisons

In this section, we evaluate model performance both in– and out–of–sample. For the former, we limit ourselves to compare models in Table 7, where we report the information criteria (AIC and BIC) and two loss functions (Mean Square Error –MSE – and Quasi-Likelihood – QLike, consistent in the sense of Patton, 2011) to evaluate the fitting capabilities of the models. For all countries, the best performing model (the one in bold) is the L-ACM. The only exception is Spain, where the L-ACM has better performance in terms of the information criteria, even if the LI-ACM is marginally better in terms of MSE.

The next step is to compute the Model Confidence Set (MCS, Hansen et al., 2011) to compare the one-step-ahead out-of-sample forecasting performance across the estimated models in terms of the same MSE and QLike loss functions at the 10% significance level.

For this purpose, we split the sample at the end of a year and we compute the one-step-ahead out-of-sample forecasts for the following year. The choice of the splitting dates is driven by the need to exclude the most important quantitative easing programme, the EAPP, so that the first subsample considered stops at the end of 2014. We then consider each additional year in turn until 2018, in order to consider the full period of the EAPP.

As shown in Table 8, both loss functions provide similar results, confirming that the proxy–augmented models capture the features of the EAPP in delivering an improved forecasting performance. This outcome is more apparent when considering the subsamples individually. Before the EAPP announcement, the MCS results in the 2015 column of Table 8 show, for example, that all the models enter the best set of models (according to QLike, represented by the symbol ○) in 3 out of 4 cases, and even that the AMEM is the best model (●) in France, Germany and Spain and it belongs to the best set in Italy, for which the best model is the ACM (results are more mixed for the MSE).

Similar results derive from the forecasting period ending in 2016. For both loss functions, this time, the AMEM is the best model in 3 out of 4 cases, whereas in Spain such a role goes to the AMEMX: no component model is present in the best set in Germany, where the AMEM is the only model belonging to the best set (together with the Li-ACM if we consider the MSE).

Results change drastically, if we refer to the forecasting period ending in 2017, where the ECB purchased assets were € 80 billion per month until March 2017 and € 60 billion per month until the end of the year, causing a remarkable change in the ECB balance sheet composition. This time the policy augmented models have a higher forecasting power and the AMEM is always excluded from the best set of models. Importantly, focusing on the QLike, the ACM enters the best set in 3 out of 4 cases, whereas the Li-ACM is the best model for Germany and Spain.

This very model becomes the best across all cases considering the forecasting periods ending in 2018 and 2019, respectively. The difference in the results of these two sub-samples is in the best set of models, which is larger in the last year, possibly due to the fact that no APP was implemented until November 2019, when the BCE established a new APP of € 20 billion per month. Overall, however, unconventional policies seem to have played a crucial role in reducing stock market volatility.

Table 7: Model comparisons via Information Criteria (AIC and BIC) and forecasting capability (MSE and MAE) – Sample period: June 1, 2009 - December 31, 2019. Best model in bold.

	AMEM	**AMEMX**	**ACM**	**L-ACM**	**Li-ACM**
			France		
Loglik	-7825.053	-7793.936	-7778.073	**-7776.201**	-7777.387
AIC	5.830	5.809	5.797	**5.796**	5.797
BIC	5.841	5.824	5.815	**5.814**	5.815
MSE	29.531	29.117	28.844	**28.627**	28.639
QLike	0.068	0.066	**0.065**	**0.065**	**0.065**
			Germany		
Loglik	-7592.516	-7570.601	-7557.797	**-7555.038**	-7555.658
AIC	5.657	5.642	5.633	**5.631**	5.632
BIC	5.668	5.658	5.651	**5.649**	**5.649**
MSE	23.551	23.269	23.096	**22.93**	22.939
QLike	0.054	0.053	**0.052**	**0.052**	**0.052**
			Italy		
Loglik	-7721.272	-7665.273	-7646.329	**-7646.068**	-7646.473
AIC	5.753	5.713	**5.699**	**5.699**	**5.699**
BIC	5.764	5.728	**5.717**	**5.717**	**5.717**
MSE	28.171	27.634	27.209	**27.191**	27.203
QLike	0.048	0.046	**0.045**	**0.045**	**0.045**
			Spain		
Loglik	-8116.961	-8066.502	-8052.907	**-8046.278**	-8046.668
AIC	6.048	6.011	6.002	**5.997**	**5.997**
BIC	6.059	6.027	6.02	**6.015**	**6.015**
MSE	42.048	40.918	40.462	39.461	**39.439**
QLike	0.056	0.054	**0.053**	**0.053**	**0.053**

Table 8: The Model Confidence Set results. P-value 10%

France

	2015	2016	2017	2018	2019
AMEM	□ ●	■ ●			
AMEMX	□	□ ○	■ ●		□ ○
ACM		□ ○	○		□
L-ACM	■ ○	□ ○			□
Li-ACM	□ ○	□ ○	○	■ ●	■ ●

Germany

	2015	2016	2017	2018	2019
AMEM	■ ●	■ ●		○	
AMEMX	□ ○				
ACM	□ ○				
L-ACM	□ ○				□ ○
Li-ACM	□ ○	□	■ ●	■ ●	■ ●

Italy

	2015	2016	2017	2018	2019
AMEM	□ ○	■ ●			
AMEMX	□ ○	□ ○	■ ●	■	□ ●
ACM	■ ●	□ ○	○		□ ○
L-ACM	□ ○	□ ○			□ ○
Li-ACM	□ ○	□ ○		□ ●	■ ○

Spain

	2015	2016	2017	2018	2019
AMEM	●	□ ○			
AMEMX	○	■ ●	■ ○	○	
ACM		○	○	○	
L-ACM	○	○	○		□
Li-ACM	■ ○	□ ○	□ ●	■ ●	■ ●

Note: □(○) indicates models belonging to the best set according to the MSE(QLike) loss function; ■(●) represents the best model.

4.3 How long does volatility take to converge to its unconditional level?

We are now in a position to address two substantive questions: i) how long do quantitative easing policies affect stock market volatility? ii) What would be the volatility response to a higher quantitative easing shock? To suggest an answer, we adopt a multi-step forecasting procedure.

We focus on the sub-sample for the estimation period ending in 2017, with the following year used as the out–of–sample period: using the multi–step ahead forecasting formulas detailed in the Appendix A.2, we get results shown in Figure 5. As expected, the duration of the effect - measured as the number of days the volatility needs to reach the unconditional level, i.e. the unconditional mean[6] - depends on the country: what emerges is an average duration between 41 business days (estimated in Italy via Li-ACM) and 96 business days (from the ACM in Germany). Furthermore, regardless of the duration, once the unconventional policy effect is completely absorbed by the market, volatility converges to its unconditional level, which does not depend on the different model specifications; however, due to the different way they account for the policy effect a slight difference between models is present, more marked for the AMEM, which neglects those effects altogether. As expected, in all the cases the convergence path is upward sloping, with volatility reaching a higher level when the downward impact of unconventional policies ceases. Such an upward profile is shared by the AMEM as well, given that volatility is lower than its unconditional level at the beginning of the forecasting period.

Finally, we analyse how volatility would have responded to a higher quantitative easing shock. For this purpose we have increased the quantitative easing proxies by one standard deviation (0.26). Taking the previous multi-step forecasting values as a baseline scenario, we have chosen to represent the results in the form of the difference between the two scenarios in Figure 6. The role played by unconventional policies in reducing stock market volatility emerges clearly: a higher unconventional policy shock would have had a stronger downward effect, with volatility that would have been lower down to 2.5 points in debt–troubled countries (2 points in France and Germany) relative to the baseline scenario.

[6]For practical purposes, convergence is considered achieved when subsequent forecasts do not differ by more than 1 basis point of volatility.

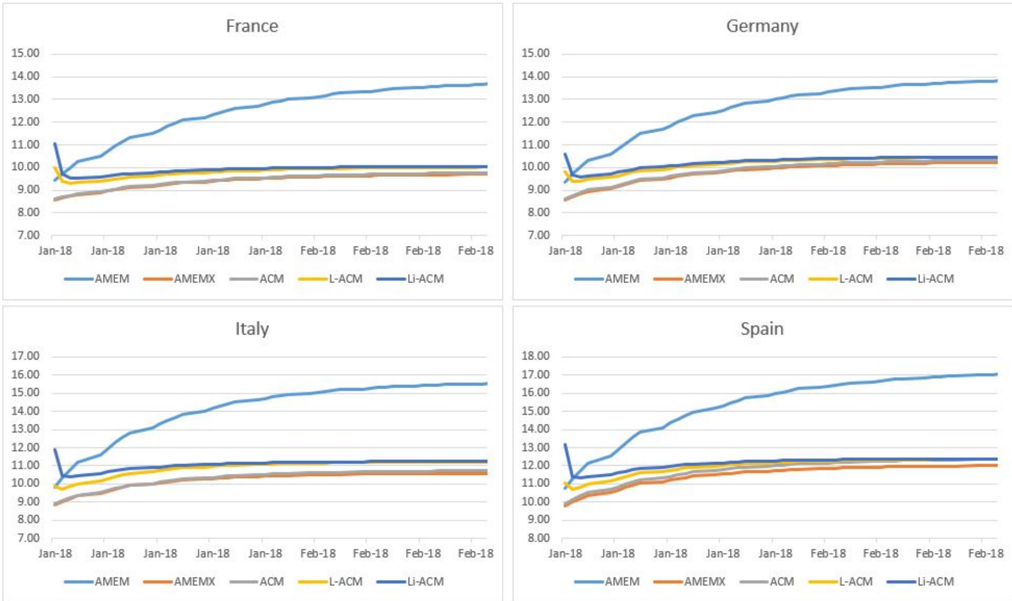

Figure 5: Multi-step forecast. Proxy: UMP/TA. Sample period: June 1, 2009 to December 31, 2017. Forecast Period: January 1, 2018 - December 31, 2018

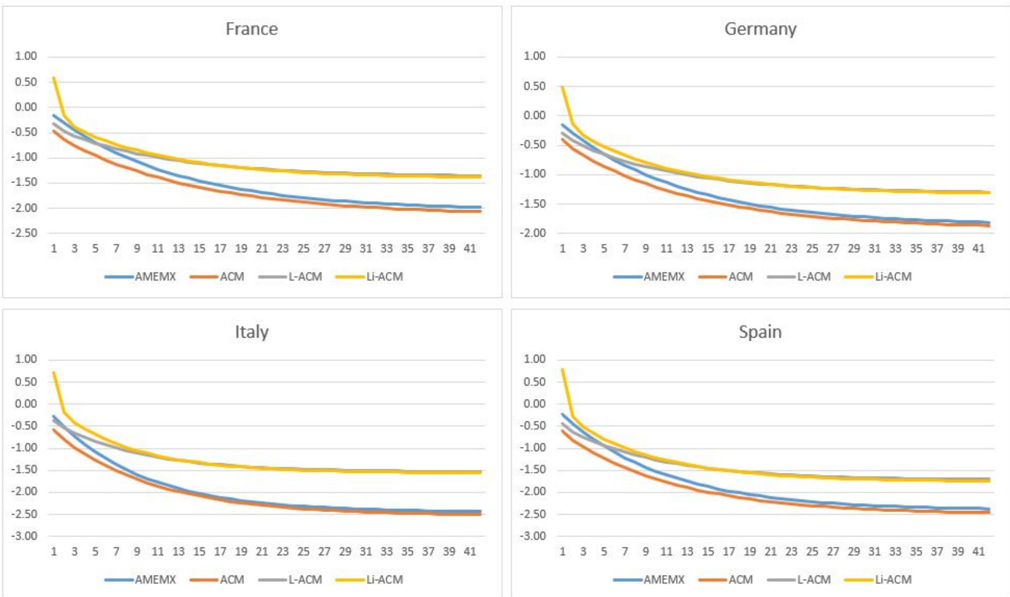

Figure 6: Impulse Response Function: the effect on volatility of a 1 standard deviation unconventional policy shock. Sample period: June 1, 2009 to December 31, 2017. Forecast Period: January 1, 2018 - December 31, 2018

5 Concluding Remarks

In this paper, we analyzed how unconventional monetary policies by the ECB affected realized volatility. To do so, we suggested a number of models accommodating the presence of a component aimed at capturing the part of volatility depending directly on QE policies, next to pure, so to speak, volatility dynamics: such components may combine additively or multiplicatively. As shown by our results, what matters for the effectiveness of these policies is the balance sheet composition (as argued by Curdia and Woodford, 2011). Indeed, an increase in securities held by ECB for monetary policy purposes relative to total asset reduces volatility in both debt–troubled countries (Italy and Spain, in our analysis) and in benchmark countries (France and Germany), with the former generally beneficing more. However, our policy proxies do not allow us to distinguish the specific effect of each implemented policy, so that we cannot identify which of these extraordinary measures is more effective. This, of course, is an issue worth pursuing in future analysis, as well as the possibility to control also for spillovers among countries in a multivariate context, which could also highlight the presence of a common component across markets.

By evaluating the out–of–sample forecasting performance of our models through the Model Confidence Set procedure, the importance of the EAPP emerges as stressing the role played by our proxy for forecasting purposes. This is also confirmed by the multi-step forecast procedure, since we found, on the one hand, that the effects of unconventional policies in lowering volatility last for at most 90 business days, and, on the other, that shocking unconventional policies by one standard deviation lowers stock market volatility by up to 2.72 points.

Exploiting the component structure of our models, we can extract a separate and distinctive signal related to the policy effects on volatility. In economic terms, such results document that the unconventional monetary policy has a mitigating effect on market volatility at times of distress, when the interest rates are close to the zero lower bound, as a further channel for central banks for restoring the proper functioning of the economy.

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Appendices

A.1 Marginal effects

In this appendix we derive, for each model, the marginal effects of the policy variables on the volatility in terms of first partial derivatives of $\mu_{t+\tau}$ with respect to x_{t-1} and Δ_t, as illustrated in Section 3 (Table 1).

ACM

$$RV_t = \mu_t \epsilon_t$$

$$\begin{aligned}
\mu_t &= \varsigma_t + \xi_t \\
\mu_t &= \omega + (\alpha + \gamma D_{t-1})RV_{t-1} + \beta\varsigma_{t-1} + \delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1}
\end{aligned}$$

$$\begin{aligned}
\mu_{t+1} &= \omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \psi\xi_t \\
&= \omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \\
&\quad + \psi[\delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1}] \\
&= \omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t + \delta[(x_t - x^\star) + \psi(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+1} - \Delta^\star) + \psi(\Delta_t - \Delta^\star)] + \psi^2\xi_{t-1}
\end{aligned}$$

$$\begin{aligned}
\mu_{t+2} &= \omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1} + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \psi\xi_{t+1} \\
&= \omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1} + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \\
&\quad + \psi\{\delta[(x_t - x^\star) + \psi(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+1} - \Delta^\star)\psi(\Delta_t - \Delta^\star) + \psi^2\xi_{t-1}\} \\
&= \omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1} + \delta[(x_{t+1} - x^\star) + \psi(x_t - x^\star) + \\
&\quad + \psi^2(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+2} - \Delta^\star) + \psi(\Delta_{t+1} - \Delta^\star) + \psi^2(\Delta_t - \Delta^\star)] + \psi^3\xi_{t-1}
\end{aligned}$$

$$\vdots$$

$$\begin{aligned}
\mu_{t+\tau} &= \omega + (\alpha + \gamma D_{t+\tau-1})RV_{t+\tau-1} + \beta\varsigma_{t+\tau-1} + \delta[(x_{t+\tau-1} - x^\star) + \cdots + \psi^\tau(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+\tau} - \Delta^\star) + \cdots + \psi^\tau(\Delta_t - \Delta^\star)] + \psi^{\tau+1}\xi_{t-1}.
\end{aligned}$$

By taking the first partial derivatives, we get the marginal effect of the implementation variable x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
δ	$\psi\delta$	$\psi^2\delta$	$\cdots$	$\psi^\tau\delta$

and the marginal effect of the announcement variable Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
φ	$\psi\varphi$	$\psi^2\varphi$	$\cdots$	$\psi^\tau\varphi$

L-ACM

$$RV_t = \mu_t \epsilon_t$$

$$\begin{aligned}
\mu_t &= 2\varsigma_t \frac{exp(\xi_t)}{1+exp(\xi_t)} \\
&= 2\varsigma_t \frac{exp[\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_t]}{1+exp[\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_t]}
\end{aligned}$$

$$\begin{aligned}
\mu_{t+1} &= 2\varsigma_{t+1} \frac{exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi\xi_t]}{1+exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi\xi_t]} \\
&= 2\varsigma_{t+1}[\frac{exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi(\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_{t-1})]}{1+exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi(\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_{t-1})]}] \\
&= 2\varsigma_{t+1}[\frac{exp[\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}]}{1+exp[\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}]}]
\end{aligned}$$

$$\begin{aligned}
\mu_{t+2} &= 2\varsigma_{t+2} \frac{exp[\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\xi_{t+1}]}{1+exp[\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\xi_{t+1}]} \\
&= 2\varsigma_{t+2} \\
&\quad [\frac{exp(\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\{\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}\})}{1+exp(\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\{\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}\})}] \\
&= 2\varsigma_{t+2} \\
&\quad [\frac{exp(\delta[(x_{t+1}-x^\star)+\psi(x_t-x^\star)+\psi^2(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+2}-\Delta^\star)+\psi(\Delta_{t+1}-\Delta^\star)+\psi^2(\Delta_t-\Delta^\star)]+\psi^3\xi_{t-1})}{1+exp(\delta[(x_{t+1}-x^\star)+\psi(x_t-x^\star)+\psi^2(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+2}-\Delta^\star)+\psi(\Delta_{t+1}-\Delta^\star)+\psi^2(\Delta_t-\Delta^\star)]+\psi^3\xi_{t-1})}] \\
&\vdots \\
&= 2\varsigma_{t+\tau}[\frac{exp(\delta[(x_{t+\tau-1}-x^\star)+\cdots+\psi^\tau(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+\tau}-\Delta^\star)+\cdots+\psi^\tau(\Delta_t-\Delta^\star)]+\psi^{\tau+1}\xi_{t-1})}{1+exp(\delta[(x_{t+\tau-1}-x^\star)+\cdots+\psi^\tau(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+\tau}-\Delta^\star)+\cdots+\psi^\tau(\Delta_t-\Delta^\star)]+\psi^{\tau+1}\xi_{t-1})}]
\end{aligned}$$

By taking the first partial derivatives, we get the marginal effect of the implementation variable x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$2\varsigma_t\delta\frac{exp(\xi_t)}{[1+exp(\xi_t)]^2}$	$2\varsigma_{t+1}\psi\delta\frac{exp(\xi_{t+1})}{[1+exp(\xi_{t+1})]^2}$	$2\varsigma_{t+2}\psi^2\delta\frac{exp(\xi_{t+2})}{[1+exp(\xi_{t+2})]^2}$	$\cdots$	$2\varsigma_{t+\tau}\psi^\tau\delta\frac{exp(\xi_{t+\tau})}{[1+exp(\xi_{t+\tau})]^2}$

and the marginal effect of the announcement variable Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$2\varsigma_t\varphi\frac{exp(\xi_t)}{[1+exp(\xi_t)]^2}$	$2\varsigma_{t+1}\psi\varphi\frac{exp(\xi_{t+1})}{[1+exp(\xi_{t+1})]^2}$	$2\varsigma_{t+2}\psi^2\varphi\frac{exp(\xi_{t+2})}{[1+exp(\xi_{t+2})]^2}$	$\cdots$	$2\varsigma_{t+\tau}\psi^\tau\varphi\frac{exp(\xi_{t+\tau})}{[1+exp(\xi_{t+\tau})]^2}$

Li-ACM

$$
\begin{aligned}
RV_t &= \mu_t \epsilon_t \\
\mu_t &= \varsigma_t \xi_t \\
&= [\omega + (\alpha + \gamma D_{t-1}) RV_{t-1} + \beta \varsigma_{t-1}][\delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi \xi_{t-1}] \\
\mu_{t+1} &= [\omega + (\alpha + \gamma D_t) RV_t + \beta \varsigma_t][\delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \psi \xi_t] \\
&= [\omega + (\alpha + \gamma D_t) RV_t + \beta \varsigma_t]\{\delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \\
&\quad + \psi[\delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi \xi_{t-1}]\} \\
&= [\omega + (\alpha + \gamma D_t) RV_t + \beta \varsigma_t]\{\delta[(x_t - x^\star) + \psi(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+1} - \Delta^\star) + \psi(\Delta_t - \Delta^\star)] + \psi^2 \xi_{t-1}]\} \\
\mu_{t+2} &= [\omega + (\alpha + \gamma D_{t+1}) RV_{t+1} + \beta \varsigma_{t+1}][\delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \psi \xi_{t+1}] \\
&= [\omega + (\alpha + \gamma D_{t+1}) RV_{t+1} + \beta \varsigma_{t+1}]\{\delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \\
&\quad + \psi[\delta[(x_t - x^\star) - \psi(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+1} - \Delta^\star) + \psi(\Delta_t - \Delta^\star)] + \psi^2 \xi_{t-1}]\} \\
&= [\omega + (\alpha + \gamma D_{t+1}) RV_{t+1} + \beta \varsigma_{t+1}]\{\delta[(x_{t+1} - x^\star) + \psi(x_t - x^\star) + \psi^2(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+2} - \Delta^\star) + \psi(\Delta_{t+1} - \Delta^\star) + \psi^2(\Delta_t - \Delta^\star)] + \psi^3 \xi_{t-1}]\} \\
&\vdots \\
\mu_{t+\tau} &= [\omega + (\alpha + \gamma D_{t+\tau-1}) RV_{t+\tau-1} + \beta \varsigma_{t+\tau-1}]\{\delta[(x_{t+\tau-1} - x^\star) + \psi^\tau(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+\tau} - \Delta^\star) + \cdots + \psi^\tau(\Delta_t - \Delta^\star)] + \psi^{\tau+1} \xi_{t-1}]\}.
\end{aligned}
$$

By taking the first partial derivatives, we get the marginal effect of x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$\delta\varsigma_t$	$\psi\delta\varsigma_{t+1}$	$\psi^2\delta\varsigma_{t+2}$	$\cdots$	$\psi^\tau\delta\varsigma_{t+\tau}$

and the marginal effect of Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$\varphi\varsigma_t$	$\psi\varphi\varsigma_{t+1}$	$\psi^2\varphi\varsigma_{t+2}$	$\cdots$	$\psi^\tau\varphi\varsigma_{t+\tau}$

AMEMX

$$
\begin{aligned}
RV_t &= \mu_t \epsilon_t \\
\mu_t &= \omega + (\alpha + \gamma D_{t-1}) RV_{t-1} + \beta \mu_{t-1} + \delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star)
\end{aligned}
$$

$$
\begin{aligned}
\mu_{t+1} &= \omega + (\alpha + \gamma D_t) RV_t + \beta \mu_t + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) \\
&= \omega + (\alpha + \gamma D_t) RV_t + \beta[\omega + (\alpha + \gamma D_{t-1}) RV_{t-1} + \beta \mu_{t-1} + \\
&\quad + \delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star)] + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) \\
&= \omega(1+\beta) + \alpha(RV_t + \beta RV_{t-1}) + \gamma(D_t RV_t + \beta D_{t-1} RV t - 1) + \beta^2 \mu_{t-1} + \\
&\quad + \delta[(x_t - x^\star) + \beta(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+1} - \Delta^\star) + \beta(\Delta_t - \Delta^\star)]
\end{aligned}
$$

$$
\begin{aligned}
\mu_{t+2} &= \omega + (\alpha + \gamma D_{t+1}) RV_{t+1} + \beta \mu_{t+1} + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) \\
&= \omega + (\alpha + \gamma D_{t+1}) RV_{t+1} + \beta[\omega(1+\beta) + \alpha(RV_t + \beta RV_{t-1}) + \\
&\quad + \gamma(D_t RV_t + \beta D_{t-1} RV t - 1) + \beta^2 \mu_{t-1} + \delta[(x_t - x^\star) + \beta(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+1} - \Delta^\star) + \beta(\Delta_t - \Delta^\star)] + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) \\
&= \omega(1+\beta+\beta^2) + \alpha(RV_{t+1} + \beta RV_t + \beta^2 RV_{t-1}) + \\
&\quad + \gamma(D_{t+1} RV_{t+1} + \beta D_t RV_t + \beta^2 D_{t-1} RV t - 1) + \beta^3 \mu_{t-1} + \\
&\quad + \delta[(x_{t+1} - x^\star) + \beta(x_t - x^\star) + \beta^2(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+2} - \Delta^\star) + \beta(\Delta_{t+1} - \Delta^\star) + \beta^2(\Delta_t - \Delta^\star)]
\end{aligned}
$$

$$\vdots$$

$$
\begin{aligned}
\mu_{t+\tau} &= \omega(1 + \beta + \cdots + \beta^t au) + \alpha(RV_{t+\tau-1} + \cdots + \beta^\tau RV_{t-1}) + \\
&\quad + \gamma(D_{t+\tau-1} RV_{t+\tau-1} + \cdots + \beta^\tau D_{t-1} RV t - 1) + \beta^{\tau+1} \mu_{t-1} + \\
&\quad + \delta[(x_{t+\tau-1} - x^\star) + \cdots + \beta^\tau(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+\tau} - \Delta^\star) + \cdots + \beta^\tau(\Delta_t - \Delta^\star)].
\end{aligned}
$$

By taking the first partial derivatives, we get the marginal effect of the implementation variable x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
δ	$\beta\delta$	$\beta^2\delta$	$\cdots$	$\beta^\tau\delta$

and the marginal effect of the announcement variable Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
φ	$\beta\varphi$	$\beta^2\varphi$	$\cdots$	$\beta^\tau\varphi$

A.2 Multi-step forecasting

In this appendix we present the formula we used to perform the Multi–step forecasting procedure.

ACM

$$
\begin{aligned}
RV_T &= \mu_T \epsilon_T \\
\mu_T &= \varsigma_t + \xi_t \\
&= \omega + \alpha RV_{T-1} + \beta\varsigma_{T-1} + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) + \psi\xi_{T-1} \\
\mu_{T+1} &= \omega + \alpha RV_T + \beta\varsigma_T + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) + \psi\xi_T \\
&= \omega + \alpha RV_T + \beta(\omega + \alpha RV_{T-1} + \beta\varsigma_{T-1}) + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) + \\
&\quad + \psi[\delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \bar{\Delta}^\star) + \psi\xi_{T-1})] \\
&= \omega(1+\beta) + \beta^2\varsigma_{T-1} + \alpha(\beta RV_{T-1} + RV_T) + \delta[\psi(x_{T-1} - x^\star) + (x_T - x^\star)] + \\
&\quad + \varphi[\psi(\Delta_T - \Delta^\star) + (\Delta_{T+1} - \Delta^\star)] + \psi^2\xi_{T-1} \\
&\vdots \\
\mu_{T+\tau} &= \omega(1+\beta+\beta^2+..+\beta^\tau) + \beta^{\tau+1}\varsigma_{T-1} + \alpha\sum_{i=-1}^{\tau-1}\beta^{\tau-1+i}RV_{T-i} + \\
&\quad + \delta\sum_{i=-1}^{\tau-1}\psi^{\tau-1+i}(x_{T-i} - \bar{x}) + \varphi\sum_{i=0}^{\tau}\psi^{\tau-i}(\Delta_{T+i} - \bar{\Delta}) + \psi^\tau\xi_{T-1}.
\end{aligned}
$$

Similarly to the previous case, for $\tau \to \infty$ the unconditional mean is given by:

$$\mu = [\tfrac{1}{1-(\alpha+\frac{\gamma}{2})(\frac{1}{1-\beta})}][\tfrac{\omega}{1-\beta} + \tfrac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]$$

L-ACM

$$RV_T = \mu_T \epsilon_T$$

$$\begin{aligned}
\mu_T &= 2\varsigma_T[\frac{exp(\xi_T)}{(1+exp(\xi_T))}] \\
&= 2[\omega + \alpha RV_{T-1} + \beta\varsigma_{T-1}]\{\frac{exp[\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1}]}{1+exp[\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1}]}\}
\end{aligned}$$

$$\begin{aligned}
\mu_{T+1} &= 2[\omega + \alpha RV_T + \beta\varsigma_T]\{\frac{exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi\xi_T]}{1+exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi\xi_T]}\} \\
&= 2[\omega + \alpha RV_T + \beta(\omega + \alpha RV_{T-1} + \beta\varsigma_{T-1})] \\
&\quad \{\frac{exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi(\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1})]}{1+exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi(\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1})]}\} \\
&= 2[\omega(1+\beta) + \beta^2\varsigma_{T-1} + \alpha(\beta RV_{T-1} + RV_T)] \\
&\quad \{\frac{exp[\delta(\psi(x_{T-1}-x^\star)+(x_T-x^\star))+\varphi(\psi(\Delta_T-\Delta^\star)+(\Delta_{T+1}-\Delta^\star))+\psi^2\xi_{T-1}]}{1+exp[\delta(\psi(x_{T-1}-x^\star)+(x_T-x^\star))+\varphi(\psi(\Delta_T-\Delta^\star)+(\Delta_{T+1}-\Delta^\star))+\psi^2\xi_{T-1}]}\}
\end{aligned}$$

$$\begin{aligned}
\mu_{T+\tau} &= 2[\omega(1+\beta+\beta^2+..+\beta^\tau) + \beta^{\tau+1}\varsigma_{T-1} + \alpha\sum_{i=-1}^{\tau-1}\beta^{\tau-1+i}RV_{T-i}] \\
&\quad \{\frac{exp[\delta\sum_{i=-1}^{\tau-1}\psi^{\tau-1+i}(x_{T-i}-x^\star)+\varphi\sum_{i=0}^{\tau}\psi^{\tau-i}(\Delta_{T+i}-\Delta^\star)+\psi^\tau\xi_{T-1}]}{1+exp[\delta\sum_{i=-1}^{\tau-1}\psi^{\tau-1+i}(x_{T-i}-x^\star)+\varphi\sum_{i=0}^{\tau}\psi^{\tau-i}(\Delta_{T+i}-\Delta^\star)+\psi^\tau\xi_{T-1}]}\}
\end{aligned}$$

for $\tau \to \infty$

$$\mu = \frac{\{2(\frac{\omega}{1-\beta})[\frac{exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}{1+exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}]\}}{1-2(\frac{\alpha+\frac{\gamma}{2}}{1-\beta})[\frac{exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}{1+exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}]}$$

Li-ACM

$$
\begin{aligned}
RV_T &= \mu_T \epsilon_T \\
\mu_T &= \varsigma_T \xi_T \\
&= [\omega + \alpha RV_{T-1} + \beta \varsigma_{T-1}][(1-\psi) + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) + \psi \xi_{T-1}] \\
\mu_{T+1} &= [\omega + \alpha RV_T + \beta \varsigma_T][(1-\psi) + \delta(x_T - x^s tar) + \varphi(\Delta_{T+1} + \Delta^\star) + \psi \xi_T)] \\
&= [\omega + \alpha RV_T + \beta(\omega + \alpha RV_{T-1} + \beta \varsigma_{T-1})] \\
&\quad \{(1-\psi)) + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) + \psi[(1-\psi) + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) + \\
&\quad + \psi \xi_{T-1})\} \\
&= [\omega(1+\beta) + \beta^2 \varsigma_{T-1} + \alpha(\beta RV_{T-1} + RV_T)] \\
&\quad [\psi(1-\psi) + \delta(x_t - x^\star) + \psi(x_{T-1} - x^\star)) + \varphi((\Delta_{T+1} - \Delta^\star) - \psi(\Delta_T - \Delta^\star) + \psi^2 \xi_{T-1})]
\end{aligned}
$$

$$
\begin{aligned}
\mu_{T+\tau} &= [\omega(1+\beta+\beta^2+..+\beta^\tau) + \beta^{\tau+1}\varsigma_{T-1} + \alpha \sum_{i=-1}^{\tau-1} \beta^{\tau-1+i} RV_{T-i}] \\
&\quad [(1-\psi)(1+\psi+\psi^2+..+\psi^\tau) + \delta \sum_{i=-1}^{\tau-1} \psi^{\tau-1+i}(x_{T-i} - x^\star) + \\
&\quad + \varphi \sum_{i=0}^{\tau} \psi^{\tau-i}(\Delta_{T+i} - \Delta^\star) + \psi^\tau \xi_{T-1}]
\end{aligned}
$$

for $\tau \to \infty$

$$
\mu = [\tfrac{\omega}{1-\beta} + \tfrac{1-\psi-\delta(x_{T-1}-\bar{x})-\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]\{\tfrac{1}{1-(\frac{\alpha+\frac{\gamma}{2}}{1-\beta})[\frac{1-\psi-\delta(x_{T-1}-\bar{x})-\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}\}
$$

AMEMX

$$
\begin{aligned}
RV_T &= \mu_T \epsilon_T \\
\mu_T &= \omega + \alpha RV_{T-1} + \beta\mu_{T-1} + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) \\
\mu_{T+1} &= \omega + \alpha RV_T + \beta\mu_T + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) \\
&= \omega + \alpha RV_T + \beta[\omega + \alpha RV_{T-1} + \beta\mu_{T-1} + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star)] + \\
&\quad + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) \\
&= \omega(1+\beta) + \beta^2\mu_{T-1} + \alpha(\beta RV_{T-1} + RV_T) + \delta[\beta(x_{T-1} - x^\star) + (x_T - x^\star)] + \\
&\quad + \varphi[\beta(\Delta_T - \Delta^\star) + (\Delta_{T+1} - \Delta^\star)] \qquad .
\end{aligned}
$$

$$\vdots$$

$$
\begin{aligned}
\mu_{T+\tau} &= \omega(1 + \beta + \beta^2 + .. + \beta^\tau) + \beta^{\tau+1}\mu_{T-1} + \alpha \sum_{i=-1}^{\tau-1} \beta^{\tau-1+i} RV_{T-i} + \\
&\quad + \delta \sum_{i=-1}^{\tau-1} \beta^{\tau-1+i}(x_{T-i} - x^\star) + \varphi \sum_{i=0}^{\tau} \beta^{\tau-i}(\Delta_{T+i} - \Delta^\star)
\end{aligned}
$$

For $\tau \to \infty$ the process converges to the unconditional mean:

$$\mu = \frac{\omega+\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\alpha-\beta-\frac{\gamma}{2}}$$

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