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IMPROVING THE FORECASTING OF DYNAMIC CONDITIONAL CORRELATION: A VOLATILITY DEPENDENT APPROACH

Edoardo Otranto

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CRENoS – CAGLIARI
VIA SAN GIORGIO 12, I-09100 CAGLIARI, ITALIA
TEL. +39-070-6756406; FAX +39-070- 6756402

CRENoS - SASSARI
VIA TORRE TONDA 34, I-07100 SASSARI, ITALIA
TEL. +39-079-2017301; FAX +39-079-2017312

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Via Is Mirrionis, 1
09123 Cagliari
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Improving the Forecasting of Dynamic Conditional Correlation: a Volatility Dependent Approach

Edoardo Otranto
University of Sassari and CRENoS

Abstract

Forecasting volatility in a multivariate framework has received many contributions in the recent literature, but problems in estimation are still frequently encountered when dealing with a large set of time series. The Dynamic Conditional Correlation (DCC) modeling is probably the most used approach; it has the advantage of separating the estimation of the volatility of each time series (with great flexibility, using single univariate models) and the correlation part (with the strong constraint imposing the same dynamics to all the correlations). We propose a modification to the DCC model, providing different dynamics for each correlation, simply hypothesizing a dependence on the volatility structure of each time series. This new model implies adding only two parameters with respect to the original DCC model. Its performance is evaluated in terms of out-of-sample forecasts with respect to the DCC models and other multivariate GARCH models. The results on four data sets seem to favor the new model.

Keywords: dynamic conditional correlation, GARCH distance, Multivariate GARCH, out-of-sample forecasts.
JEL Classification: C32, C53, G10.

1 Introduction

Forecasting volatility is an important issue with several financial applications, such as risk estimation, asset allocation, derivative pricing. It is well known that financial volatilities and correlations of different assets are subject to co-movements; moreover, not only the conditional variance, but also the conditional correlation seems not to be constant through time for many empirical applications. For these reasons a multivariate approach seems more appropriate (see, for example, Audrino and Bühlmann, 2004).

The large production of multivariate volatility models (for a review, see Bauwens et al., 2006 and Silvennoinen and Teräsvirta, 2009) has not had the same level of success in practical applications because of the trade-off between flexibility and parsimony of the models; in practice, the estimation of a multivariate model with a large set of assets is possible only imposing strong restrictions on the number of parameters. For this reason a popular model, such as the VEC model (Bollerslev et al. 1988), is frequently applied in simple form such as the Scalar VEC or the Diagonal VEC, in which similar dynamics are imposed on several assets.

The Dynamic Conditional Correlation (DCC) model of Engle (2002) could be considered an important turning point in the multivariate volatility modelling. In fact, due to its ability to separate the estimation of the volatility and the correlation parts (Engle and Sheppard, 2001), it is possible to estimate several univariate GARCH models (one for each time series) to represent the volatility and a single GARCH model to represent the correlation dynamics. In this way the model can be applied to a large set of time series with a small computational effort. On the other hand, imposing the same dynamics on all the correlations is a strong restriction. Greater flexibility could be obtained using the approach of Billio et al. (2006), who consider different dynamics for groups of correlations, but the choice of these groups is well subjective (see Bauwens et al., 2006). Otranto (2010) proposed an algorithm to select these groups; anyway some constraint on the matrices of the parameters driving the correlation dynamics of each group (matrices of rank 1) has to be imposed.

The idea proposed in this paper is very simple: time series with similar volatility structure have similar correlation dynamics. This idea was already implicit in other approaches, such as the multivariate stochastic volatility models (Ghysels et al., 1996), in which no dynamics was provided for correlations, but they depend on the dynamics of the corresponding conditional variances (see Bauwens et al., 2006). For this purpose we reparameterize the coefficients of the correlation dynamics imposing a dependence on a measure of distance between pairs of GARCH models. Adding only two parameters to the original DCC model, we can estimate a flexible model with different dynamics for all the correlations. The information derived by the comparison between the volatility structure of two series of returns can help to forecast their correlation. We compare the forecasts of this model with those obtained by other Conditional Correlation (CC) models, using four sets of financial indices. The choice of the "best" model in this framework is not easy because the asset returns do not contain sufficient information to identify such a volatility model (Audrino and Barone-Adesi, 2006); moreover, comparing our model with the other CC models, we see that they only differ in the correlation part, whereas the volatility part is the same, so the differences can not be large. For this reasons we provide several comparisons, preferably in terms of statistical tests, that Clements et al. (2009) have shown to be more effective in forecasting with respect to the comparisons in terms of portfolio allocation. Anyway, we show also some comparison using theoretical portfolios, in which the expected returns are the same for all the models adopted. This choice will provide a correct comparison among different correlation matrices (Engle and Colacito, 2006). The results show a certain evidence in favor of the new approach.

The paper is organized as follows: in the next section we describe the new model proposed, whereas in section 3 a comparison among four multivariate models is performed in terms of out-of-sample forecasts, using a 20-variate Italian data set; moreover, synthetic results relative to other three data sets, composed by 7, 10 and 30 time series respectively, are also shown. Some final remarks will conclude the paper.

2 A Volatility Dependent DCC Model

Let $\boldsymbol{r}_t$ a $(n \times 1)$ vector of returns of n financial time series ($t = 1, \dots, T$). Let us indicate with $\boldsymbol{\mu}$ the mean and $\boldsymbol{H}_t$ the conditional covariance matrix of $\boldsymbol{r}_t$.

Following Bollerslev (1990), the time-varying conditional covariance matrix can be decomposed in:

$$\boldsymbol{H}_t = \boldsymbol{S}_t \boldsymbol{R}_t \boldsymbol{S}_t, \tag{2.1}$$

where $\boldsymbol{S}_t$ is a diagonal matrix containing the conditional standard deviations (modeled by univariate GARCH models) and $\boldsymbol{R}_t$ is a time-varying positive definite matrix of correlations. A general multivariate class of CC models can be written in the following way (Engle, 2002):

$$\begin{aligned} \boldsymbol{R}_t &= \tilde{\boldsymbol{Q}}_t^{-1} \boldsymbol{Q}_t \tilde{\boldsymbol{Q}}_t^{-1}, \\ \boldsymbol{Q}_t &= (\boldsymbol{\iota}_n \boldsymbol{\iota}_n' - \boldsymbol{A} - \boldsymbol{B}) \odot \boldsymbol{R} + \boldsymbol{A} \odot \boldsymbol{u}_{t-1} \boldsymbol{u}_{t-1}' + \boldsymbol{B} \odot \boldsymbol{Q}_{t-1} \\ \tilde{\boldsymbol{Q}}_t &= \operatorname{diag}(\boldsymbol{Q}_t^{1/2}) \end{aligned} \tag{2.2}$$

where $\odot$ is the Hadamard product, $\boldsymbol{\iota}_n$ a n-vector of ones and $\boldsymbol{A}$ and $\boldsymbol{B}$ are $(n \times n)$ matrices of coefficients, $\boldsymbol{R}$ is the unconditional correlation matrix, $\boldsymbol{u}_t = \boldsymbol{S}_t^{-1} \boldsymbol{r}_t$. Note that we need to rescale the matrix $\boldsymbol{Q}_t$, as in the first equation of (2.2), because it does not directly provide a correlation matrix; in fact the elements on the diagonal of $\boldsymbol{Q}_t$ are not constrained to be equal to one. Note also that the presence of $(\boldsymbol{\iota}_n \boldsymbol{\iota}_n' - \boldsymbol{A} - \boldsymbol{B})$ in the second equation of (2.2) corresponds to the idea of variance targeting explained in Engle and Mezrich (1996).

Of course this representation implies estimating $2n(n-1)$ coefficients and it is impracticable already for $n > 5$; for this reason we need some simple re-parameterization of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.

The Conditional Constant Correlation (CCC) model (Bollerslev, 1990) is obtained by (2.2) putting $\boldsymbol{A} = \boldsymbol{B} = 0$. In other words, the correlation matrix is constant along time.

In the DCC model proposed by Engle (2002), $\boldsymbol{A}$ and $\boldsymbol{B}$ are two non negative scalar coefficients (call them a and b; $a + b < 1$).

A consistent estimation of the DCC model can be easily obtained because it is possible to separate the estimation of the variance equations and the one of the correlation matrix, and, under mild conditions, the covariance matrix is positive definite (see Engle and Sheppard, 2001). In particular, using the two-step estimation procedure of Engle and Sheppard (2001), the variance part can be obtained estimating n univariate GARCH models (one for each time series).

It is evident that, in spite of a large flexibility for the estimation of the volatility part (a different GARCH is estimated for each series), the imposition of the same correlation dynamics is a strong constraint. A possibility could be to consider rank 1 matrices obtained as $\boldsymbol{A} = \boldsymbol{a}\boldsymbol{a}'$ and $\boldsymbol{B} = \boldsymbol{b}\boldsymbol{b}'$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are $(n \times 1)$ vectors of coefficients. We can call this model the vector diagonal (Rank 1) DCC (R1DCC) model. If n is too large, it is possible to consider only k different elements in $\boldsymbol{a}$ and $\boldsymbol{b}$ (Billio et al., 2006), using the algorithm proposed in Otranto (2010) to detect the series following the same dynamics.

Our proposal is to hypothesize a form of dependence of the correlation dynamics on the similarity of the volatility structures; it is logical to imagine that series with similar volatility are more correlated compared to series with different volatility. Starting from this hypothesis, we can re-parameterize the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ in (2.2) in the following way:

$$\boldsymbol{A} = \{g_A(d_{ij})\}; \quad \boldsymbol{B} = \{g_B(d_{ij})\}$$

where $g_A(d_{ij})$ and $g_B(d_{ij})$ are functions of a distance measure d_{ij} between the volatilities of the i-th and j-th series. An appropriate choice for the distance matrix could be the GARCH distance proposed in Otranto (2008), which is an extension of the AR metrics (Piccolo, 1990) to the GARCH case. For the simple case of GARCH(1,1), this distance is given by (see Otranto, 2010):[1]

$$d_{ij} = \left[\frac{\alpha_i^2}{1-\beta_i^2} + \frac{\alpha_j^2}{1-\beta_j^2} - \frac{2\alpha_i\alpha_j}{1-\beta_i\beta_j} \right]^{1/2} \tag{2.3}$$

where (α_i, β_i) and (α_j, β_j) are the GARCH coefficients of the i-th and j-th series of returns respectively.

In Table 1 we show the five number summary of the distributions of the correlations between the columns of the distance matrices $\boldsymbol{D} = \{d_{ij}\}$ and the correlation matrix of four data sets of daily financial returns (Yahoo finance source). The data set G7 includes the returns of the financial general indices of the G-7 member countries from 4 November 2003 to 14 July 2009 (1484 observations for each series); the data set ASIA includes the returns of 10 general indices of the main Asian markets from 5 January 2000 to 29 May 2009 (2452 observations for each series); the data set ITALY contains the 20 sector indices of the Italian Mibtel general index from 4 January 2000 to 1 October 2008 (2251 observations for each series; this data set will be analyzed in more detail in section 3); the data set DOW contains the series of returns of the 30 components of Dow Jones index from 19 June 2001 to 16 July 2009 (2030 observations for each series). The table clearly shows the inverse relationship between the GARCH distance of two series and their correlation, in particular for ASIA and G7. Moreover, all the correlation coefficients come out negative.

[1] For the general GARCH(p,q) case see Otranto (2008). In our applications we will use the GARCH(1,1) case, which, as noted by Bollerslev et al. (1992), fits in an excellent way a wide range of financial data.

Table 1: Five number summary of the distribution of the correlation between the columns of the GARCH distance matrix and the correlation matrix of four data sets.

	G7	ASIA	ITALY	DOW
Minimum	-0.854	-0.861	-0.822	-0.722
First Quartile	-0.770	-0.804	-0.423	-0.356
Median	-0.756	-0.720	-0.389	-0.284
Third Quartile	-0.738	-0.569	-0.369	-0.246
Maximum	-0.621	-0.409	-0.314	-0.131

The functions $g_A(\cdot)$ and $g_B(\cdot)$ have to respect the constraints $(a_{ij} + b_{ij}) < 1$ for each $i, j = 1, \ldots, n$, where a_{ij} and b_{ij} are the elements of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively. Every specification that satisfies this constraint is a valid candidate. We choose the logistic functional form:

$$\begin{aligned} a_{ij} &= \frac{exp(\phi_A+\theta_A d_{ij})}{1+exp(\phi_A+\theta_A d_{ij})+exp(\phi_B+\theta_B d_{ij})} \\ b_{ij} &= \frac{exp(\phi_B+\theta_B d_{ij})}{1+exp(\phi_A+\theta_A d_{ij})+exp(\phi_B+\theta_B d_{ij})} \end{aligned} \tag{2.4}$$

Note that the DCC model is a particular case of (2.4), where $\theta_A = \theta_B = 0$, and that the model proposed contains only 2 new parameters with respect to the DCC specification. We call this model Volatility Dependent DCC (VDDCC).

The model proposed can be estimated with the two-step procedure proposed by Engle and Sheppard (2001). In fact, the full likelihood function can be split into two parts, the first one relative to the estimation of the volatility part (the GARCH coefficients) and the second part relative to the estimation of the correlation part, conditional on the estimation of the GARCH parameters. In this way the distance matrix $\boldsymbol{D}$ is estimated after the first step and used in the second step.

3 Comparison of models

One of the main purposes, using financial volatility models, is the ability to forecast the covariance matrix, because it will be used in the investment strategies, such as portfolio allocation, evaluation of the risk, classification of time series with similar co-movements, etc. In this section we concentrate on the ITALY data set, constituted by the following 20 sectorial indices of the Italian Mibtel general index: food, insurance, cars, banks, paper, chemical, construction, distribution, media, electronics, finance misc., holding, real estate, plant machine, industrial misc., mineral metals, finance services, public utility, textile, transport tourism.

We have estimated four models belonging to the CC family: a DCC, a VDDCC, a R1DCC and a CCC.[2] For the R1DCC model, to avoid the heavy estimation of 40 parame-

[2] We have also estimated alternative multivariate GARCH models not belonging to the CC class: a scalar VEC, a Diagonal VEC (Bollerslev et al., 1988), a Generalized Orthogonal GARCH (van der Weide, 2002). They are less flexible with respect to the CC models because they modelize simultaneously the conditional variances and the conditional covariances with constraints in the matrices of coefficients. Their results are clearly worse than the other models for all the indicators and tests used for comparison, so we do not show

Table 2: Data set ITALY: Estimates of the GARCH coefficients of the 20 univariate time series composing the data set.

Series	μ	γ	α	β
food	0.068	0.025	0.081	0.919
insurance	0.038	0.021	0.105	0.881
cars	0.036	0.090	0.132	0.843
banks	0.044	0.009	0.074	0.922
paper	-0.157	1.491	0.623	0.136
chemical	0.009	0.015	0.097	0.899
construction	0.058	0.036	0.152	0.826
distribution	-0.006	0.049	0.170	0.830
media	-0.019	0.011	0.096	0.904
electronics	0.048	0.013	0.069	0.925
finance misc.	-0.040	1.057	0.276	0.623
holding	0.077	0.032	0.149	0.837
real estate	0.053	0.054	0.203	0.789
plant machine	0.094	0.073	0.125	0.845
industrial misc.	0.018	0.088	0.164	0.809
mineral metals	0.052	0.034	0.080	0.901
finance services	0.083	0.077	0.155	0.837
public utility	0.019	0.015	0.099	0.890
textile	0.062	0.023	0.085	0.902
transport tourism	0.059	0.066	0.170	0.77

ters, we have applied the algorithm described in Otranto (2010) to identify the series following the same DCC dynamics. This procedure consists of an agglomerative algorithm that inserts in the same clusters the series having a non-significant distance (in terms of DCC parameters, analogous to the distance (2.3)) at a certain significance level (we use a test size equal to 0.05). We have obtained the following 5 groups of series (in the order in which they enter in the procedure):

Group 1: electronics, industrial misc.

Group 2: media, finance misc., paper

Group 3: construction, real estate

Group 4: insurance, finance service, banks, distribution, plant machine, cars, mineral metals, holding, transport tourism, food.

Group 5: chemical, textile, public utility

The first step of the estimation procedure consists of estimating the 20 univariate GARCH(1,1) models, which are equal for each approach:

$$r_{i,t} = \mu_i + \epsilon_{i,t}$$
$$h_{i,t} = \gamma_i + \alpha_i \epsilon^2_{i,t-1} + \beta_i h_{i,t-1}$$

The GARCH coefficients are shown in Table 2 (to save space we do not show the

them.

standard errors); from them it will be possible to derive the distance matrix $\boldsymbol{D}$ by (2.3) and the corresponding $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices for the VDDCC model.

The estimates of the correlation part are shown in Table 3; the standard errors are calculated using the *sandwich* covariance matrix illustrated in Engle and Sheppard (2001) and Engle (2002).

We verify the goodness of fit of each model analyzing the standardized residuals $\boldsymbol{z}_t = \boldsymbol{H}_t^{-1/2}(\boldsymbol{r}_t - \boldsymbol{\mu})$, using two criteria (see Engle and Sheppard, 2001, and Audrino and Barone-Adesi, 2006). A first criterion calculates the percentage of the 20 series belonging to the data set having variance in a confidence interval of one (we use a confidence level of 95%); a second criterion calculates the percentage of the 400 series, obtained by the cross-products $\boldsymbol{z}_t\boldsymbol{z}_t'$, for which the Ljung-Box test accepts the null hypothesis of no correlation (we use a significance level of 5%). The results of such tests are shown in Table 3. All models provide standardized residuals having unit variance; the Ljung-Box test rejects the null hypothesis in 18% (20% for CCC) of cases. Anyway, in practical cases, this percentage failing is always more than the nominal size of the test and it is consistent with the results obtained by Engle and Sheppard (2001) and Audrino and Barone-Adesi (2006).

The VDDCC model outperforms the other models in terms of likelihood function and AIC and BIC criteria. Moreover, being the DCC model nested in VDDCC, the likelihood ratio test rejects the null of equal models at the conventional significance levels.

We can not compare directly the VDDCC and R1DCC models via a likelihood ratio test because they are not nested; anyway it is possible to verify if each parameter in $\boldsymbol{A}$ of VDDCC is equal to the corresponding parameter in $\boldsymbol{a}\boldsymbol{a}'$ in R1DCC, and each parameter in $\boldsymbol{B}$ of VDDCC is equal to the corresponding parameter in $\boldsymbol{b}\boldsymbol{b}'$ in R1DCC. In practice we derive the covariance matrix of the estimated parameters present in the lower triangular matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ using the delta method.[3] The Wald test is performed using the following statistic:

$$W = \boldsymbol{m}\boldsymbol{\delta}'(\boldsymbol{m}\boldsymbol{\Omega}\boldsymbol{m}')^{-1}\boldsymbol{m}\boldsymbol{\delta} \tag{3.1}$$

where $\boldsymbol{\delta}$ is a 2×1 vector containing an element of $\boldsymbol{A}$ ($\boldsymbol{B}$) and the corresponding element in $\boldsymbol{a}\boldsymbol{a}'$ ($\boldsymbol{b}\boldsymbol{b}'$); $\boldsymbol{m} = [1, -1]$; $\boldsymbol{\Omega}$ is a (2×2) diagonal matrix containing the variance of the parameters in $\boldsymbol{\delta}$. The statistic (3.1) follows a chi-square distribution with 1 degree of freedom.

Comparing the $n(n-1) = 380$ pairs of coefficients, we obtain that only 37 of them (19 in matrix A and 18 in matrix B) can be considered different at a 5% significance level. In practice, the two models provide matrices that have 90% of similar elements, but for the VDDCC model we have used 4 coefficients, whereas for R1DCC we used 10 coefficients. Moreover, the log-likelihood function and the information criteria favor the more parsimonious model.

[3]The elements on the diagonal are excluded because, rescaling $\boldsymbol{Q}_t$ as in the first equation of (2.2), they provide always the value 1 in the conditional correlation.

Table 3: Data set ITALY: Estimates of the correlation part of alternative models belonging to the CC family (standard errors in parentheses) and goodness-of-fit criteria.

		DCC		
a	b			
0.016	0.938			
(0.002)	(0.008)			
% of series with variance 1			% Ljung-Box accepted	
100(20/20)			82.50(330/400)	
log-lik	AIC	BIC		
-84555.25	75.164	75.271		
		VDDCC		
ϕ_A	θ_A	ϕ_B	θ_B	
-0.867	-0.974	3.093	0.940	
(0.042)	(0.037)	(0.149)	(0.262)	
% of series with variance 1			% Ljung-Box accepted	
100(20/20)			82.25(329/400)	
log-lik	AIC	BIC		
-84532.05	75.145	75.257		
		R1DCC		
a_1	a_2	a_3	a_4	a_5
0.118	0.119	0.123	0.128	0.132
(0.006)	(0.011)	(0.009)	(0.006)	(0.009)
b_1	b_2	b_3	b_4	b_5
0.979	0.965	0.964	0.968	0.970
(0.002)	(0.006)	(0.006)	(0.004)	(0.004)
% of series with variance 1			% Ljung-Box accepted	
100(20/20)			82.50(330/400)	
log-lik	AIC	BIC		
-84543.21	75.161	75.288		
		CCC		
% of series with variance 1			% Ljung-Box accepted	
100(20/20)			80.25(321/400)	
log-lik	AIC	BIC		
-85104.85	75.651	75.752		

Table 4: Data set ITALY: Out-of-sample MSE.

DCC	VDDCC	R1DCC	CCC
12.218	12.203	12.252	12.302

Table 5: Data set ITALY: p-values of the Diebold-Mariano test to verify the equality of means of pairs of the out-of-sample squared forecast errors.

	VDDCC	R1DCC	CCC
DCC	4.98E-08	6.86E-13	1.40E-16
VDDCC		3.87E-21	5.29E-30
R1DCC			6.13E-09

3.1 Forecasting Performance in terms of statistical tests

The VDDCC model seems better with respect to the other CC models in terms of likelihood functions. But what about its forecasting performance? As said, the main purpose of this representation is to use the information derived from the first step estimation (the volatility part) to improve the estimation of the correlation part, hypothesizing a dependence of the correlation dynamics on the similarity of the volatility structure. For this reason we have selected an out-of-sample forecast data set, using the last $f = 400$ observations of the original data. We have re-estimated the models adding a new observation and obtaining the one-step ahead forecast for the successive trading day. A first simple comparison can be made in terms of Mean Squared Error (MSE), which, in this multivariate framework, assumes the form:

$$\frac{1}{n(n+1)/2}\frac{1}{f}\sum_{t=1}^{f}[vech(\hat{\boldsymbol{H}}_{T^*+t}) - vech(\boldsymbol{r}_{T^*+t} - \hat{\boldsymbol{\mu}})(\boldsymbol{r}_{T^*+t} - \hat{\boldsymbol{\mu}})'] \tag{3.2}$$

where $vech(\cdot)$ is the operator that stacks the lower triangular portion of a $n \times n$ matrix as a $n(n+1)/2 \times 1$ vector; $T^* = T - f$; the *hat* indicates the estimated elements in (3.2). The results are shown in Table 4. We can note that the VDDCC model has the lowest MSE, whereas DCC seems to have better performance with respect to R1DCC and CCC.

To verify if these results are significantly different, we use the Diebold and Mariano (1995) statistic to test the null hypothesis of no difference in the accuracy of two competing forecasts (Harvey et al., 1997). In practice we verify if the mean of the f squared forecast errors is zero. In Table 5 we show the results for each pair of models. We can note that all the pairs of mean squared forecast errors are significantly different at each size level, so we can conclude that the ranking obtained in terms of MSE can be considered valid and VDDCC presents the best performance.

As noted by Audrino and Barone-Adesi (2006), in a volatility framework, it is difficult to judge the best model in terms of forecasts when the differences of MSE are small. In fact, in (3.2) we compare the forecast of the covariance matrix with a noisy estimate based on returns values. In other words, the MSE criterion allows to discriminate among forecasts whose performance is different by orders of magnitude. For this reason, we use also other methods to evaluate the forecasting performance of the alternative models in terms of statistical tests.

Clements et al. (2009) make a comparison between techniques for evaluating multivariate volatility forecasts based on statistical methods and techniques based on economic applications (such as portfolio allocation). Their simulation results show that the statistical methods are more effective and, in particular, indicate the Model Confidence Set (MCS) approach of Hansen et al. (2003) as a useful tool to discriminate among different forecasting methods. This approach compares directly each pair of forecasts, so it does not require a benchmark forecast.

Briefly, the MCS approach consists in trimming the set of candidate models. First, all the sequences of all loss differentials between model i and j are computed:

$$l_{ij,t} = L_{i,t} - L_{j,t} \qquad t = 1, \ldots, f; \quad i \neq j$$

where $L_{i,t}$ is a loss function. Clements et al. (2009) suggest the use of the Quasi-Likelihood loss function:

$$qll_{ij,t} = [log|\boldsymbol{H}_{i,t}| + (\boldsymbol{r}_{i,t} - \boldsymbol{\mu}_i)'\boldsymbol{H}_{i,t}^{-1}(\boldsymbol{r}_{i,t} - \boldsymbol{\mu}_i)] - [log|\boldsymbol{H}_{j,t}| + (\boldsymbol{r}_{j,t} - \boldsymbol{\mu}_j)'\boldsymbol{H}_{j,t}^{-1}(\boldsymbol{r}_{j,t} - \boldsymbol{\mu}_j)] \tag{3.3}$$

Moreover, Audrino and Bühlmann (2003) and Audrino and Barone-Adesi (2006) consider also the direction of the Quasi-Likelihood differences, defining a dichotomic loss differential:

$$dl_{ij,t} = \begin{cases} 1 & \text{if } qll_{ij,t} \leq 0 \\ -1 & \text{if } qll_{ij,t} > 0 \end{cases} \tag{3.4}$$

At each step the null hypothesis of equal predictive ability:

$$E(l_{ij,t}) = 0 \quad \text{for each} \quad i \neq j \tag{3.5}$$

is tested for a set of models $\mathcal{M}$. This is made using the range statistic:

$$T_R = \max_{i,j \in \mathcal{M}} \frac{|\bar{l}_{ij}|}{\widehat{var}(\bar{l}_{ij})^{1/2}}$$

or the semi-quadratic statistic (less conservative):

$$T_{SQ} = \sum_{i,j \in \mathcal{M}} \frac{\bar{l}_{ij}^2}{\widehat{var}(\bar{l}_{ij})}$$

where $\bar{l}_{ij}$ is the mean of $l_{ij,t}$, whereas its variance is obtained from a block-bootstrap procedure (see Hansen et al., 2003, and Becker and Clements, 2008, for details). The first test is for the full set of candidate models; if the null (3.5) is rejected, the worst performing model is eliminated by the set of candidate models. This is identified as the model i such that:

$$i = \max_{i \in \mathcal{M}} \frac{\bar{l}_{i\cdot}}{\widehat{var}(\bar{l}_{i\cdot})^{1/2}}$$

where

$$\bar{l}_{i\cdot} = \frac{\sum_{j \in \mathcal{M}} \bar{l}_{ij}}{m-1}$$

Table 6: Data set ITALY: MCS results for individual forecasts. The first row represents the first model removed, down to the best performing model in the last row.

$qll_{ij,t}$				$dl_{ij,t}$			
T_R		T_{SQ}		T_R		T_{SQ}	
Model	p-value	Model	p-value	Model	p-value	Model	p-value
DCC	0.0000	DCC	0.0000	DCC	0.0000	DCC	0.0000
CCC	0.0000	CCC	0.0000	R1DCC	0.0000	R1DCC	0.0000
R1DCC	0.0003	R1DCC	0.0003	CCC	0.0000	CCC	0.0000
VDDCC	1	VDDCC	1	VDDCC	1	VDDCC	1

and m is the number of models in $\mathcal{M}$. Also the variance of $\bar{l}_{i\cdot}$ is evaluated via block-bootstrap.

In Table 6 the results of the MCS procedure are shown (we use 3000 bootstrap samples to estimate the variance of $\bar{l}_{ij}$ and $\bar{l}_{i\cdot}$). We can note that, in despite of the MSE results, in all the cases (test T_R and test T_{SQ}; differential loss function (3.3) or (3.4)), the DCC model is the first model excluded by the procedure, whereas the VDDCC is considered the best one.

3.2 Forecasting performance in terms of portfolio allocation

We mentioned that the evaluation of forecasts in terms of portfolio allocation has some drawbacks in terms of efficacy (Clements et al., 2009). Moreover the results depend on the weights of the portfolio assets, more related to the mean of returns than their variances and correlations (Chopra and Ziemba, 1993). Engle and Colacito (2006) propose to consider a set of theoretical expected returns equal for all the competitive models, so that the comparisons will depend only on the differences in correlation matrices. The problem, in this case, is that the four competitive models we are analyzing have the same conditional variance and differ only for correlations; in other words, we can not expect significant differences among the forecasting performances in an asset allocation framework. Anyway, this approach can provide some tool to confirm (or contradict) the results obtained in the previous subsection. We follow the approach of Engle and Colacito (2006); they show that the realized volatility is smallest for the correctly specified covariance matrix for any vector of expected returns. They suggest to select arbitrary vectors of expected returns, then construct optimal portfolio weights with the alternative covariance models and to calculate the sample variance of each portfolio. The strategy with the smallest covariance for each vector of expected returns will be the best strategy. The key problem here is the choice of the vectors of expected returns; the main experiments of Engle and Colacito (2006) only regard two assets and many alternatives can be used. In the case of high order asset allocation, the choice of an appropriate vector of expected returns is not easy. For example, Engle and Colacito (2006), considering a portfolio composed by 21 stocks and 13 bonds, select only hedging portfolios, obtained by setting one entry equal to 1 and the others equal to zero; in this way the asset with 1 is hedged against all other assets. We follow this idea and create 20 hedging portfolios with the same weights used by Engle and Colacito (2006), obtained by a classical variance minimization problem subject to a

Table 7: Data set ITALY: Comparison of volatilities for each theoretical portfolio

Portfolio	DCC	VDDCC	R1DCC	CCC
food	100.67	100.00	102.12	104.06
insurance	102.09	101.43	100.00	101.28
cars	100.24	100.10	100.00	102.00
banks	100.00	100.03	100.15	101.33
paper	101.10	100.00	100.76	100.05
chemical	100.11	100.00	101.23	103.81
construction	100.00	100.11	100.28	104.64
distribution	100.39	100.09	101.66	100.00
media	100.93	101.21	100.00	101.43
electronics	100.08	100.00	100.60	100.80
finance misc	100.30	100.00	100.59	100.50
holding	100.00	100.13	100.49	104.46
real estate	100.00	100.33	100.79	103.39
plant machine	100.56	100.00	101.35	102.72
industrial misc	100.00	100.11	100.91	101.86
mineral metals	100.48	100.00	100.43	100.32
finance services	100.31	100.00	100.55	100.35
public utility	100.04	100.00	100.22	102.25
textile	100.15	100.00	100.98	101.44
transport tourism	100.31	100.00	101.62	103.57

Table 8: Data set ITALY: t-values of the joint Diebold-Mariano test to compare squared returns of theoretical portfolios.

	DCC	VDDCC	R1DCC	CCC
DCC		2.542	-2.190	-1.655
VDDCC	-2.542		-2.841	-2.235
R1DCC	2.190	2.841		-0.783
CCC	1.655	2.235	0.783	

required return equal to 1:

$$\boldsymbol{w}_t = \frac{\boldsymbol{H}_t^{-1}\boldsymbol{r}}{\boldsymbol{r}'\boldsymbol{H}_t^{-1}\boldsymbol{r}} \qquad t = T^* + 1, \ldots, T^* + f$$

where $\boldsymbol{r}$ is the vector of expected returns.

In Table 7 we show the sample standard deviations of each portfolio for the 20 cases (the hedged asset is indicated in the first column), setting the lowest standard deviation equal to 100; in this way a number like $(100 + x)$ means that, knowing the true covariance matrix, an $x\%$ higher return could be required. We can note that in 5 cases the best covariance is shown by DCC, in 11 cases by VDDCC, in 3 cases by R1DCC and in 1 case by CCC. Anyway, in many cases the differences are small.

To understand if the differences are significant and to resume the 20 different results, we can use a Diebold and Mariano (1995) procedure to test differences jointly for the

20 expected returns. In practice, we compare the squared returns of two theoretical portfolios, dividing their difference by the geometric mean of the two variance estimators (weighted version of the test; see Engle and Colacito, 2006). The t-values of the Diebold-Mariano statistics are shown in Table 8.

It is useful to consider the sign of the t-value when the null is rejected; in fact, the statistic in position (i, j) $(i, j = 1, \dots, 4)$ is constructed as the difference of the squared realized returns of the methods indicated in row i and column j: a positive number is evidence in favor of the method in the column. From Table 8 it is possible to note that, in despite of the equality of the variances, the forecasting performance of the VDDCC model is significantly better with respect to the other models; in other terms, the better forecasting of conditional correlations seems to contribute to a significant better allocation of assets in the portfolio.

3.3 Other data sets

We have performed the same experiments, illustrated for the data set ITALY, for the other three data sets. In Tables 9-11 we have synthesized these results. For the data sets G7 and ASIA we have estimated a full R1DCC model, whereas for data set DOW we have selected the series with similar correlation dynamics.[4] The number of out-of-sample forecasts is 400 for G7 and ASIA and 100 for DOW, which contains 2030×30 data, implying a certain computational effort.

The VDDCC model outperforms the other models in terms of information criteria (and also in terms of likelihood ratio test, where it can be applied) for the three data sets. Also in terms of MSE the better performance of the VDCC model seems clear; only in the ASIA data set the MSE of DCC is lower, but the corresponding Diebold-Mariano test accepts the null of equal mean forecasting error.

The MCS approach favors the VDDCC model, which is always the model belonging to the best set; only the T_R test, using the quasi-likelihood loss function, in some cases shows a not significant difference with respect to the other models, whereas, for the $dl_{ij,t}$ differential loss function, we observe the same result for all the data sets (VDDCC the best model, DCC the worst).

The analysis of theoretical portfolios in general shows small and not significant differences among the four models, but, as said previously, it is difficult to detect significant differences in the performance. Anyway, the sign of the Diebold and Mariano statistic favors the VDDCC model (a part in the data set G7, where only the elements in the column relative to DCC are all positive).

4 Final remarks

In this work we have developed a new model, belonging to the class of conditional correlation models. The idea is that the structure of volatility of each series contains information to forecast the correlation dynamics. The analysis of four data sets seems to support this

[4]The Otranto (2010) procedure has detected 2 groups with similar correlation dynamics; the first one containing the assets United Tech and Exxon Mobile CP, the second all the rest.

Table 9: Data set G7: Synthesis of experiments to evaluate the forecasting performance of CC models.

Likelihood functions				
	DCC	VDDCC	R1DCC	CCC
log-lik	-15642.37	-15618.93	-15619.78	-15735.97
AIC	21.122	21.093	21.108	21.245
BIC	21.229	21.207	21.258	21.345
MSE				
	DCC	VDDCC	R1DCC	CCC
	104.07	103.64	104.24	104.30
Diebold-Mariano p-value to test equal MSE				
		VDDCC	R1DCC	CCC
	DCC	3.43E-08	0.0613	0.0567
	VDDCC	0	0.0001	0.0004
	R1DCC	0	0	0.2558
MCS results using $qll_{ij,t}$ as differential loss function				
Model removed	T_R p-value		Model removed	T_{QS} p-value
DCC	0.0000		DCC	0.0000
CCC	0.0000		CCC	0.0000
R1DCC	0.0633		R1DCC	0.0633
VDDCC	1		VDDCC	1
MCS results using $dl_{ij,t}$ as differential loss function				
Model removed	T_R p-value		Model removed	T_{QS} p-value
DCC	0.00000		DCC	0.0000
R1DCC	0.0000		R1DCC	0.0000
CCC	0.0000		CCC	0.0000
VDDCC	1		VDDCC	1
Joint Diebold-Mariano statistic to test equal squared returns of theoretical portfolios				
	DCC	VDDCC	R1DCC	CCC
DCC		-0.130	-1.307	-1.125
VDDCC	0.130		-1.061	-0.953
R1DCC	1.307	1.061		0.013
CCC	1.125	0.953	-0.013	

Table 10: Data set ASIA: Synthesis of experiments to evaluate the forecasting performance of CC models.

Likelihood functions				
	DCC	VDDCC	R1DCC	CCC
log-lik	-50436.14	-50401.24	-50404.80	-50568.67
AIC	41.173	41.146	41.157	41.280
BIC	41.272	41.250	41.285	41.374
MSE				
	DCC	VDDCC	R1DCC	CCC
	79.835	79.838	80.117	80.532
Diebold-Mariano p-value to test equal MSE				
		VDDCC	R1DCC	CCC
	DCC	0.471	3.90E-06	4.76E-07
	VDDCC		9.29E-05	1.88E-05
	R1DCC			0.002
MCS results using $qll_{ij,t}$ as differential loss function				
Model removed	T_R p-value		Model removed	T_{QS} p-value
DCC	0.0253		DCC	0.0027
R1DCC	0.0253		R1DCC	0.0040
CCC	0.0253		CCC	0.0043
VDDCC	1		VDDCC	1
MCS results using $dl_{ij,t}$ as differential loss function				
Model removed	T_R p-value		Model removed	T_{QS} p-value
DCC	0.0000		DCC	0.0000
R1DCC	0.0000		R1DCC	0.0000
CCC	0.0000		CCC	0.0000
VDDCC	1		VDDCC	1
Joint Diebold-Mariano statistic to test equal squared returns of theoretical portfolios				
	DCC	VDDCC	R1DCC	CCC
DCC		0.231	-1.002	-3.378
VDDCC	-0.231		0.001	-2.194
R1DCC	1.002	-0.001		-4.558
CCC	3.378	2.194	4.558	

Table 11: Data set DOW: Synthesis of experiments to evaluate the forecasting performance of CC models.

		Likelihood functions		
	DCC	VDDCC	R1DCC	CCC
log-lik	-132487.06	-132429.53	-132487.03	-132542.82
AIC	130.649	130.595	130.651	130.702
BIC	130.987	130.938	130.994	131.034
		MSE		
	DCC	VDDCC	R1DCC	CCC
	360.549	360.192	361.898	361.596
		Diebold-Mariano p-value to test equal MSE		
		VDDCC	R1DCC	CCC
	DCC	4.43E-12	0.123	0.178
	VDDCC		0.070	0.107
	R1DCC		0.000	0.019
		MCS results using $qll_{ij,t}$ as differential loss function		
Model removed	T_R p-value		Model removed	T_{QS} p-value
DCC	0.0150		DCC	0.0000
CCC	0.0150		CCC	0.0003
R1DCC	0.0150		R1DCC	0.0043
VDDCC	1		VDDCC	1
		MCS results using $dl_{ij,t}$ as differential loss function		
Model removed	T_R p-value		Model removed	T_{QS} p-value
DCC	0.0000		DCC	0.0000
R1DCC	0.0000		R1DCC	0.0000
CCC	0.0000		CCC	0.0000
VDDCC	1		VDDCC	1
		Joint Diebold-Mariano statistic to test equal squared returns of theoretical portfolios		
	DCC	VDDCC	R1DCC	CCC
DCC		0.447	-0.717	-0.860
VDDCC	-0.447		-0.699	-0.849
R1DCC	0.717	0.699		-1.224
CCC	0.860	0.849	1.224	

idea. The main advantage of the VDDCC model is in the possibility to re-parameterize the correlation matrix using only 4 parameters (2 more than the DCC model).

Of course the success of the model depends on the data set utilized; in some cases the dependence of correlation on the distance matrix with elements (2.3) is stronger with respect to others. Moreover, the logit function (2.4) could be more appropriate in some case with respect to others. Future works could be addressed to experiment alternative distance measures and alternative functions that link distance and correlation.

References

[1] Audrino, F. & Barone-Adesi, G. (2006). Average conditional correlation and tree structures for multivariate GARCH models. *Journal of Forecasting*, 25, 579-600.

[2] Audrino, F. & Bühlmann, P. (2004). Synchronizing multivariate financial time series. *Journal of Risk*, 6, 81-106.

[3] Becker, R. & Clements, A. (2008). Are combination forecasts of S&P 500 volatility statistically superior? *International Journal of Forecasting*, 24, 122-133.

[4] Billio, M., Caporin, M. & Gobbo, M. (2006). Flexible Dynamic Conditional Correlation Multivariate GARCH models for asset allocation. *Applied Financial Economics Letters*, 2, 123-130.

[5] Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH approach. *Review of Economics and Statistics*, 72, 498-505.

[6] Bollerslev, T., Chou, R. Y. & Kroner, K. F. (1992). ARCH modeling in finance: A review of the theory and empirical evidence. *Journal of Econometrics*, 52, 559.

[7] Bollerslev, T., Engle, R. F. & Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. *The Journal of Political Economy*, 96, 116-131.

[8] Chopra, V. & Ziemba, W. (1993). The effect of errors in means, variances and covariances on optimal portfolio choice. *Journal of Portfolio Management*, 19, 6-11.

[9] Clements, A., Doolan, M., Hurn, S. & Becker, R. (2009). On the efficacy of techniques for evaluating multivariate volatility forecasts. NCER Working Paper Series 41, Financial Economic Program.

[10] Engle, R. F. (2002). Dynamic coditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, 20, 339-350.

[11] Engle, R. F. & Colacito, R. (2006). Testing and valuing dynamic correlations for asset allocation. *Journal of Business and Economic Statistics*, 24, 238-253.

[12] Engle, R. F. & Mezrich, J. (1996). GARCH for groups. *Risk*, 9, 36-40.

[13] Engle, R. F. & Sheppard, K. (2001). Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. Mimeo, UCSD.

[14] Ghysels, E., Harvey, A. & Renault, E. (1996). Stochastic volatility. In: Maddala, G. S. & Rao, C. R. (Eds.), *Handbook of Statistics*. Amsterdam: Elsevier Science, pp. 119-191.

[15] Hansen, P., Lunde, A. & Nason, J. (2003). Choosing the best volatility models: the model confidence set approach. *Oxford Bulletin of Economic and Statistics*, 65, 839-861.

[16] Harvey, D., Leybourne, S. & Newbold, P. (1997). Testing the equality of prediction mean squared errors. *International Journal of Forecasting* 13, 281-291.

[17] Otranto, E. (2008). Clustering heteroskedastic time series by model-based procedures. *Computational Statistics and Data Analysis*, 52, 4685-4698.

[18] Otranto, E. (2010). Identifying Financial Time Series with Similar Dynamic Conditional Correlation. *Computational Statistics and Data Analysis*, 54, 1-15.

[19] Piccolo, D. (1990). A distance measure for classifying ARIMA models. *Journal of Time Series Analysis*, 11, 153-164.

[20] van der Weide, R. (2002). GO-GARCH: a multivariate generalized orthogonal GARCH model. *Journal of Applied Econometrics*, 17, 549-564.

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