



ON FINANCIAL FRICTIONS AND FIRM MARKET POWER

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On financial frictions and firm market power*

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Abstract

We build a static general-equilibrium model with monopolistically competitive firms that borrow funds from competitive banks in an economy subject to financial frictions. These frictions are due to non verifiability of both ex post firm returns and managerial effort. Market power has opposing effects. On one side, firms' pricing over marginal cost reduces output compared to perfect competition. On the other, by increasing firms' profitability, market power reduces the impact of financial frictions. The resulting tradeoff is ambiguous. We show that, other things equal, there exists an optimal positive level of market power that maximizes welfare. Such optimal degree of market power increases with moral hazard and decreases with the efficiency of firm liquidation following bankruptcy.

Keywords: Market power, moral hazard, bankruptcy, liquidation.

Jel Classification: E44, G21, G33.

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1 Introduction

There is an extensive literature that looks at the macroeconomic interaction between the financial and the real sectors of an economy. Brunnermeier *et al.* (2011) provide a comprehensive survey. Within that literature, this paper deals with the role of firm's market power in shaping the structural macroeconomic consequences of financial frictions. Firm's market power, other things equal, leads to prices higher than marginal costs, which reduces output compared to perfect competition, causing a welfare loss. However, by increasing firms' profitability, market power could have a beneficial impact mitigating the adverse effects that financial frictions could have on firms' activity. Our contribution is to explore such trade off in a static general equilibrium model in which firms are monopolistically competitive and subject to financial frictions due to asymmetric information. We find that, in the presence of bankruptcy costs and moral hazard resulting from such informational imperfections, there is an optimal degree of market power that maximizes welfare. That is, a fully competitive production sector would be detrimental to firms' survival and to households' welfare.

Our analysis could be extended to dynamic frameworks to study how firm's market power shapes the effects of financial frictions on business cycle dynamics and long run growth. The literature on the theory and empirics of the relationship between finance and growth largely neglects the role of firm's market power.¹ Embedding our setup in a growth model by including a source of endogenous Total Factor Productivity (TFP) growth would allow to assess how firm's market power affects economic growth depending on the development of financial institutions capable of reducing moral hazard and the efficiency of the bankruptcy procedures. As it is well known since seminal work by La Porta *et al.* (1998), underdeveloped legal systems with poor protection of creditors' rights result in low financial development and more relevant financial frictions. In turn, the extensive literature on finance and growth shows that financial development significantly affects economic growth (Levine, 2005). Our results suggest that, in less developed environments (where bankruptcy losses and moral hazard are likely to be more relevant), firms' market power might reduce the effects of financial frictions and be growth conducive.²

From a business cycle perspective, the vast literature on financial frictions and short term fluctuations starting with Bernanke and Gertler (1989) does not provide a systematic assessment of the role of firm's market power either. Exporting our setup in a business cycle model would allow to study how market power affects the role of financial frictions in shaping business cycle fluctuations by affecting the persistence and the amplification of short run shocks.

Our benchmark is a model with uncertainty due to demand idiosyncratic shocks. Households choose

¹See Levine (2005) and Carré and L'oeillet (2018) for comprehensive surveys.

²On a somehow related issue, the literature on the charter value hypothesis looks at the effects of banks' market power on the stability of the financial system (see Freixas, 2010).

how much labor to supply, how much to consume, the composition of consumption in terms of varieties, and their amount of bank deposits. Firms produce final goods by employing labor. Since they have no financial resources, they demand external finance provided by banks that compete *à la* Bertrand. We depart from the benchmark model by introducing two sources of financial frictions: non observability and verifiability of both firm's ex post returns and firms' management effort. Non verifiability of ex post returns implies the use of standard debt contracts in the financing of firms. As a consequence, bankruptcy emerges in equilibrium, generating a loss of welfare that depends on the efficiency of the liquidation process. Non verifiability of firm's management effort generates the possibility of moral hazard. Firm's management might have the incentive to choose low effort to seek private benefits, even though this affects negatively firms' prospects by increasing the probability of adverse demand shocks. We provide a full characterization of the equilibrium and the conditions for its existence, both in case moral hazard does not play a role, and in case it does and economic activity is constrained as a result.

We calibrate the model in order to match evidence from US data related to the external finance premium on commercial and industrial loans, and the rate of exit (default) of total private establishments. We use the calibrated model to do comparative statics on the effects of firm's market power and its interplay with the efficiency of the liquidation process following bankruptcy. As it emerges from the numerical simulations, a general feature of the model is that firm's market power reduces significantly the effects of financial frictions. The trade off between this effect and the efficiency loss caused by firms pricing over the marginal cost is not trivial. There exist an optimal level of market power that maximizes welfare. Such optimal level of market power increases with the size of the bankruptcy losses due to the inefficiency of the liquidation process.

Our paper contributes to the extensive literature on misallocation and macroeconomics in the presence of financial frictions. Several contributions have focused on the long-run consequences of misallocation. For instance, Piketty (1997) shows that credit rationing due to moral hazard leads to the possibility of low output and growth regimes, with high interest rates and higher wealth inequality. Galor and Zeira (1993) discuss the possibility of non-ergodic wealth distributions emerging as a consequence of exogenous credit constraints in the presence of non-convex production technologies. Banerjee and Newmann (1993) provide similar multiplicity to the one studied by Piketty (1997) in a model with exogenous interest rates. Banerjee and Moll (2010) argue that capital misallocation resulting from credit constraints can be very persistent with quantitatively important consequences. Moll (2014) shows that financial frictions have long-run lasting consequences with transitory productivity shocks, while if shocks are sufficiently persistent, self-financing could still eventually be a good substitute for external finance. Buera *et al.* (2011) develop a quantitative model to show that financial frictions distort the allocation of capital and talent across firms, with adverse sizable consequences on TFP. They also show that this channel explains a substantial part of cross-country variation in TFP, output per worker, and other measures of productivity. Buera and Shin (2013) study how

misallocation due to financial frictions affects significantly the speed of convergence after reforms triggering an efficient allocation of resources. In the same vein, Midrigan and Yi Xu (2014) show that financial frictions reduce TFP by distorting entry and technology adoption decisions, as well as by generating dispersion in the return to capital across producers. The importance of financial deepening for TFP growth also emerges as a strong result in the analysis by Jeong and Townsend (2007).

Complementary to the above literature, we explore how market power affects the allocation of resources and, therefore, aggregate output and welfare, by interacting with financial frictions due to asymmetric information. Related to market power, Galle (2019) constructs a general equilibrium model to analyze how competition affects the efficiency of capital allocation across heterogeneous firms in the presence of financial constraints. He argues that lower market power results in lower markups, but also slows down capital accumulation, thereby reducing the positive steady state impact of increased competition. Their empirical findings based on data from India confirm such predictions. In the same vein, Jungherr and Strauss (2017) study the effect of market power in a growth model of a small open economy characterized by exogenous financial constraints. Higher market power brings higher earnings, which result in higher self-financing of investment. In the presence of borrowing constraints, the higher self-financing strengthens capital accumulation. Their empirical testing based on South Korea micro data confirms such prediction. In Galle (2019) and Jungherr and Strauss (2017), the negative effect of competition in the presence of financial constraints stems from the fact that more competition leads to lower profits, which reduce self-financing. Differently, in our model, the negative effect of competition is due to the fact that less competition increases firm's profitability and makes firms less likely to go bankrupt. As a result, financial frictions due to moral hazard and bankruptcy losses are mitigated. Furthermore, in our model financial frictions are fully microfounded and their effects are explicitly derived from the rational behavior of both firms and banks.

Also related to our paper, Silva (2019) analyzes how the interaction between heterogeneous financial frictions and monopolistic competition influence the transmission of monetary policy into investment and corporate cash holdings.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, we characterize the general equilibrium. A baseline calibration of the model parameters is proposed in Section 4. Next, the quantitative analysis begins in Section 5 examining the interaction between a low market power and high private benefits to bring pervasive moral hazard. The analysis continues in Section 6 with a discussion of the general equilibrium effects of financial frictions depending on firms' market power and the liquidation technology and in Section 7 commenting the welfare effects of suboptimal market power using the household consumption equivalence. Section 8 concludes the paper with a summary of its main results.

2 The model

We consider an economy with households, firms and banks. Households receive salaries in exchange of labor and dividends from banks and firms, using these sources of income to consume final goods. Firms use labor and external finance to produce varieties of consumption goods. Banks issue deposits and finance firms' operations.

2.1 Households

The economy is populated by a continuum of size one of households who live for one period. They choose how much labor to supply, how much income to consume, and the composition of their consumption in terms of varieties, where the latter choice is separable from the former two. They also decide their demand for bank deposits. Their utility function is

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{n^{1+\gamma}}{1+\gamma} \quad (1)$$

where, $\sigma, \gamma > 0$ are the elasticities of the marginal utility with respect to the consumption index, c , and labor, n , respectively, and $\psi > 0$ is a scale parameter that measures the relative weight of labor disutility. Taking the Dixit and Stiglitz (1977) aggregation scheme, the consumption index is

$$c = \left[\int_0^1 e(i) c(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

where $c(i)$ is consumption of variety i , $e(i)$ is a variety-specific shock, and $0 < \theta < 1$ is the inverse of Dixit and Stiglitz (1977)'s elasticity of substitution, which is also a measure of firm's market power. We assume that $e(i)$ is uniformly distributed between $[\underline{\epsilon}, \bar{\epsilon}]$, with $\bar{\epsilon} > \underline{\epsilon} > 0$, and density function $\frac{1}{\bar{\epsilon}-\underline{\epsilon}}$, so that its expected value and variance are

$$E[e(i)] = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \frac{1}{\bar{\epsilon}-\underline{\epsilon}} e(i) de(i) = \frac{\bar{\epsilon} + \underline{\epsilon}}{2} \quad (2)$$

$$Var[e(i)] = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \frac{1}{\bar{\epsilon}-\underline{\epsilon}} \left(e(i) - \frac{\bar{\epsilon} + \underline{\epsilon}}{2} \right)^2 de(i) = \frac{1}{12} (\bar{\epsilon} - \underline{\epsilon})^2 \quad (3)$$

Individuals make their consumption decisions after the realization of the idiosyncratic shock, therefore they face no uncertainty. Accordingly, the inverse demand function for variety i is found to be³

$$\frac{p(i)}{p} = e(i) \left(\frac{c(i)}{c} \right)^{-\theta} \quad (4)$$

where, the idiosyncratic shock $e(i)$ brings an exogenous demand shift and

$$p = \left[\int_0^1 p(i)^{\frac{\theta-1}{\theta}} e(i)^{1/\theta} di \right]^{\frac{\theta}{\theta-1}} \quad (5)$$

³See technical Appendix for the proof.

is the Dixit-Stiglitz price index.

The sources of income for the households are the salary, wn , at a competitive real wage, w ; firm's dividends, d_f ; banks' dividends, d_b ; and bank deposits, dep , pay a risk-free interest rate, r , which is equal to the opportunity cost of deposits.⁴ Households decide the level of consumption, c , labor supply, n , and deposits, dep , so to solve the following maximization problem:⁵

$$\begin{aligned} \max_{\{c, n, dep\}} \quad & \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{n^{1+\gamma}}{1+\gamma} \\ \text{s.to:} \quad & c \leq wn + d_f + d_b + rdep \\ dep \quad & \leq \quad wn \end{aligned}$$

The first order conditions are

$$\begin{aligned} c^{-\sigma} + \lambda &= 0 \\ -\psi n^\gamma - (\lambda + \varphi) w &= 0 \\ \lambda r + \varphi &= 0 \\ wn + d_f + d_b + rdep &= c \\ dep &= wn \end{aligned}$$

where λ and φ are, respectively, the Lagrangian multipliers of the budget constraint and the deposits' constraint. Hence, the optimal labor supply schedule satisfies

$$w = \frac{\psi n^\gamma c^\sigma}{1+r} \tag{6}$$

and the budget constraint can be rewritten as follows

$$c = (1+r)wn + d_f + d_b \tag{7}$$

2.2 Financial intermediation and financial frictions

Firms hire labor and pay wages before they produce. Accordingly, they need external financial resources, which are supplied by competitive banks that issue nominal claims to firms in exchange for a future promise of repayment. Workers deposit their salaries in the banks.

⁴We could provide a full analysis of household's decision to deposit within banks *versus* alternative risk-free assets. However, it is clear that so long as r exceeds the return on alternative assets, households will choose deposits. Therefore, in equilibrium r will be equal to the return on the alternative asset. We impose this result upfront to simplify the exposition.

⁵In principle, we should also explicitly assume that households have an endowment of time equal to x , and therefore impose the constraint that the amount of labor they choose to offer, n , cannot exceed, x . Rather than doing that, we solve the model under the implicit assumption that the unconstrained level of labor supply is lower than x . This is always verified for x sufficiently large.

We introduce two sources of friction in the economy: non verifiability of ex post firm returns and moral hazard. Non verifiability of ex post returns implies that firms' external finance provided by banks takes the form of standard debt (i.e. loans).⁶ As a result, bankruptcy emerges, which is a source of inefficiency so long as some of the firm's value is lost in the process of liquidation upon default.

We model moral hazard by assuming that the distribution of the idiosyncratic shock, $e(i)$, is affected by the level of effort exerted by firm's management, which can either be high or low and it is not contractible. In particular, we assume that the level of effort exerted determines the lower bound, \underline{e} , of the support of the distribution of $e(i)$, which equals \underline{e}_H if firm's management exerts high effort, and \underline{e}_L in case it exerts low effort, with $\underline{e}_L < \underline{e}_H$. As a consequence, the expected firm's revenue would decline in the presence of low effort, while its variance would increase.

However, we assume that if firm's management exerts low effort it appropriates private benefits by misusing funds as in Tirole (2006). In other words, exerting low effort enables the managers to focus on distracting resources away from the firm, thereby generating a private benefit. Accordingly, we model private benefits as a fraction b of the size of firm's loan, and independent of the realization of the idiosyncratic shock.⁷ That is, a higher volume of activity, that implies more financial resources borrowed by the firm, generates higher private benefits if the firm's managements exerts low effort. Thus, also as in Tirole (2006), in our model private benefits enter the expected payoff in an additive way.

2.3 Firms

The economy is populated by a continuum of size one of firms operating in a monopolistically competitive market. Firms are operated by management that appropriates profits and private benefits. Each firm produces one differentiated variety of consumption good, facing the Dixit-Stiglitz inverse demand constraint (4). The optimal choice of output produced takes place before observing the realization of the idiosyncratic demand shock, $e(i)$. Regarding the production technology, firm i produces, $y(i)$; using labor, $n(i)$; according to the following linear production function:

$$y(i) = An(i) \tag{8}$$

where $A > 0$ is a productivity parameter. Since firms do not have internal financial resources, they should get external finance in order to pay salaries, $wn(i)$. In particular, given the production function (8), in order to produce $y(i)$, firm i needs to demand a loan of size $wy(i)/A$. Accordingly, given the interest rate of loans, r_f , the total cost for firm i , producing $y(i)$, is $(1 + r_f) wy(i)/A$.

⁶See Townsend (1979) and Bolton and Dewatripont (2005, chapter 5).

⁷An alternative way of introducing moral hazard is to measure the private benefit in terms of effort disutility.

Recalling the demand function (4), the total real revenue for firm i is

$$\frac{p(i)}{p}y(i) = e(i) \left(\frac{c(i)}{c} \right)^{-\theta} y(i)$$

We note that firm's revenues depend on the realization of the idiosyncratic shock, $e(i)$. Using market-clearing conditions at firm level, $c(i) = y(i)$, we obtain

$$\frac{p(i)}{p}y(i) = e(i)c^\theta y(i)^{1-\theta} \quad (9)$$

Firm i 's bankruptcy occurs if total revenues are lower than payments due to banks. The firm is otherwise solvent. Accordingly, the firm goes bankrupt if and only if

$$e(i)c^\theta y(i)^{1-\theta} < (1 + r_f) w \frac{y(i)}{A}$$

This brings the following definition of the critical shock⁸

$$\widehat{e}(i) = \max \left\{ \underline{\epsilon}, (1 + r_f) \frac{w}{A} y(i)^\theta c^{-\theta} \right\} \quad (10)$$

Firm i goes bankrupt if $\underline{\epsilon} < e(i) < \widehat{e}(i)$, being solvent otherwise. In the event of bankruptcy, the firm earns zero, and the bank appropriates the liquidation value of the firm (to be defined below). Therefore, firm's management has the incentive to declare bankruptcy if and only if $e(i) < \widehat{e}(i)$, and to repay the loan otherwise.

2.3.1 Firm's optimal output choice conditional on effort

Since the representative firm i chooses how much to produce, $y(i)$, before observing the realization of the shock, $e(i)$, it does so in order to maximize its expected profit conditional on effort. If the firm chooses high managerial effort, $\underline{\epsilon}_H$, expected profits (weighted by the respective probabilities of loan repayment and default) are

$$E(\pi(i)|\underline{\epsilon}_H) = \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} \int_{\widehat{e}(i)}^{\bar{\epsilon}} \frac{1}{\bar{\epsilon} - \widehat{e}(i)} \left(e(i)c^\theta y(i)^{1-\theta} - (1 + r_f) w \frac{y(i)}{A} \right) de(i) + \frac{\widehat{e}(i) - \underline{\epsilon}_H}{\bar{\epsilon} - \underline{\epsilon}_H} 0$$

where solving out the integral and dropping the null second term, we reach

$$E(\pi(i)|\underline{\epsilon}_H) = \frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y(i)^{1-\theta} - \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1 + r_f) \frac{w y(i)}{A} \quad (11)$$

The first order condition for the choice of output produced, $y(i)$, then is

$$\frac{\bar{\epsilon} + \widehat{e}(i)}{2} c^\theta y^{-\theta}(i) - \frac{1}{(1 - \theta)} \frac{(1 + r_f) w}{A} = 0 \quad (12)$$

⁸In practical terms, the value of the critical shock $\widehat{e}(i)$ implied by (10) is $(1 + r_f) \frac{w}{A} y(i)^\theta c^{-\theta}$, unless market power θ becomes sufficiently high to give a value of $\widehat{e}(i)$ below its lower bound $\underline{\epsilon}$. For those cases, (10) sets $\widehat{e}(i) = \underline{\epsilon}$.

Firms operate in monopolistic competition and the marginal revenue (left-hand side in (12)) is a constant mark-up, $1/(1 - \theta)$, over the marginal cost, $(1 + r_f)w/A$. Solving equation (12), for $y(i)$, we find

$$y(i) = \left(\frac{(1 - \theta) A (\bar{\epsilon} + \hat{e}(i))}{2w(1 + r_f)} \right)^{1/\theta} c \quad (13)$$

Conditional on low effort, $\underline{\epsilon}_L$, firm i expected profits including private benefits are

$$E(\pi(i)|\underline{\epsilon}_L) = \frac{\bar{\epsilon}^2 - \hat{e}^2(i)}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y(i)^{1-\theta} - \left(\frac{(\bar{\epsilon} - \hat{e}(i))}{(\bar{\epsilon} - \underline{\epsilon}_L)} (1 + r_f) - b \right) \frac{wy(i)}{A} \quad (14)$$

which implies the following first order condition for output, $y(i)$,

$$\frac{(1 - \theta) (\bar{\epsilon}^2 - \hat{e}^2(i))}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y(i)^{-\theta} - \left(\frac{(\bar{\epsilon} - \hat{e}(i))}{(\bar{\epsilon} - \underline{\epsilon}_L)} (1 + r_f) - b \right) \frac{w}{A} = 0 \quad (15)$$

Solving for $y(i)$ yields

$$y(i) = \left[\frac{(1 - \theta) A (\bar{\epsilon} + \hat{e}(i))}{2w \left(1 + r_f - b \frac{\bar{\epsilon} - \underline{\epsilon}_L}{\bar{\epsilon} - \hat{e}(i)} \right)} \right]^{\frac{1}{\theta}} c \quad (16)$$

2.3.2 Effort choice and incentive compatibility

Let us now analyze the effort choice made by the firm's management for a given level of production. The essence of the moral hazard problem is that effort cannot be monitored and it is not contractible. Hence, if banks are willing to finance a firm conditional on firm's management exerting high effort, the loan contract should preserve the incentives of the firm's management to do so. In other words, firm's expected profits conditional on high effort (9) should be higher than firm's expected profits conditional on low effort (12), i.e. $E(\pi(i)|\underline{\epsilon}_H) \geq E(\pi(i)|\underline{\epsilon}_L)$. Therefore, the incentive compatibility constraint (ICC) takes the following form

$$\frac{\bar{\epsilon} - \hat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} \left(\frac{\bar{\epsilon} + \hat{e}(i)}{2} c^\theta y^{1-\theta}(i) - (1 + r_f) \frac{wy(i)}{A} \right) \geq \frac{\bar{\epsilon} - \hat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_L} \left(\frac{\bar{\epsilon} + \hat{e}(i)}{2} c^\theta y^{1-\theta}(i) - \frac{(1 + r_f)wy(i)}{A} \right) + b \frac{wy(i)}{A} \quad (17)$$

Using equation (12) to substitute in (17) results, after some algebra, in the following inequality:

$$r_f \geq \frac{b(1-\theta)(\bar{\epsilon} - \underline{\epsilon}_H)(\bar{\epsilon} - \underline{\epsilon}_L)}{(\bar{\epsilon} - \hat{e}(i))(\underline{\epsilon}_H - \underline{\epsilon}_L)\theta} - 1 \quad (18)$$

The ICC is satisfied in equilibrium if r_f exceeds the threshold defined by the right-hand side of (18).⁹

⁹The standard result on the effects of moral hazard finds that a higher interest rate reduces the incentives to exert high effort (Tirole, 2006, chapter 3). Such relationship holds in our original expression of ICC, (17), as the marginal cost of a higher r_f is greater for a firm that exerts high effort (left-hand side) than for a firm that exerts low effort (right-hand side). Remarkably, the equilibrium expression (18) goes on the opposite direction: a higher interest rate would prevent firms from deviating to low effort. The reason is that we have incorporated the optimal choice of firm-level output.

2.4 Banks

The economy is populated by a continuum of size one of banks, which finance firms' operations. Banks provide loans and issue deposits paying a real risk-free interest rate, r . Firms borrow from banks by means of a standard debt contract at the real interest rate, r_f . Accordingly, the bank receives a return r_f per unit of loan if firm i is able to repay, and appropriates the liquidation value of the firm in the event of bankruptcy. We assume that a fraction $(1 - \tau)$ of firm revenues, with $\tau \in [0, 1]$, is destroyed during the process of liquidation upon default. The expected value of a loan of size $l(i)$ extended to a firm i exerting effort $\underline{\epsilon}$ is

$$\frac{\bar{\epsilon} - \hat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}} \int_{\hat{\epsilon}(i)}^{\bar{\epsilon}} \left(\frac{1}{\bar{\epsilon} - \hat{\epsilon}(i)} (1 + r_f) l(i) \right) d\epsilon(i) + \frac{\hat{\epsilon}(i) - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} \tau \int_{\underline{\epsilon}}^{\hat{\epsilon}(i)} \frac{1}{\hat{\epsilon}(i) - \underline{\epsilon}} e(i) c^\theta y(i)^{1-\theta} d\epsilon(i) \quad (19)$$

According to equation (19), banks' expected profits account for both the return upon firm survival and the expected liquidation value upon bankruptcy (the fraction τ of revenues from sales). Taking the integral in the latter term, we get the liquidation value

$$lv = \tau \frac{\hat{\epsilon}^2(i) - \underline{\epsilon}^2}{2(\bar{\epsilon} - \underline{\epsilon})} y(i)^{1-\theta} c^\theta \quad (20)$$

which implies this loss of value

$$loss = (1 - \tau) \frac{\hat{\epsilon}^2(i) - \underline{\epsilon}^2}{2(\bar{\epsilon} - \underline{\epsilon})} y(i)^{1-\theta} c^\theta \quad (21)$$

We note that, if the loan size is the same across firms, equations (20) and (21) also measure aggregate expected values. Also, since we have no aggregate uncertainty, by the law of large numbers, such aggregate expected values are the same as the ex post aggregate realizations.

A necessary condition for banks to be willing to finance firm i is that the expected return of the loan is greater or equal than the cost of issuing the corresponding deposit, i.e.

$$\frac{\bar{\epsilon} - \hat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}} (1 + r_f) l(i) + lv \geq (1 + r) l(i) \quad (22)$$

where the left-hand side is obtained by taking the integral in (19), and the right-hand side measures the cost of the deposit. Note that $\frac{\bar{\epsilon} - \hat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}}$ is the probability of repayment (firm's survival). We assume that banks set the interest rate r_f and compete *à la* Bertrand, serving any demand at the interest rate they set.

3 General equilibrium

We now characterize the general equilibrium and discuss the existence conditions. First, let us describe the timing of events:

Stage 0. Given the relative prices, the risk-free interest rate and the real wage, households choose consumption and consumption varieties, demand deposits and supply labor. Depending on wages and the

interest rate on loans, firms demand labor and loans, and choose production. Banks issue deposits at the risk-free interest rate, choose the loan interest rate and supply loans.

Stage 1. Firms' management chooses effort.

Stage 2. Idiosyncratic shocks take place and payoffs are realized.

Having described the timing of events, we define the equilibrium as follows:

Definition 1. *Given the exogenous risk-free interest rate, r , a general equilibrium is:*

- *a set of relative prices $p(i)/p$ for $i \in [0, 1]$; a loan interest rate, r_f ; and a real wage, w ;*
 - *a set of quantities for the varieties of consumption goods, $y(i)$ for $i \in [0, 1]$; labor, n ; deposits, dep ; and loans, l ;*
 - *a managerial effort level, $\underline{\epsilon}$;*
- such that agents are playing their best strategies and the markets for goods, labor, deposits and loans clear.*

In principle, we have three types of equilibrium:

1. Unconstrained equilibrium with high effort. In this case, the ICC given by inequality (18) is not binding and competition among banks brings r_f down to a level such that banks make zero expected profits.
2. Constrained equilibrium with high effort. In this case, the ICC is binding, and banks make positive expected profits.
3. Equilibrium with low effort. The ICC does not apply since banks lend independently of firm's management effort choice, and competition among banks brings r_f down to a level such that banks make zero expected profits (conditional on low effort).

3.1 Unconstrained equilibrium with high effort

Let us start analyzing the unconstrained equilibrium with high effort. We solve for the general equilibrium of the model under the assumption that the ICC is slack, and then study the necessary and sufficient conditions for this conjecture to hold. First, we find the equilibrium value of the critical realization of the shock, $\widehat{e}(i)$. Plugging the output optimality condition (12) in equation (10), with $\widehat{e}(i) > \underline{\epsilon}_H$, after some algebra we obtain

$$\widehat{e}(i) = \frac{(1 - \theta)}{(1 + \theta)} \bar{\epsilon} \quad (23)$$

Therefore, the critical shock goes down if firm's market power increases (higher θ) or the maximum value of the shock falls (lower $\bar{\epsilon}$).

Since banks compete *à la* Bertrand, in an equilibrium with high effort, they undercut each other, so long as the ICC is satisfied. Accordingly, in an unconstrained equilibrium with high effort, banks' equilibrium interest rate is such that they make zero expected profits. This implies that their participation constraint (22) must hold with strict equality

$$\frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1 + r_f) \frac{y(i)w}{A} + \tau \frac{\widehat{\epsilon}^2(i) - \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} y(i)^{1-\theta} c^\theta = (1 + r) \frac{y(i)w}{A}$$

where, in (22), we have substituted for $lv(i)$ using (20) and $l(i) = y(i)w/A$. We can rewrite the above expression plugging the optimality condition (12) to get, after some algebra

$$r_f = \frac{\bar{\epsilon} - \underline{\epsilon}_H}{\bar{\epsilon} - \widehat{\epsilon}(i)} \frac{(1 + r)}{1 + \tau \left(\frac{\widehat{\epsilon}^2(i) - \underline{\epsilon}_H^2}{(1-\theta)(\bar{\epsilon}^2 - \widehat{\epsilon}^2(i))} \right)} - 1 \quad (24)$$

where the equilibrium value of $\widehat{\epsilon}(i)$ is given by (23).

In the unconstrained equilibrium, banks make zero profits, so that $d_b = 0$. As for firms' dividends, they are equal to firms' expected profits¹⁰

$$d_f = \theta c^\theta y(i)^{1-\theta} \frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)}$$

Substituting for d_f in the household budget constraint, (7), and also using $d_b = 0$, we get

$$(1 + r) \frac{wy(i)}{A} + \theta c^\theta y(i)^{1-\theta} \frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} = c \quad (25)$$

Plugging the labor market-clearing condition, $n = y(i)/A$, in the optimal labor supply schedule (6) we have

$$\psi \left(\frac{y(i)}{A} \right)^\gamma c^\sigma = w \quad (26)$$

Finally, we study the necessary and sufficient conditions for the ICC to be slack. First of all, given the incentive compatibility constraint (18), the interest rate on loans, r_f , should satisfy

$$r_f \geq \frac{b(1-\theta)(\bar{\epsilon} - \underline{\epsilon}_H)(\bar{\epsilon} - \underline{\epsilon}_L)}{(\bar{\epsilon} - \widehat{\epsilon}(i))(\underline{\epsilon}_H - \underline{\epsilon}_L)\theta} - 1 \equiv r_f^{\min} \quad (27)$$

Using the equilibrium value of the critical shock, $\widehat{\epsilon}(i)$, given in (23), substituting for r_f using equation (24), and rearranging terms to solve analytically for the cutoff value of b , we obtain

$$b \leq \frac{\theta(1 + r)}{1 - \theta + \tau \left(\frac{(1-\theta)^2 \bar{\epsilon}^2 - \underline{\epsilon}_H^2 (1+\theta)^2}{4\theta \bar{\epsilon}^2} \right)} \frac{(\underline{\epsilon}_H - \underline{\epsilon}_L)}{(\bar{\epsilon} - \underline{\epsilon}_L)} \quad (28)$$

It is immediate to verify that the right-hand side of the above condition is an increasing function of θ , which takes value zero for $\theta = 0$. Accordingly, there exist a critical value for θ , such that the above condition

¹⁰This expression for d_f is obtained by substituting the optimality condition (12) into the equation that defines the expected profit conditional on firms' management exerting high effort (11).

is satisfied if and only if θ exceeds such critical value. Therefore, in order for the economy to be in an unconstrained equilibrium, firm's market power should be sufficiently high. Moreover, the level of firm's market power needed for an unconstrained equilibrium to exist increases with the rate of private benefits b , i.e. with the relevance of the moral hazard problem. The role of firm liquidation is captured by the effect of the parameter τ on the right hand side of (28). A more efficient liquidation technology (higher τ) would require a lower private benefit rate to prevent moral hazard. The reason is that firms have more incentives to deviate to low effort when liquidation upon default is less costly and the interest rate of lending is lower.

However, the existence of an equilibrium with high effort, independently of whether constrained or not, requires an additional necessary condition. Namely, firm's management should have no incentive to deviate from high effort to low effort.¹¹ Given a high effort equilibrium, firm's management has no incentive to deviate if

$$\pi(i)|_{\underline{\epsilon}_{H \rightsquigarrow L}} \leq \pi(i)|_{\underline{\epsilon}_H}$$

where

$$\pi(i)|_{\underline{\epsilon}_H} = \frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} \left(\frac{\bar{\epsilon} + \widehat{\epsilon}(i)}{2} c^\theta y^{1-\theta}(i) - (1 + r_f) \frac{wy(i)}{A} \right)$$

and

$$\pi(i)|_{\underline{\epsilon}_{H \rightsquigarrow L}} = \frac{\bar{\epsilon} - \widehat{\epsilon}_L(i)}{(\bar{\epsilon} - \underline{\epsilon}_L)} \left(\frac{\bar{\epsilon} + \widehat{\epsilon}_L(i)}{2} c^\theta y_L^{1-\theta}(i) - \frac{(1 + r_{f,L})wy_L(i)}{A} \right) + b \frac{wy_L(i)}{A}$$

This condition must be verified taking the aggregate variables (real wage and aggregate consumption) evaluated at the equilibrium with high effort. As for the expected profits in case of deviation, $\pi(i)|_{\underline{\epsilon}_{H \rightsquigarrow L}}$, these are obtained considering the optimal output if firm's management deviates to low effort, $y_L(i)$; the related interest rate of the loan, $r_{f,L}$; and the critical shock, $\widehat{\epsilon}_L(i)$.¹²

3.2 Constrained equilibrium with high effort

Let us now analyze the constrained equilibrium with high effort. Banks undercut each other until the ICC is binding. Therefore, the value of r_f associated with the unconstrained equilibrium with high effort, should be strictly lower than the minimum value of r_f such that the ICC is satisfied. Accordingly, a necessary condition for a constrained equilibrium is that the private benefit rate is high enough to violate condition (29).

Given equation (27) and substituting for the equilibrium value of $\widehat{\epsilon}(i)$, which is still given by equation (23), the equilibrium interest rate satisfies

$$r_f = \frac{b(1-\theta^2)(\bar{\epsilon} - \underline{\epsilon}_H)(\bar{\epsilon} - \underline{\epsilon}_L)}{2\theta^2\bar{\epsilon}(\underline{\epsilon}_H - \underline{\epsilon}_L)} - 1 \quad (29)$$

¹¹Note that if firms deviate to low effort, they anticipate that, in the subsequent subgame, banks will undercut each other charging them an interest rate such that banks make zero expected profits conditional on firms' management exerting low effort.

¹²See the technical Appendix for the equations that determine $y_L(i)$, $r_{f,L}$ and $\widehat{\epsilon}_L(i)$.

The equilibrium values of wages, production, and consumption, are found as before (see technical Appendix). In the constrained equilibrium, banks make strictly positive profits, so that bank's dividends are given by

$$d_B = \left[\frac{\bar{\epsilon} - \hat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}} (1 + r_f) - (1 + r) \right] w \frac{y}{A} + lv$$

where lv is the aggregate liquidation value obtained from (20).

3.3 Equilibrium with low effort

Let us first focus on the equilibrium value of the critical realization of the shock, $\hat{e}(i)$. Combining equations (10), with $\underline{\epsilon}_L < \hat{e}(i)$, and (15) yields

$$\frac{1 - \theta}{2} \left(\frac{\bar{\epsilon}^2 - \hat{e}(i)^2}{\bar{\epsilon} - \underline{\epsilon}_L} \right) = \left(\frac{\bar{\epsilon} - \hat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_L} - \frac{b}{1 + r_f} \right) \hat{e}(i) \quad (30)$$

The above condition gives the equilibrium value of $\hat{e}(i)$ as an implicit function of the equilibrium value of r_f . As for the equilibrium value of r_f , we note that in any unconstrained equilibrium with low effort, banks undercut each other until they make zero expected profits, that is

$$\frac{\bar{\epsilon} - \hat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_L} (1 + r_f) \frac{y(i)w}{A} + \tau \frac{\hat{e}^2(i) - \underline{\epsilon}_L^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} y(i)^{1-\theta} c^\theta = (1 + r) \frac{y(i)w}{A}$$

Using the expression for optimal output with low effort (16), we can rewrite the above condition as follows

$$\frac{(1 + r_f)(\bar{\epsilon} - \hat{e}(i))}{\bar{\epsilon} - \underline{\epsilon}_L} + \frac{\tau}{1 - \theta} \left(\frac{(1 + r_f)(\hat{e}^2(i) - \underline{\epsilon}_L^2)}{(\bar{\epsilon} + \hat{e}(i))(\bar{\epsilon} - \underline{\epsilon}_L)} - \frac{b(\hat{e}^2(i) - \underline{\epsilon}_L^2)}{\bar{\epsilon}^2 - \hat{e}^2(i)} \right) = 1 + r \quad (31)$$

which implicitly states a relationship between the equilibrium values of r_f and $\hat{e}(i)$. The equilibrium values of r_f and $\hat{e}(i)$ are found combining (30) and (31). The equilibrium values of wages, production, and consumption are then found as discussed in the technical Appendix.

For the existence of the low-effort equilibrium, it must be checked that firm's management has no incentive to deviate from low effort to high effort. The formal condition is

$$\pi(i)|_{\underline{\epsilon}_L \rightsquigarrow H} \leq \pi(i)|_{\underline{\epsilon}_L}$$

where

$$\pi(i)|_{\underline{\epsilon}_L \rightsquigarrow H} = \frac{\bar{\epsilon} - \hat{e}_H(i)}{\bar{\epsilon} - \underline{\epsilon}_H} \left(\frac{\bar{\epsilon} + \hat{e}_H(i)}{2} c^\theta y_H^{1-\theta}(i) - (1 + r_{f,H}) \frac{wy_H(i)}{A} \right)$$

is firm's expected profit if firm's management deviates from low effort to high effort (given the equilibrium values of real wage and aggregate demand), and

$$\pi(i)|_{\underline{\epsilon}_L} = \frac{\bar{\epsilon} - \hat{e}(i)}{(\bar{\epsilon} - \underline{\epsilon}_L)} \left(\frac{\bar{\epsilon} + \hat{e}(i)}{2} c^\theta y^{1-\theta}(i) - \frac{(1 + r_f)wy(i)}{A} \right) + b \frac{wy(i)}{A}$$

is the equilibrium expected profit if firm's management exerts low effort. The equations that determine $y_L(i)$, $r_{f,L}$ and $\hat{e}_L(i)$ are shown in the technical Appendix.

The analytical solutions for both the unconstrained and the constrained equilibria have been derived in the technical Appendix. They are too complicated to explore the model analytically. In addition, we have not been able to derive an analytical solution for the low effort equilibrium. Subsequently, our strategy to understand the implications of market power and bankruptcy costs for production and welfare will be based on numerical simulations. Hence, we introduce in the next section a baseline calibration of the parameters of the model and in Sections 5, 6 and 7 we conduct exercises that modify the values of key parameters to discuss their quantitative effects on output, consumption, default rates or welfare.

4 Calibration

We set the baseline calibration of the model so that the economy is in a general unconstrained equilibrium with high effort. The calibration takes into account realistic values for both the external finance premium and the probability of default, defining a model for annual observations. Table 1 reports the calibrated parameters of the model.

Table 1. Calibration of parameters

A	Labor productivity	1.00
θ	Inverse Dixit-Stiglitz elasticity	0.34
σ	Elasticity of consumption	1.25
γ	Elasticity of labor	2.00
ψ	Weight of labor disutility	0.68
r	Risk-free interest rate	0.01
b	Private benefits rate	0.03
$\bar{\epsilon}$	Upper bound shock	1.40
$\underline{\epsilon}_H$	Lower bound shock with high effort	0.60
$\underline{\epsilon}_L$	Lower bound shock with low effort	0.51
τ	Liquidation value per unit of firm revenue	0.80

Labor productivity is normalized at $A = 1.0$ so that labor and output produced are identical. The inverse of the Dixit-Stiglitz demand elasticity is set at $\theta = 0.34$. This implies that firm's dividends bring 31% of total income in the steady state, whereas labor income contributes to 66% of it.

Household preferences are modulated with an elasticity of utility with respect to consumption $\sigma = 1.25$, which is close to the common case of log preferences on consumption. The elasticity of the disutility with respect to labor is $\gamma = 2.0$ to bring a Frisch elasticity of labor supply at 0.5, which is consistent with the empirical evidence (Altonji, 1986; Domeij and Flodén, 2006). The scale parameter that captures the weight

on labor disutility in the utility function is set at $\psi = 0.68$ in order to normalize output to one in the unconstrained equilibrium with high effort. The risk-free real interest rate, r is fixed at 1%, which is a reasonable assumption for a model that delivers annual observations.

The private benefits rate is set at $b = 0.03$, which implies that firm's management exerting low effort collects 3% of the total value of the loan. The shock distribution is bounded between $\bar{\epsilon} = 1.4$ and $\underline{\epsilon}_H = 0.6$ in the case of high managerial effort, which results in an expected value of the shock equal to 1. When firm's management exerts low effort, the lower bound slips down to $\underline{\epsilon}_L = 0.51$, i.e. 85% of the value assigned under high effort. The fraction of the value of firm revenues that banks are able to recover in the event of loan default is set at 80%, i.e. $\tau = 0.8$. The equilibrium values of the endogenous variables resulting from the baseline calibration are provided in Table 2 below.

Table 2. Equilibrium values of the endogenous variables

Output, y	1.0000
Consumption, c	0.9782
Probability of default, d	0.1119
Interest rate of loans, r_f	0.0393
Critical shock, $\hat{e}(i)$	0.6896
Real wage, w	0.6585
Labor, n	1.0000
Loan value, l	0.6585
Liquidation value, lv	0.0573
Bankruptcy loss, $loss$	0.0240
Expected profit, $E(\pi(i))$	0.3131
Social welfare, U	-4.2500

The probability of default, d , is 11.19%, and the annual real interest rate of loans, r_f , is 3.93%, giving a spread with respect to the risk-free asset of 2.93%. This number is close to the average gap between the Commercial and Industrial Loan rates over the intended Federal Funds rate for the US economy in the period between 1993 and 2017, which is 2.32%.¹³ The probability of default also matches the annual rate of exit of US total private establishments in the same period, which equals 11.69%.¹⁴

¹³Source: Survey of Terms of Business Lending released by the Board of Governors of the Federal Reserve System (<http://www.federalreserve.gov/releases/e2/e2chart.htm>).

¹⁴The average rate of exit has been computed using the quarterly series of establishment deaths included in the Business Employment Dynamics report released by the Bureau of Labor Statistics (BLS).

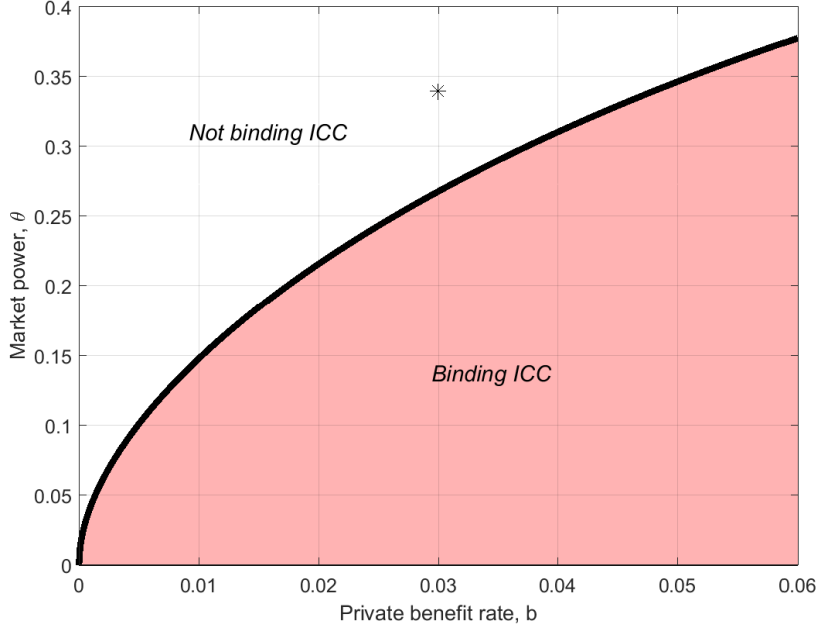


Figure 1: Constrained *versus* unconstrained equilibrium with high effort depending on θ and b .

As we mentioned before, the above solution satisfies the necessary and sufficient conditions for an unconstrained equilibrium with high effort to exist. In particular, the minimum value of θ necessary for the existence of such equilibrium is equal to 0.2678. Moreover, firm's expected profits if firm's management exerts low effort are equal to 0.3014. This is strictly lower than equilibrium expected profits, which are equal to 0.3131, making that deviation unprofitable.

5 Pervasiveness of moral hazard

In this section we analyze how, in the presence of moral hazard, the equilibrium with high effort is either constrained or not. We know from previous discussions that a necessary condition for an unconstrained general equilibrium to exist is that condition (28) holds. Conversely, if (28) does not hold, moral hazard is pervasive, and then the equilibrium with high effort is constrained.

Based on the baseline calibration of the model, we will now discuss how the pervasiveness of moral hazard depends on: (i) firms' market power, θ ; (ii) the rate of private benefits, b ; and (iii) the effect that low effort has on the variability of the shock, proxied by $(\underline{\epsilon}_H - \underline{\epsilon}_L)$. These are the model elements that mostly affect the intensity of the moral hazard problem. Figure 1 displays the combinations of market power and private benefit rate that lead to either an unconstrained or a constrained equilibrium with high effort.

Similarly, Figure 2 shows the combinations of market power and the effect that low effort has on the

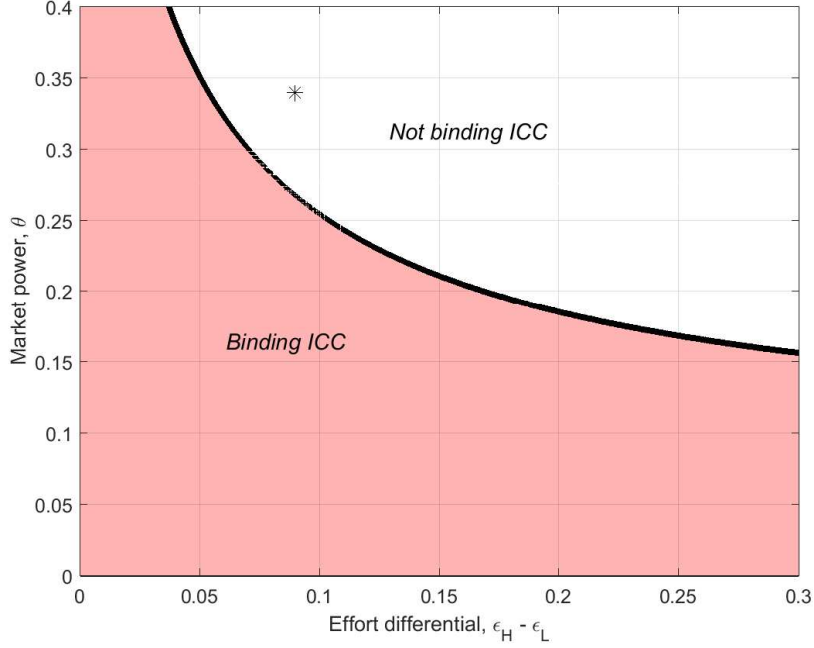


Figure 2: Constrained *versus* unconstrained equilibrium with high effort depending on θ and $\epsilon_H - \epsilon_L$.

variability of the idiosyncratic shock that lead to either an unconstrained or a constrained equilibrium. The values plotted in Figures 1 and 2 have been obtained for the baseline calibration of the model. In both graphs the symbol “*” represents the baseline calibration values for θ , b and $(\epsilon_H - \epsilon_L)$.

For each rate of private benefits, Figure 1 plots the line related to the minimum level of market power required for the equilibrium to be unconstrained. This line marks the frontier between two regions. In one region, characterized by a high θ combined with a sufficiently low b , the ICC is not binding. In the other region, characterized by a low θ combined with a sufficiently high b , the ICC is binding. Therefore, if either the private benefit rate is high enough, or the firm’s market power is low enough, the economy enters a constrained equilibrium due to moral hazard. In the first case, the high private benefits collected if firm’s management exerts low effort make moral hazard pervasive and Bertrand-type competition across banks becomes less effective. Banks optimal strategy is to keep the interest rate, r_f , above the level that would imply zero expected profits, which preserves the incentive of firm’s management to exert high effort.

A constrained equilibrium may also emerge due to a low market power. For a given level of the private benefit rate, if firms have less market power, their expected profits in case of high effort go down, and private benefits associated with low effort become more attractive. This strengthens the incentive for firm’s management to exert low effort. In this situation, banks’ optimal strategy is to keep the interest rate at a higher level. Clearly, in the absence of private benefits, $b = 0$, the unconstrained equilibrium is the only equilibrium for any degree of firm market power, as moral hazard disappears.

For each level of the difference $(\underline{\epsilon}_H - \underline{\epsilon}_L)$, Figure 2 plots the line related to the minimum level of market power required for the ICC to be not binding. Equivalently to Figure 1, this line marks the frontier between two regions. In one region, characterized by a high θ combined with a sufficiently low difference $(\underline{\epsilon}_H - \underline{\epsilon}_L)$, the ICC is not binding. In the other region, characterized by a low θ combined with a sufficiently high difference $(\underline{\epsilon}_H - \underline{\epsilon}_L)$, the ICC is binding.

While more market power results in higher expected profits, a larger difference $(\underline{\epsilon}_H - \underline{\epsilon}_L)$ implies a larger drop in firm's expected profits if firm's management exerts low effort. Therefore, the unconstrained equilibrium with high effort turns more likely both with a higher value of θ and a higher value of $(\underline{\epsilon}_H - \underline{\epsilon}_L)$. The intuition is that both large market power and large profits losses due to low effort reduce the case for moral hazard as firm's management has less incentive to exert low effort.

6 General equilibrium effects

In this section, we analyze how the macroeconomic variables react to the two sources of financial friction contemplated in our model: bankruptcy and moral hazard. Bankruptcy causes an inefficiency due to the fact that liquidation upon default results in a loss of value so long as $\tau < 1$. Moral hazard causes an inefficiency because, when it is pervasive, the equilibrium with high effort is constrained and characterized by a higher interest rate. We explore the general equilibrium effects of these two frictions for alternative values in the following structural parameters: (i) market power, θ , and; (ii) efficiency of the liquidation technology, τ .

6.1 Market power

Using the baseline calibration, Figure 3 displays the equilibrium values of the key endogenous variables of our model as θ moves from 0.20 to 0.45. We identify three regions delimited by solid vertical lines. From left to right, the first region corresponds to an unconstrained equilibrium with low managerial effort, the second region corresponds to a constrained equilibrium with high effort and pervasive moral hazard, and the third region corresponds to an unconstrained equilibrium with high managerial effort. The economy is in an unconstrained equilibrium with high effort for values of θ between 0.268 and 0.450, where firm's expected profit is maximum and the probability of default is zero for values of θ higher than 0.4 (see values to the right of the vertical dashed lines in Figure 3). In such unconstrained equilibrium, an increase in θ reduces output while firm's dividends and welfare, measured as the level of households' utility (1), continue to increase. Moreover, consumption, loan interest rate, probability of firm default, and firm's liquidation value keep falling, while bank's expected profits (bank dividends) stay at zero. The fact that welfare continues to grow, even if consumption is falling, can be explained by the reduction in disutility from labor and the lower bankruptcy losses. For values of θ between 0.258 and 0.268, the economy is in a constrained equilibrium

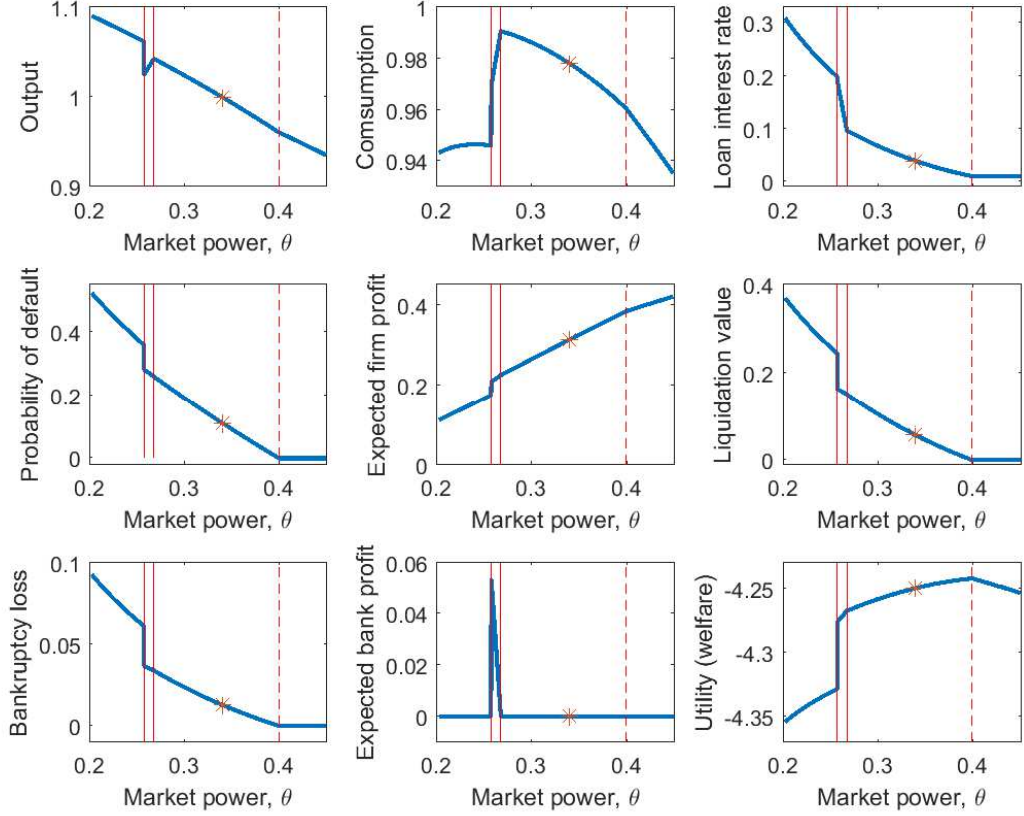


Figure 3: General equilibrium effects depending on market power, θ

with high effort. Remarkably, given that the ICC is binding, banks charge the minimum interest rate. An increase in θ increases output, consumption and welfare as the cost of borrowing, the probability of default and the bankruptcy losses keep falling. Finally, for θ smaller than 0.258, the economy is an unconstrained equilibrium where firm's management prefers to exert low effort.

Low market power may lead to an equilibrium with low effort. This region of low effort equilibrium, plotted in Figure 3 for values of $\theta \in [0.200, 0.258]$, is characterized by a relatively high level of output and the lowest levels of consumption and welfare. Output is relatively high due to the lower level of market power as well as to the fact that firm's management enjoys private benefits, which positively depend on output. Consumption falls significantly as the default rate soars and dividends are substantially reduced. Household utility (welfare) is the lowest of the three possible equilibria due to low consumption and high labor supply.

As Figure 3 shows, increasing market power brings a hump-shaped response of consumption. Unlike in a frictionless economy, market power has a beneficial impact because it mitigates the effects of financial frictions. Conditional on high effort, more market power means higher expected profits, a lower probability of

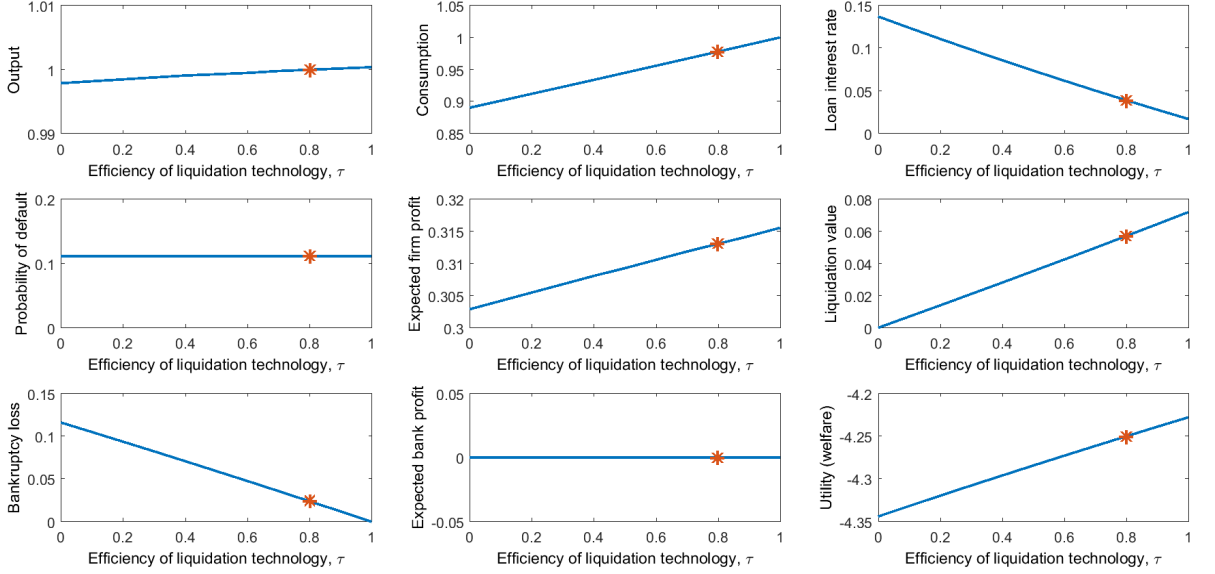


Figure 4: General equilibrium effects depending on liquidation technology, τ

default and, more importantly, a lower incidence of moral hazard. If this effect is greater than the efficiency loss from firm's mark-up, a higher market power may turn socially desirable because the reduction of output may be lower than the reduction of bankruptcy losses. Figure 3 indicates that the value of θ that yields the highest level of welfare is 0.4. Such optimal value of θ might change with alternative parameterizations of the model, as discussed in Section 7.

6.2 Effects of liquidation technology

We now turn to the analysis of how the efficiency of the banks' liquidation technology affects the economy. Figure 4 shows the behavior of different macroeconomic aggregates for $\tau \in [0, 1]$. The case in which $\tau = 0$ refers to an economy in which banks are unable to recover any of the loan value in the event of firm's bankruptcy. The case in which $\tau = 1$ refers to an economy in which liquidation upon bankruptcy is fully efficient and no value is lost in the liquidation following default, so that the bank collects all the firm revenue. For values of $\tau \in (0, 1)$, keeping the baseline calibration values for the other parameters, the economy is always in an unconstrained equilibrium with high effort. The values associated with the baseline calibration ($\tau = 0.8$) are marked with an asterisk "*" in Figure 4.

As we can see, the liquidation value grows with τ . A more efficient liquidation technology means that, in the event of bankruptcy, less value is lost in the liquidation process, and banks are able to recover a larger part of the value of the loan they issued to the bankrupt firm. Accordingly, the loan interest rate falls with τ . Note, also, that both consumption and output grow in τ . The increase in output is due to the fact that

a better liquidation technology means lower interest rates and therefore a lower marginal costs for firms, which induces them to produce more. Meanwhile, consumption rises because of the increase in both labor income and firm dividends collected by the households. Figure 4 also shows that welfare increases with the efficiency of liquidation, and its maximum value is attained at the fully efficient liquidation technology ($\tau = 1.0$). Note that the positive effect of higher consumption on the utility dominates the negative effect of higher labor supply.

7 Welfare analysis

In this section, we evaluate how market power and the efficiency of liquidation in the event of default affect welfare. Figure 5 displays the welfare costs for alternative values of θ (across horizontal axis) and τ (across plots), with vertical lines defining the three aforementioned equilibrium regions. Welfare costs are measured in terms of the percentage of forgone consumption with respect to the benchmark economy in which liquidation is efficient, $\tau = 1$, and household utility reaches its maximum value. Table 3 reports these welfare costs for several values of market power, θ , and efficiency of liquidation, τ . For the baseline calibration values of $\theta = 0.34$ and $\tau = 0.8$, the welfare cost is 3.34% of output. Table 4 shows the value of θ that minimizes the welfare costs for a given value of τ . Welfare costs behave differently with respect to θ , depending on τ . As it can be seen from Figure 5, and from Tables 3 and 4:

1. For values of $\tau < 0.837$, the welfare cost is minimum at the market power value of $\theta = 0.4$ (see dashed vertical line in Figure 5), which implies a probability of firm default equal to 0 and, therefore, an interest rate of the loan equal to the risk-free interest rate. Reducing market power (moving to the left in the plots of Figure 5 and in Table 3) decreases welfare because of the reduction in consumption that comes with higher firm default and borrowing costs. Increasing market power (moving to the right in the plots of Figure 5 and in Table 3) also decreases welfare because of the reduction in consumption resulting from a higher firm mark-up and a lower labor income. However, for values of $\tau \in [0.837, 1]$, the minimum welfare cost is not found at the value of θ that eliminates firm default. In this case, having less market power is socially desirable even though it brings some bankruptcies. In particular, welfare cost first decreases and then increases, so that there exist a unique value of optimal market power. For example, when $\tau = 0.9$ welfare is maximized for $\theta = 0.341$ as it can be seen in Figure 5 and Table 4. When liquidation technology is highly efficient, the trade-off between the welfare gain from increasing competition and the welfare loss from higher default rates favours the latter. As a special case, for $\tau = 1$, the optimal market power is at the corner point between the unconstrained and the constrained equilibrium with high effort (see Figure 5). Therefore, welfare is maximized at the minimum level of market power required to prevent the pervasiveness of moral hazard. The

corresponding value of $\theta = 0.274$ brings a global optimum level of market power and the welfare cost is null.

2. For any given value of $\theta < 0.4$, welfare costs are a decreasing function of τ . Lower values of τ mean less efficient liquidation, which entails higher welfare costs.¹⁵ Once market power is high enough to eliminate firm default ($\theta \geq 0.4$), the welfare cost remains the same at different values of τ .

Table 3. Welfare cost of financial frictions (% of output)

	θ					
	0.20	0.26	0.30	0.34	0.40	0.45
τ	0.75	17.61	7.24	5.28	3.91	2.49
	0.80	14.59	5.91	4.30	3.34	2.49
	0.85	11.50	9.32	3.32	2.77	2.49
	0.90	8.34	7.14	2.34	2.20	2.49
	0.95	5.10	4.93	1.35	1.63	2.49
	1.00	1.79	2.69	0.36	1.06	2.49

Table 4. Optimal market power and efficiency of liquidation

	θ^*	Welfare cost (% of output)
$\tau = 0.75$	0.400	2.49
$\tau = 0.80$	0.400	2.49
$\tau = 0.85$	0.396	2.48
$\tau = 0.90$	0.341	2.20
$\tau = 0.95$	0.273	1.28
$\tau = 1.00$	0.274	0.00

The economic interpretation of the observed relationship between welfare cost and market power is the following. A higher level of market power implies higher expected firm's profits and therefore (i) makes exerting low effort less attractive for firm's management, and; (ii) reduces the probability of default. Accordingly, market power helps preventing moral hazard and reduces bankruptcy losses, where the latter effect is weaker the higher is the value of τ , vanishing completely for $\tau = 1$. Hence, with a fully-efficient liquidation technology ($\tau = 1$), the only positive effect of market power is to prevent moral hazard. This is

¹⁵There is a special case reported in Table 3. For $\theta = 0.26$, welfare cost rises as τ changes from 0.80 to 0.85. This breaks down the continuity in the welfare gains of a more efficient liquidation technology. The reason of this special case is that the equilibrium of the economy switches from unconstrained to constrained due to pervasive moral hazard. This shows that lower bankruptcy costs bring an incentive for firms to deviate to low effort. As condition (28) implies, a higher τ would require a lower private benefit rate b to prevent pervasive moral hazard.

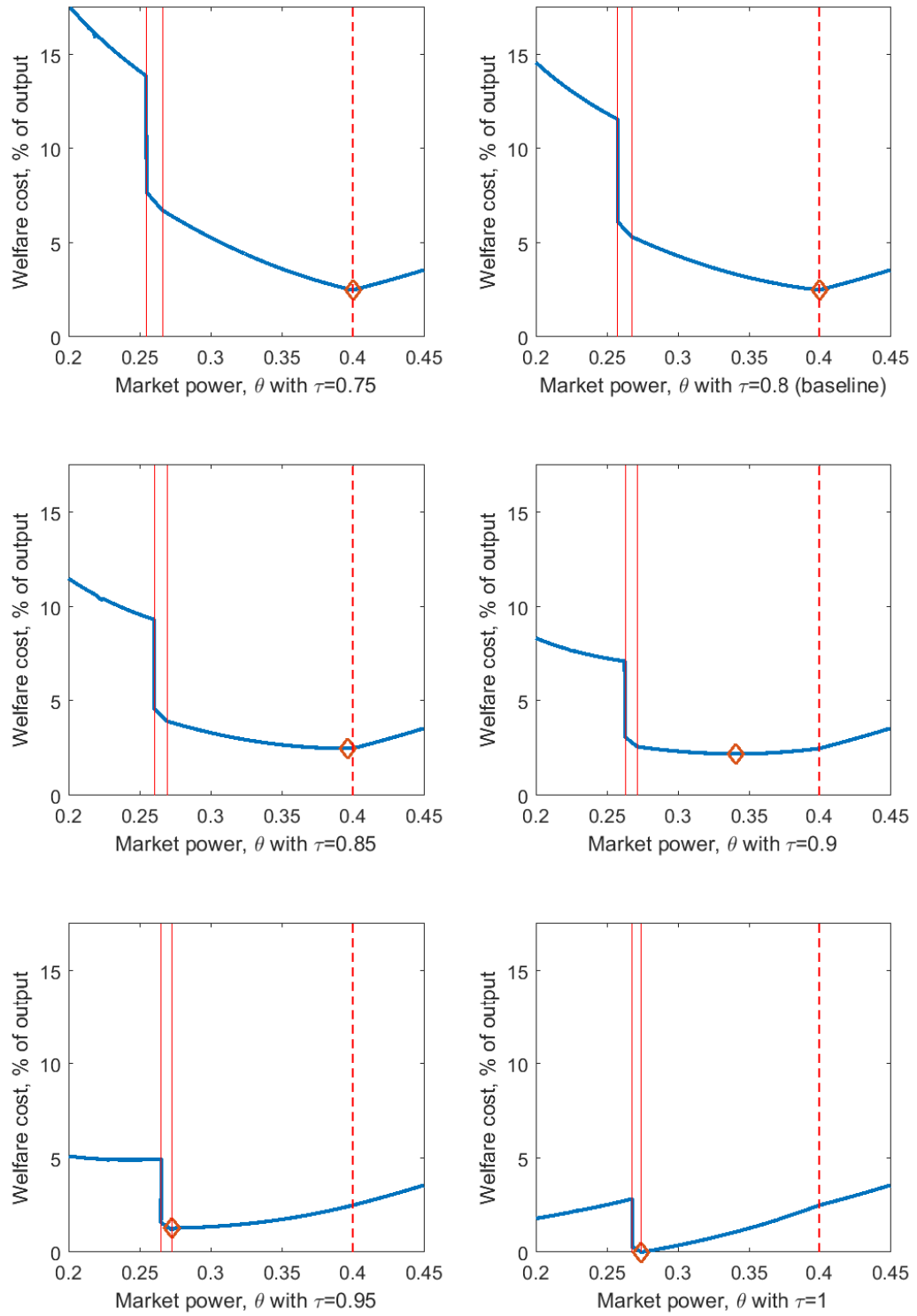


Figure 5: Welfare cost of financial frictions

the reason why the optimal value of market power in this case is equal to 0.274, which is the minimum level of θ necessary to sustain an unconstrained equilibrium with high effort. As τ goes down, market power also contributes positively to welfare by reducing losses related to bankruptcy. This explains why the optimal value of market power increases above 0.274 to reach the no-default case when $\theta = 0.4$. If liquidation technology brings a severe bankruptcy loss, the socially desirable market power is large at $\theta = 0.4$.

In all cases, the optimal degree of market power is the one that balances the positive effect of a higher θ , due to the mitigation of moral hazard and the reduction of bankruptcy costs, with the standard negative effect that results from a higher firm's mark-up and lower production compared to a perfectly competitive market.

8 Conclusions

We developed a static general equilibrium model with monopolistically competitive producers and financial frictions due to the non verifiability of firms' managerial decisions and of ex post profitability. Our analysis shows that, aside from the standard negative effect, which adversely affects welfare, market power mitigates financial frictions. The resulting trade off between benefits and costs of increasing competition is not trivial. We find that, generally, the presence of financial frictions might require some degree of market power in order to maximize welfare and prevent pervasive moral hazard. Deviations from such optimal degree of market power may have significant welfare effects. For example, the welfare cost of financial frictions in the calibrated model amounts to a 3.34% loss of output, associated with a firm's default rate of 11% and a loan interest rate almost 3% above the risk-free rate. As documented in the paper, this cost increases substantially when the economy faces pervasive moral hazard or when the efficiency of liquidation is low. Therefore, in economies with underdeveloped financial systems, in which financial frictions are relevant, promoting competition in the production sector might be detrimental to macroeconomic performance. In order to benefit from competition, these economies should undertake reforms, including that of the legal system, that foster the development of financial institutions to reduce moral hazard and make firm liquidation more efficient.

Our setup could also be extended to dynamic frameworks to analyze how the interplay of market power and financial frictions affects the entry and exit decisions of firms along the business cycle, as well as the long run growth process.

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Technical Appendix

1. Optimal choice of the amount consumed of variety i

Households choose $c(i)$ by solving the following maximization problem:

$$\begin{aligned} \max_{\{c(i)\}} \quad & c = \left[\int_0^1 e(i) c(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \\ \text{s.to:} \quad & pc = \int_0^1 p(i) c(i) di \end{aligned}$$

The associated first order conditions are (being λ the Lagrange multiplier):

$$\begin{aligned} c^\theta e(i) c(i)^{-\theta} &= \lambda p(i) \quad \forall i \in [0, 1] \\ pc &= \int_0^1 p(i) c(i) di \end{aligned}$$

which leads to (4) using the price index (5).

2. Income equation (overall resources constraint)

This section derives the overall resources constraint in the three possible equilibria of the model. The exercise begins at the household budget constraint

$$(1+r)wn + d_f + d_b = c \tag{A1}$$

and substitutes equilibrium expressions for both firm dividend, d_f , and bank dividend, d_b , to obtain an equation that relates the sources of income to the uses of income. As this equation comes in aggregate terms, it can be identified as the overall resources constraint.

Unconstrained equilibrium with high effort

The unconstrained equilibrium with high effort results in null bank dividends, $d_b = 0$, and the following amount of firm dividends (using symmetric firm behavior for production, $y(i) = y$)

$$d_f = \frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y^{1-\theta} - \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1+r_f) w \frac{y}{A}$$

Inserting both d_b and d_f in the household budget constraint, (A1), gives

$$(1+r)wn + \frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y^{1-\theta} - \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1+r_f) wn = c \tag{A2}$$

Bertrand-type competition leads to the equilibrium interest rate of the loan

$$(1+r_f) = \frac{\bar{\epsilon} - \underline{\epsilon}_H}{\bar{\epsilon} - \widehat{e}(i)} \left(1 + r - \frac{lv}{l} \right)$$

which can be inserted in (A2) to obtain

$$\frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y^{1-\theta} - \frac{lv}{l} wn = c \quad (\text{A3})$$

Since the aggregate loan size is equal to the effective labor cost, $l = wn$, we can rewrite (A3) as the following overall resources constraint

$$\frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} y \left(\frac{c}{y} \right)^\theta + lv = c$$

The sources of income are the revenue from firms that survive and the liquidation value from firms that go bankrupt. All income is spent on consumption goods.

Constrained equilibrium with high effort (pervasive moral hazard)

The constrained equilibrium occurs when the ICC is binding and the interest rate of the loan set by the banks is at

$$r_f = \frac{b(1-\theta)(\bar{\epsilon} - \underline{\epsilon}_H)(\bar{\epsilon} - \underline{\epsilon}_L)}{(\bar{\epsilon} - \widehat{\epsilon}(i))(\underline{\epsilon}_H - \underline{\epsilon}_L)^\theta} - 1$$

which turns higher than the one that would result from Bertrand-type competition. The analytical solution for the critical value of the idiosyncratic shock, $\widehat{\epsilon}(i)$, relies on its definition, (10), and the first order condition of the firm, (12), which lead to

$$\widehat{\epsilon}(i) = \frac{(1-\theta)}{(1+\theta)} \bar{\epsilon},$$

and this solution for the interest rate of the constrained equilibrium.

$$r_f = \frac{b(1-\theta)(1+\theta)(\bar{\epsilon} - \underline{\epsilon}_H)(\bar{\epsilon} - \underline{\epsilon}_L)}{2\theta^2 \bar{\epsilon}(\underline{\epsilon}_H - \underline{\epsilon}_L)} - 1$$

Subsequently, bank profits are positive and the dividend collected by the household is

$$d_b = \frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1 + r_f) l + lv - (1 + r) l \quad (\text{A4})$$

Taking the household budget constraint, $c = (1 + r) wn + d_f + d_b$, and inserting the bank dividend, (A4), we get

$$(1 + r) wn + d_f + \left(\frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1 + r_f) - (1 + r) \right) l + lv = c, \quad (\text{A5})$$

where using $l = wn$ yields

$$d_f + \left(\frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1 + r_f) - (1 + r) \right) l + lv = c$$

Firm dividend is

$$d_f = \frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y^{1-\theta} - \frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1 + r_f) w \frac{y}{A} \quad (\text{A6})$$

The optimal amount of output is given by

$$y(i) = \left(\frac{(1-\theta) A (\bar{\epsilon} + \widehat{\epsilon}(i))}{2w(1+r_f)} \right)^{1/\theta} c$$

Using the symmetric equilibrium, $y(i) = y$, and solving for $(c/y)^\theta$ brings

$$(c/y)^\theta = \frac{2w(1+r_f)}{(1-\theta)A(\bar{\epsilon} + \widehat{e}(i))} \quad (\text{A7})$$

Plugging (A7) in (A6) and putting terms together on $w\frac{y}{A}$, it is obtained

$$d_f = \frac{\theta}{1-\theta} \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1+r_f) w \frac{y}{A}$$

which can be substituted in (A5) to get, after some simplifying manipulations,

$$\frac{1}{1-\theta} \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}} (1+r_f) w \frac{y}{A} + lv = c$$

It can be noticed that (A7) implies $\frac{(1+r_f)w}{(1-\theta)A} = \left(\frac{c}{y}\right)^\theta \frac{(\bar{\epsilon} + \widehat{e}(i))}{2}$, which can be used in the previous expression to reach

$$\frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon})} y \left(\frac{c}{y}\right)^\theta + lv = c$$

that is an identical expression to the one found in the unconstrained general equilibrium.

Equilibrium with low effort

Bank competition *a la* Bertrand leads to zero profits in an equilibrium with low effort. Thus, bank dividend is null, $d_b = 0$. As for firm profit, revenues are negatively affected by the lower expected value of the idiosyncratic shock ($\underline{\epsilon}_L < \underline{\epsilon}_H$) and a private benefit is collected as a fraction b of the size of the loan. This brings the following firm dividend

$$d_f = \frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y^{1-\theta} - \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_L} (1+r_f) w \frac{y}{A} + bl$$

where using the equilibrium conditions, $l = wn = w\frac{y}{A}$, we have

$$d_f = \frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y^{1-\theta} - \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_L} \left(1 + r_f - b \frac{\bar{\epsilon} - \underline{\epsilon}_L}{\bar{\epsilon} - \widehat{e}(i)}\right) w \frac{y}{A} \quad (\text{A8})$$

Plugging (A8) and $d_b = 0$ in the household budget constraint (1) yields

$$(1+r)wn + \frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y^{1-\theta} - \frac{\bar{\epsilon} - \widehat{e}(i)}{\bar{\epsilon} - \underline{\epsilon}_L} \left(1 + r_f - b \frac{\bar{\epsilon} - \underline{\epsilon}_L}{\bar{\epsilon} - \widehat{e}(i)}\right) wn = c \quad (\text{A9})$$

The equilibrium interest rate of the loan that results from Bertrand-style competition is

$$(1+r_f) = \frac{\bar{\epsilon} - \underline{\epsilon}_L}{\bar{\epsilon} - \widehat{e}(i)} \left(1 + r - \frac{lv}{l}\right)$$

which can be inserted in (A9) to obtain

$$(1+r)wn + \frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y^{1-\theta} - \left(1 + r - \frac{lv}{l} - b\right) wn = c$$

and cancelling the $(1+r)wn$ terms simplifies to

$$\frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y^{1-\theta} + \left(\frac{lv}{l} + b \right) wn = c \quad (\text{A10})$$

Using $l = wn$ in (A10), the overall resources constraint becomes

$$\frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} c^\theta y^{1-\theta} + lv + bl = c$$

which shows 3 sources of income: revenue from surviving firms, liquidation value of defaulting firms and the private benefit.

3. Set of equations defining equilibrium with either high effort

General equilibrium with high effort

Optimal output	$y(i) = \left(\frac{(1-\theta)A(\bar{\epsilon} + \widehat{\epsilon}(i))}{2w(1+r_f)} \right)^{1/\theta} c$
Aggregate output	$y = y(i)$
Critical shock	$\widehat{\epsilon}(i) = \max \left\{ \underline{\epsilon}_H, (1+r_f) \frac{w}{A} y(i)^\theta c^{-\theta} \right\}$
Interest rate of the loan	$1+r_f = \max \left\{ \frac{\bar{\epsilon} - \underline{\epsilon}_H}{\bar{\epsilon} - \widehat{\epsilon}(i)} \left(1+r - \frac{lv}{l} \right), \frac{b(1-\theta)(1+\theta)(\bar{\epsilon} - \underline{\epsilon}_H)(\bar{\epsilon} - \underline{\epsilon}_L)}{2\theta^2 \bar{\epsilon}(\underline{\epsilon}_H - \underline{\epsilon}_L)} \right\}$
Liquidation value	$lv = \tau \frac{\bar{\epsilon}^2(i) - \underline{\epsilon}_L^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} y^{1-\theta} c^\theta$
Loan	$l = wn$
Production	$y = An$
Labor supply	$w = \frac{\psi n^\gamma c^\sigma}{1+r}$
Firm dividend	$d_f = \frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y(i)^{1-\theta} - \frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{(\bar{\epsilon} - \underline{\epsilon}_H)} (1+r_f) wn$
Bank dividend	$d_b = \max \left\{ 0, \frac{\bar{\epsilon} - \widehat{\epsilon}(i)}{\bar{\epsilon} - \underline{\epsilon}_H} (1+r_f) l + lv - (1+r) l \right\}$
Overall resources constraint	$\frac{\bar{\epsilon}^2 - \widehat{\epsilon}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y^{1-\theta} + lv = c$
Critical shock if deviating to low effort	$\widehat{\epsilon}_L(i) = (1+r_{f,L}) \frac{w}{A} y_L(i)^\theta c^{-\theta}$
Interest rate if deviating to low effort	$1+r_{f,L} = \frac{\bar{\epsilon} - \underline{\epsilon}_L}{\bar{\epsilon} - \widehat{\epsilon}_L(i)} \left(1+r - \frac{lv_L(i)}{l_L(i)} \right)^{1/\theta}$
Output if deviating to low effort	$y_L(i) = \left(\frac{(1-\theta)A(\bar{\epsilon} + \widehat{\epsilon}_L(i))}{2w(1+r_{f,L} - b \frac{\bar{\epsilon} - \underline{\epsilon}_L}{\bar{\epsilon} - \widehat{\epsilon}_L(i)})} \right)^{1/\theta} c$
Liquidation value if deviating to low effort	$lv_L(i) = \tau \frac{\bar{\epsilon}_L^2(i) - \underline{\epsilon}_L^2}{2(\bar{\epsilon} - \underline{\epsilon}_L)} y_L(i)^{1-\theta} c^\theta$
Loan value if deviating to low effort	$l_L(i) = w \frac{y_L(i)}{A}$

The set of 16 equations may provide a numerical solution for the 16 endogenous variables: $y(i)$, y , $\widehat{\epsilon}(i)$, r_f , c , w , n , lv , l , d_f , d_b , $y_L(i)$, $lv_L(i)$, $l_L(i)$, $\widehat{\epsilon}_L(i)$ and $r_{f,L}$. It should be noticed that the list of equations contemplates both the constrained and the unconstrained equilibrium cases as characterized by the values obtained for the interest rate of the loans, r_f , and the bank dividend, d_b .

For the existence of the high effort equilibrium, firms should not have any gain if deviating to low effort, which implies the need for satisfying this incentive constraint

$$\pi(i)|_{\underline{\epsilon}_H \rightsquigarrow L} \leq \pi(i)|_{\underline{\epsilon}_H}$$

where

$$\begin{aligned}\pi(i)|_{\underline{\epsilon}_H} &= \frac{\bar{\epsilon}-\hat{\epsilon}(i)}{\bar{\epsilon}-\underline{\epsilon}_H} \left(\frac{\bar{\epsilon}+\hat{\epsilon}(i)}{2} c^\theta y^{1-\theta}(i) - (1+r_f) \frac{wy(i)}{A} \right) \\ \pi(i)|_{\underline{\epsilon}_H \rightsquigarrow L} &= \frac{\bar{\epsilon}-\hat{\epsilon}_L(i)}{(\bar{\epsilon}-\underline{\epsilon}_L)} \left(\frac{\bar{\epsilon}+\hat{\epsilon}_L(i)}{2} c^\theta y_L^{1-\theta}(i) - \frac{(1+r_{f,L})wy_L(i)}{A} \right) + b \frac{wy_L(i)}{A}\end{aligned}$$

4. Set of equations defining equilibrium with low effort

General equilibrium in the model with low effort

Optimal output	$y(i) = \left(\frac{(1-\theta)A(\bar{\epsilon}+\hat{\epsilon}(i))}{2w(1+r_f-b\frac{\bar{\epsilon}-\underline{\epsilon}_L}{\bar{\epsilon}-\hat{\epsilon}(i)})} \right)^{1/\theta} c$
Aggregate output	$y = y(i)$
Critical shock	$\hat{\epsilon}(i) = \text{Max} \left\{ \underline{\epsilon}_L, (1+r_f) \frac{w}{A} y^\theta c^{-\theta} \right\}$
Interest rate of the loan	$1+r_f = \frac{\bar{\epsilon}-\underline{\epsilon}_L}{\bar{\epsilon}-\hat{\epsilon}(i)} \left(1+r - \frac{lv}{l} \right)$
Liquidation value	$lv = \tau \frac{\hat{\epsilon}^2(i)-\underline{\epsilon}_L^2}{2(\bar{\epsilon}-\underline{\epsilon}_L)} y^{1-\theta} c^\theta$
Loan	$l = wn$
Production	$y = An$
Labor supply	$w = \frac{\psi n^\gamma c^\sigma}{1+r}$
Firm dividend	$d_f = \frac{\bar{\epsilon}^2-\hat{\epsilon}^2(i)}{2(\bar{\epsilon}-\underline{\epsilon}_L)} c^\theta y^{1-\theta} - \frac{\bar{\epsilon}-\hat{\epsilon}(i)}{(\bar{\epsilon}-\underline{\epsilon}_L)} (1+r_f) wn + bl$
Bank dividend	$d_b = 0$
Overall resources constraint	$\frac{\bar{\epsilon}^2-\hat{\epsilon}(i)^2}{2(\bar{\epsilon}-\underline{\epsilon}_L)} c^\theta y^{1-\theta} + lv + bl = c$
Critical shock if deviating to high effort	$\hat{\epsilon}_H(i) = (1+r_{f,H}) \frac{w}{A} y_H(i)^\theta c^{-\theta}$
Interest rate if deviating to high effort	$1+r_{f,H} = \max \left\{ \frac{\bar{\epsilon}-\underline{\epsilon}_H}{\bar{\epsilon}-\hat{\epsilon}_H(i)} \left(1+r - \frac{lv_H(i)}{l_H(i)} \right), \frac{b(1-\theta)(1+\theta)(\bar{\epsilon}-\underline{\epsilon}_H)(\bar{\epsilon}-\underline{\epsilon}_L)}{2\theta^2 \bar{\epsilon}(\underline{\epsilon}_H-\underline{\epsilon}_L)} \right\}$
Output if deviating to high effort	$y_H(i) = \left(\frac{(1-\theta)A(\bar{\epsilon}+\hat{\epsilon}_H(i))}{2w(1+r_{f,H})} \right)^{1/\theta} c$
Liquidation value if deviating to high effort	$lv_H(i) = \tau \frac{\hat{\epsilon}_H^2(i)-\underline{\epsilon}_H^2}{2(\bar{\epsilon}-\underline{\epsilon}_H)} y_H(i)^{1-\theta} c^\theta$
Loan value if deviating to high effort	$l_H(i) = w \frac{y_H(i)}{A}$

The set of 16 equations may provide a numerical solution for the 16 endogenous variables: $y(i)$, y , $\hat{\epsilon}(i)$, r_f , c , w , n , lv , l , d_f , d_b , $y_H(i)$, $r_{f,H}$, $\hat{\epsilon}_H(i)$, $lv_H(i)$ and $l_H(i)$.

For the existence of the low effort equilibrium, firms should not have any gain if deviating to high effort, which implies the need for satisfying this incentive constraint

$$\pi(i)|_{\underline{\epsilon}_L \rightsquigarrow H} \leq \pi(i)|_{\underline{\epsilon}_L}$$

where

$$\begin{aligned}\pi(i)|_{\underline{\epsilon}_L} &= \frac{\bar{\epsilon}-\hat{\epsilon}(i)}{(\bar{\epsilon}-\underline{\epsilon}_L)} \left(\frac{\bar{\epsilon}+\hat{\epsilon}(i)}{2} c^\theta y^{1-\theta}(i) - \frac{(1+r_f)wy(i)}{A} \right) + b \frac{wy(i)}{A} \\ \pi(i)|_{\underline{\epsilon}_L \rightsquigarrow H} &= \frac{\bar{\epsilon}-\hat{\epsilon}_H(i)}{\bar{\epsilon}-\underline{\epsilon}_H} \left(\frac{\bar{\epsilon}+\hat{\epsilon}_H(i)}{2} c^\theta y_H^{1-\theta}(i) - (1+r_{f,H}) \frac{wy_H(i)}{A} \right)\end{aligned}$$

5. Analytical solutions

This section derives the analytical solution for the endogenous variables r_f , y , c , w , n , l , and for household welfare in the equilibrium with high effort (both unconstrained and constrained due to pervasive moral hazard). For the equilibrium with low effort, no analytical solution can be provided.

Unconstrained equilibrium with high effort

First, we take the equation (24) for the interest rate of the loan, and plug the analytical solution of $\widehat{e}(i)$ from equation (23) to obtain, after some algebra, the following equilibrium value

$$1 + r_f = \frac{(1 - \theta) \bar{\epsilon} (\bar{\epsilon} - \underline{\epsilon}_H)}{\frac{2\theta(1-\theta)}{(1+\theta)} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)} \bar{\epsilon}^2 - (1 + \theta) \underline{\epsilon}_H^2 \right)} (1 + r) \quad (\text{A11})$$

The analytical solution (A11) determines the risk premium as a mark-up over the interest payment of a risk-free asset. Such risk-premium depends on four model parameters: θ , $\bar{\epsilon}$, $\underline{\epsilon}_H$, and τ . As it can be observed, a more efficient liquidation technology (higher τ) reduces the risk premium and the interest rate of the loan.

Second, we solve for the equilibrium value of output. The overall resources constraint derived above in this technical Appendix is

$$\frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} c^\theta y^{1-\theta} + lv = c$$

which indicates that the sum of the income obtained by the firms that repay the loan, plus the liquidation value of defaulting firms, is equal to the total spending of households on consumption goods. Inserting (20), with $y(i) = y$, gives (after some rearrangements)

$$\frac{\bar{\epsilon}^2 - \widehat{e}(i)^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} + \tau \frac{\widehat{e}^2(i) - \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} = \left(\frac{c}{y} \right)^{1-\theta}$$

which implies this expression for the consumption-output ratio

$$c_y \equiv \frac{c}{y} = \left(\frac{\bar{\epsilon}^2 - (1 - \tau) \widehat{e}(i)^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{1}{1-\theta}}$$

Plugging (23) leads to the following analytical solution for c_y

$$c_y = \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2} \right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{1}{1-\theta}} \quad (\text{A12})$$

which will be found very convenient below.

The analytical solution of output, y , is found combining the labor supply function, the production function, the consumption-output ratio, and the optimal output equation. Inserting the production function $y = An$, to replace n in the labor supply function (6), yields

$$w = \frac{\psi \left(\frac{y}{A} \right)^\gamma c^\sigma}{1 + r}$$

Consumption is linked to output through the c_y ratio obtained in (A12), which can be used in the previous expression to obtain

$$w = \frac{\psi \left(\frac{y}{A}\right)^\gamma (c_y y)^\sigma}{1+r} \quad (\text{A13})$$

The optimal choice of output (13) implies

$$(c_y)^\theta = \frac{2w}{(1-\theta) A (\bar{\epsilon} + \widehat{\epsilon}(i))}$$

where inserting (23) for the critical shock $\widehat{\epsilon}(i)$, and the real wage w obtained in (A13) simplifies to

$$(c_y)^{\theta-\sigma} = \frac{(1+\theta)}{(1-\theta)} \frac{\psi y^{\gamma+\sigma} (1+r_f)}{A^{1+\gamma} \bar{\epsilon} (1+r)}$$

Solving the previous expression for output, we get

$$y = \left(\frac{(1-\theta)}{(1+\theta)} \frac{A^{1+\gamma} \bar{\epsilon} (1+r) (c_y)^{\theta-\sigma}}{\psi (1+r_f)} \right)^{\frac{1}{\gamma+\sigma}} \quad (\text{A14})$$

Finally, substituting for c_y and $1+r_f$ in (A14), using respectively (A12) and (A11), we obtain

$$y = \left(\frac{A^{1+\gamma}}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta-\sigma}{1-\theta}} \frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - \underline{\epsilon}_H^2 \right)}{(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{1}{\gamma+\sigma}} \quad (\text{A15})$$

Third, we can easily obtain the analytical solution of consumption by recalling the relationship $c_y = c/y$, and plugging both (A15) and (A12), to have (after simplification)

$$c = \left(\frac{A^{1+\gamma}}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta+\gamma}{1-\theta}} \frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - \underline{\epsilon}_H^2 \right)}{(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{1}{\gamma+\sigma}}$$

Fourth, the analytical solution of the real wage comes from the labor supply curve (A13), using (A14) to replace output and (A12) to replace c_y to obtain (after some simplification)

$$w = \frac{A}{(1+r)(1+\theta)} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta}{1-\theta}} \frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - (1+\theta) \underline{\epsilon}_H^2 \right)}{(\bar{\epsilon} - \underline{\epsilon}_H)} \quad (\text{A16})$$

Fifth, the analytical solution for labor can be rapidly reached from the linear production technology, $n = y/A$, and substituting the analytical solution for output, (A15) to reach

$$n = \left(\frac{A^{1+\sigma}}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta-\sigma}{1-\theta}} \frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - \underline{\epsilon}_H^2 \right)}{(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{1}{\gamma+\sigma}} \quad (\text{A17})$$

Sixth, the size of the loan is the product between the real wage and the amount of labor employed, $l = wn$. Using both (A16) and (A17) gives (after some algebra)

$$l = \frac{A^{1+\frac{1+\sigma}{\gamma+\sigma}}}{(1+r)(1+\theta)\psi^{\frac{1}{\gamma+\sigma}}} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta}{1-\theta} + \frac{\theta-\sigma}{1-\theta} \frac{1}{\gamma+\sigma}} \left(\frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - \underline{\epsilon}_H^2 \right)}{(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{1 + \frac{1}{\gamma+\sigma}}$$

Seventh, household welfare can be expressed in its analytical form as well, by plugging the solutions of both c and n in the utility function to obtain

$$U = \frac{\left(\frac{A^{1+\gamma}}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta+\gamma}{1-\theta}} \frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - \underline{\epsilon}_H^2 \right)}{(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{1-\sigma}{\gamma+\sigma}}}{1 - \sigma} - \psi \frac{\left(\frac{A^{1+\sigma}}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta-\sigma}{1-\theta}} \frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - \underline{\epsilon}_H^2 \right)}{(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{1+\gamma}{\gamma+\sigma}}}{1 + \gamma r}$$

Constrained equilibrium with high effort

First, as discussed in Subsection 3.2 of the paper, the equilibrium interest rate is given by (29).

Second, for the analytical solution of output, we follow the same steps as in the unconstrained case to obtain

$$y = \left(\frac{(1-\theta)}{(1+\theta)} \frac{A^{1+\gamma} \bar{\epsilon} (1+r) (c_y)^{\theta-\sigma}}{\psi(1+r_f)} \right)^{\frac{1}{\gamma+\sigma}}$$

where inserting (29) for the loan interest rate and (A12) for the consumption to output ratio results in¹⁶

$$y = \left(\frac{A^{1+\gamma} (1+r)}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta-\sigma}{1-\theta}} \frac{2\bar{\epsilon}^2 \theta^2 (\underline{\epsilon}_H - \underline{\epsilon}_L)}{b(1+\theta)^2 (\bar{\epsilon} - \underline{\epsilon}_H) (\bar{\epsilon} - \underline{\epsilon}_L)} \right)^{\frac{1}{\gamma+\sigma}} \quad (\text{A18})$$

Third, for the analytical solution of consumption, we can take $c = c_y y$ use the analytical expression of output (A18) and the value of c_y implied by (A12), to reach (after some simplification)

$$c = \left(\frac{A^{1+\gamma} (1+r)}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2}\right) \bar{\epsilon}^2 - \tau \underline{\epsilon}_H^2}{2(\bar{\epsilon} - \underline{\epsilon}_H)} \right)^{\frac{\theta+\gamma}{1-\theta}} \frac{2\bar{\epsilon}^2 \theta^2 (\underline{\epsilon}_H - \underline{\epsilon}_L)}{b(1+\theta)^2 (\bar{\epsilon} - \underline{\epsilon}_H) (\bar{\epsilon} - \underline{\epsilon}_L)} \right)^{\frac{1}{\gamma+\sigma}}$$

¹⁶The ratio of output in the unconstrained equilibrium (??) to output in the constrained equilibrium (??) is

$$\frac{\frac{2\theta(1-\theta)}{(1+\theta)^2} \bar{\epsilon}^2 + \frac{\tau}{2} \left(\frac{(1-\theta)^2}{(1+\theta)^2} \bar{\epsilon}^2 - \underline{\epsilon}_H^2 \right)}{\frac{2(1+r) \bar{\epsilon}^2 \theta^2 (\underline{\epsilon}_H - \underline{\epsilon}_L)}{b(1+\theta)^2 (\bar{\epsilon} - \underline{\epsilon}_L)}}$$

which raises with both τ and b .

Fourth, regarding the real wage, w , we follow the same steps as in the unconstrained equilibrium. Hence, we take the labor supply curve (A13) and bring the value of c_y given in (A12) to replace consumption, c . This gives

$$w = \frac{\psi \left(\frac{y}{A} \right)^\gamma (c_y y)^\sigma}{1 + r}$$

Using (A12) and (A18) results in the following expression (after some simplification)

$$w = A \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2} \right) \bar{\epsilon}^2 - \tau \epsilon_H^2}{2(\bar{\epsilon} - \epsilon_H)} \right)^{\frac{\theta}{1-\theta}} \frac{2\bar{\epsilon}^2 \theta^2 (\epsilon_H - \epsilon_L)}{b(1+\theta)^2 (\bar{\epsilon} - \epsilon_H) (\bar{\epsilon} - \epsilon_L)} \quad (\text{A19})$$

Fifth, the analytical solution for labor is obtained by taking the linear production technology $n = y/A$ and inserting the solution for output (A18) to get

$$n = \left(\frac{A^{1+\sigma} (1+r)}{\psi} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2} \right) \bar{\epsilon}^2 - \tau \epsilon_H^2}{2(\bar{\epsilon} - \epsilon_H)} \right)^{\frac{\theta-\sigma}{1-\theta}} \frac{2\bar{\epsilon}^2 \theta^2 (\epsilon_H - \epsilon_L)}{b(1+\theta)^2 (\bar{\epsilon} - \epsilon_H) (\bar{\epsilon} - \epsilon_L)} \right)^{\frac{1}{\gamma+\sigma}} \quad (\text{A20})$$

Sixth, the size of the loan coincides with the labor income, $l = wn$. Using equations (A19) and (A20), we obtain (after some simplification)

$$l = A^{1+\frac{1+\sigma}{\gamma+\sigma}} \left(\frac{1+r}{\psi} \right)^{\frac{1}{\gamma+\sigma}} \left(\frac{\left(1 - \frac{(1-\tau)(1-\theta)^2}{(1+\theta)^2} \right) \bar{\epsilon}^2 - \tau \epsilon_H^2}{2(\bar{\epsilon} - \epsilon_H)} \right)^{\frac{\theta}{1-\theta} + \frac{\theta-\sigma}{1-\theta} \frac{1}{\gamma+\sigma}} \left(\frac{2\bar{\epsilon}^2 \theta^2 (\epsilon_H - \epsilon_L)}{b(1+\theta)^2 (\bar{\epsilon} - \epsilon_H) (\bar{\epsilon} - \epsilon_L)} \right)^{1+\frac{1}{\gamma+\sigma}}$$

Seventh, the value of household welfare is reached from inserting the solutions of both c and n in the utility function as in the previous case.

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