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FINANCIAL LIBERALIZATION WITH HIDDEN TRADES

Ettore Panetti

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Financial Liberalization with Hidden Trades

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Abstract
How does the availability of unregulated market-based channels for the circulation of liquidity in the financial system affect the process of financial integration? To answer this question, I develop a two-country model of banking, where the banks have access to country-specific investment technologies, and agents can borrow and lend liquidity in a hidden market. I characterize the competitive equilibria at different levels of integration (both in the banking system and in the hidden market) and show that the only level of integration which the two countries are able to coordinate is the one where the two banking systems are autarkic, but international hidden trades are possible. In contrast to the previous literature, I also find that the resulting consumption allocation is constrained efficient.

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1 Introduction

Both the theory and the practice of economics tell us that financial integration is good: it allows a better diversification of risk, increases competition, spreads the returns from the comparative advantages and enhances economies of scale. Nevertheless, as highlighted by the IMF (Abiad et al., 2008), after the peaks of the nineties, financial integration around the world has come to a halt, both in developing and developed countries. The resistances, especially at the national level, against a banking union in the EU only constitute the last example of the difficulties that the process of financial integration has recently encountered. The aim of the present work is to offer a possible explanation to this phenomenon.

The story that I have in mind is one where financial integration affects the equilibrium prices of all those market-based unregulated channels for the circulation of liquidity that have developed as a consequence of financial liberalization and capital mobility, thus creating winners and losers from integration, and hindering further expansions.

To formalize this intuition, I develop a model of financial intermediation, where financial intermediaries (or, more commonly, banks) provide insurance to their customers against the realization of a private idiosyncratic shock, which affects their liquidity needs and makes them either patient or impatient to consume (as in Diamond and Dybvig, 1983) For this purpose, the banks collect deposits, invest in a long-term asset and in a storage technology (which is equivalent to liquidity), and sign a contract with their customers that states how much they are allowed to withdraw in the future, depending on the realization of their idiosyncratic liquidity needs.

I extend this framework in two directions. First, I assume that the world is divided into two countries, labeled Home (H) and Foreign (F), that have different investment technologies: Foreign has a higher yield on the long-term asset than Home. This means that Home has a comparative advantage in the storage technology, as the opportunity costs of holding liquidity versus the long-term asset is lower than in Foreign, and Foreign has a comparative advantage in the long-term asset. This difference can stem from different regulatory environments, or different production technologies that are available in the two countries, and is introduced to rationalize the need for financial integration.

As a second extension, I instead introduce the possibility for bank depositors to trade in a hidden market. That is, the agents can borrow and lend among themselves without being observed by their banks, by issuing or buying a bond whose return is determined in equilibrium (and is, in all effects, equivalent to the interest rate of the economy). The unobservability of these trades is a standard way of introducing the concept of "non-exclusivity" of the financial contracts, and is a plausible assumption for two main reasons: first, because it is difficult to imagine that a bank can preclude its customers from contacting other intermediaries, or make its contracts contingent on that; second, because, in this way, I introduce in the model all those institutions, markets and instruments that financial liberalization and capital mobility have made available to the individual investors to bypass the traditional banking system in an anonymous way, and that have been generally labeled “new financial intermediaries” or, with a somewhat negative connotation, “shadow banking system”.

In the present environment, the non-exclusivity implies that the terms of the banking contract, by the Revelation Principle, must satisfy an incentive compatibility constraint: the present value of the consumption bundle that each depositor is entitled to receive by her bank (evaluated at the equilibrium interest rate on the hidden bond) must be independent of the realization of the individual idiosyncratic shock, so that no agent has incentives to misreport her liquidity need. This means that, in a competitive equilibrium, the presence of the hidden markets imposes a burden on the banks, which see their choice sets curbed by such a constraint (which always binds). I will distinguish four different cases, depending on whether the banking systems and the hidden markets of the two countries are integrated or not, and characterize the competitive equilibrium with hidden trades in all of them.

My first result shows that, when the banking systems of the two countries are not integrated, cross-country borrowing and lending among the depositors, despite being unobservable, do increase welfare with respect to complete autarky: this is because the banks cannot observe the behavior of their customers, but know that they can exchange resources across countries. Thus, they specialize in the asset in which they hold the comparative advantage (the storage technology for Home, and the long-term asset for Foreign) and let the depositors retrade and enjoy the gains from “hidden” financial integration.

More interestingly, I show that the availability of hidden trading opportunities halts the process of integration in the banking system. The intuition for this result is the following: in order to coordinate, the two countries need to both agree that integration is welfare-improving with respect to autarky. On one side, when the two banking systems are in autarky, but the agents are allowed to trade internationally, the banks in both countries specialize their portfolios in the asset in which they hold a comparative advantage. Then, demand and supply pin down the equilibrium interest rate in the hidden market, which must lie between the two country-specific returns on the long-term assets, for the market to clear. On the other side, in the equilibrium where the two banking systems are also integrated, all banks can invest in the long-term asset of Foreign (as it yields a higher return), and the equilibrium interest rate in the hidden market must equal this, because there must be no arbitrage opportunities between the official banking system and the hidden market. Thus, the move from financial autarky to integration, in the presence of already-integrated hidden markets, generates an increase in the interest rate, which has a different effect on the welfare of the two countries: Home, i.e. the country specializing in liquidity, is better off, because its intertemporal terms of trade have improved (they lend liquidity at a higher rate of return); Foreign, i.e. the country specializing in the long-term asset, is instead worse off, because its intertemporal terms of trade have worsened (they are borrowing liquidity at a higher rate of return). Put differently, financial integration is not welfare-improving for the whole economy, but creates a winner and a loser country, depending on their comparative advantages. The necessary agreement to coordinate financial integration breaks off.

In the second part of the paper, I analyze the constrained-efficiency of the competitive equilibrium. In that respect, the crucial assumption is that a social planner can collect the endowments of the agents in the economy, and choose the best allocation to maximize their welfare, but takes as given the level of integration of the banking systems and of the hidden markets, and must satisfy the incentive compatibility constraint as the banks do. In this environment, I show that, when cross-country hidden trades are forbidden, or when both the banking system and the hidden markets are perfectly integrated, the planner is able to improve the market allocation and offer a contract equivalent to the first best: she compresses the ex post income profiles of the agents, by cross-subsidizing the consumption of those in liquidity need, and ensures that the allocation is incentive-compatible by imposing a wedge between the return on the long-term asset and the return on the hidden bond.

In contrast, when the two official banking systems are not integrated, but international hidden trades are possible, the planner cannot improve the welfare of the agents above the level provided by the banks in the competitive equilibrium. In other words, the competitive equilibrium is constrained-efficient. Intuitively, this is because the planner, who cannot transfer the endowments of the economy from one country to the other, sets a contract such that the agents have incentives to retrade, and exploits the gains from hidden financial integration, as the banks do. This is an important result for two reasons: first, because it disproves the classical result of Jacklin (1987) and Allen and Gale (2004) who show that the possibility for agents to trade in the market distorts the efficiency of the banking equilibrium in Diamond-Dybvig environments; second, because, at all other levels of financial integration, the differences between the competitive banking equilibria and the corresponding constrained efficient allocations provide the rationale for the introduction of minimum liquidity requirements. However, when the two banking systems are separated, and cross-country hidden trades are allowed, there is no way through which the decentralized equilibrium can be improved by regulation.

The rest of the paper is organized as follows. In section 2, I summarize the literature related to the present work. In section 3, I describe the environment, and characterize the equilibrium in a closed economy. In section 4, I extend the analysis to the two-country case and analyze the interactions between the possibility of hidden trades and the process of financial liberalization. In section 5, I characterize the constrained efficient allocation of both the closed economy and the two-country case, which I use as terms of comparison to study optimal regulation in section 6. Finally, in section 7, I conclude the paper.

2 Related Literature

The present paper mainly contributes to the literature that studies the limits of financial integration. Starting from the observation of Obstfeld and Taylor (2004) that financial globalization is primarily confined to rich countries, Mishkin (2007) lists many different reasons why financial globalization has not spread in less developed countries, mostly connected to the presence of information asymmetries that the institutional framework is not able to eliminate. However, most of the literature explains that financial integration has not developed because it can have adverse consequences on global imbalances (Mendoza et al., 2009), or because it exacerbates the contagion of systemic risk (Fecht and Gruner, 2005; Fecht et al., 2012) and aggregate shocks (Allen and Gale, 2000; Castiglionesi et al., 2010). Here, I take a different stance, and instead show how financial integration can have negative welfare effects, even in the absence of shocks or contagion.

To this end, I take as a starting point the workhorse model for the positive and normative analysis of financial intermediation developed by Diamond and Dybvig (1983), where the existence of financial intermediation is fully-microfounded as a way of decentralizing the constrained efficient allocation of risk in an economy with private idiosyncratic liquidity shocks. Jacklin (1987) is the first to address the issue of how the possibility for agents to engage in market trades limits the efficiency of the banking equilibrium in these environments. More recently, Farhi et al. (2009) analyze the same concepts in a mechanism-design framework, and rationalize the imposition of minimum liquidity requirements as a way of implementing the constrained efficient outcome in decentralized environments. Here, I extend their work to a two-country environment with comparative advantages and, in contrast to their findings, I show that the presence of hidden markets does not always hinder the (constrained) efficiency of the competitive banking equilibrium.

More generally, the present work contributes to the literature on the interactions between non-exclusivity of financial contracts and risk sharing. Bisin and Guaitoli (2004) study an environment where the banks sign non-exclusive contracts with their customers, who are subject to moral hazard due to the unobservability of their actions. Castiglionesi and Wagner (2013) instead analyze the efficiency of the market allocation in an environment where the banks mutually insure against the realization of some idiosyncratic shock as an outcome of bilateral (and non-exclusive) contracting.

3 A Closed Economy

To understand the basic features of the environment that I am going to extend in the next section, I here characterize the equilibrium of a closed economy. The basic structure of the environment is similar to the Farhi et al. (2009) version of the Diamond and Dybvig (1983) model of financial intermediation.

3.1 Preferences and Technology

The economy lasts for three periods, labeled $t = 0, 1, 2$, and is populated by a unitary continuum of ex ante identical agents, who are born at date 0 with an equal endowment $e = 1$.

All agents in the economy are affected by some idiosyncratic uncertainty, which hits them in the form of a preference shock. Being ex-ante equal, in $t = 1$ every agent draws a type $\theta \in \{0, 1\}$ which is private information to herself: $\pi > 0$ is the probability of being of type 0, and $(1 - \pi)$ is the probability of being of type 1. The preference shocks are independent and identically distributed across agents so that, by the law of large numbers, the cross-sectional distribution of the types is equivalent to their probability distribution: π and $(1 - \pi)$ are the fractions of agents who turn out to be of type 0 and type 1, respectively. The role of the individual types is to affect the point in time at which the agents enjoy consumption. This happens according to the utility function $U(c_1, c_2, \theta) = (1 - \theta)u(c_1) + \beta\theta u(c_2)$. Clearly, if $\theta = 0$, the agent is willing to consume only at date 1, while if $\theta = 1$ she will consume only at date 2. As is customary in this line of research, I then refer to type-0 and type-1 agents as "early" (or impatient) and "late" (or patient) consumers, respectively. The felicity function $u(c)$ is assumed to be increasing, concave, and satisfying the Inada conditions. Moreover, the coefficient of relative risk aversion $-u''(c)c/u'(c)$ is larger than or equal to 1.

Two assets are available in the economy, which can be used to hedge against the idiosyncratic uncertainty. In line with the literature, I call "short asset" a storage technology, yielding 1 unit of consumption at $t + 1$ for each unit invested in t. The other asset, that I call "long", instead delivers $\hat{R} > 1$ (with $\beta\hat{R} > 1$) units of consumption in $t = 2$ for each unit invested in $t = 0$, and can be interpreted as the marginal rate of transformation of a production technology. The short asset is "liquid", as it provides a way of moving resources from one period to the following. The long asset instead cannot be liquidated before maturity, so it is "illiquid".[1]

The economy is also populated by a large number of banks, which operate in a perfectly competitive market with free entry. At date 0 (i.e. before the realization of the private idiosyncratic shock), the agents deposit their endowments in the banks, and sign a contract with them.[2] The contract states the amount of consumption goods that the customers are entitled to withdraw at date 1 and 2, depending on their types. I define the banking contract as $C(\theta) = \{c_1(\theta), c_2(\theta)\}$, and label X and Y the amounts of short and long assets that the banks hold at date 0, respectively. A banking contract is feasible if:

$$\pi \left[c_1(0) + \frac{c_2(0)}{\hat{R}} \right] + (1 - \pi) \left[c_1(1) + \frac{c_2(1)}{\hat{R}} \right] \leq 1. \tag{1}$$

[1] A general feature of the Diamond-Dybvig framework is the presence of a "liquidation technology", which can be employed to throw away the long asset and create extra liquidity, often at a cost. In this paper, I rule this out for simplicity, as the banks would never find it convenient to use the liquidation technology to finance early consumption.

[2] In their original contribution, Diamond and Dybvig (1983) prove that the banking contract is strictly preferred to a market allocation, where each agent chooses her own portfolio of assets, and then trades the long asset at date 1, after the realization of the idiosyncratic shock.

In the environment described so far, a social planner would solve:

$$\max_{\{c_1(\theta),c_2(\theta)\}_{\theta\in\{0,1\}}} \quad \pi u(c_1(0)) + \beta(1-\pi)u(c_2(1)), \tag{2}$$

subject to the feasibility constraint (1) and the incentive compatibility constraints:

$$c_1(0) \geq c_1(1), \tag{3a}$$
$$c_2(1) \geq c_2(0), \tag{3b}$$
$$c_2(1) \geq c_1(0). \tag{3c}$$

Remember that the realizations of the idiosyncratic types are private information. Then, by the Revelation Principle, we can focus on truth-telling mechanisms where each agent has incentives to truthfully report her liquidity need: a type-0 agent must have no incentives to report being a type-1 (equation (3a)), and a type-1 agent must have no incentives to report being a type-0 (equation (3b)), which also includes the possibility of pretending to be impatient, getting $c_1(0)$, and storing the early consumption for the following period (equation (3c)).

Diamond and Dybvig (1983) show that, at the optimum, the incentive compatibility constraints are all slack, and that the constrained efficient allocation is equivalent to the first best:

$$u'(c_1(0)) = \beta\hat{R}u'(c_2(1)), \tag{4a}$$
$$\pi c_1(0) + (1-\pi)\frac{c_2(1)}{\hat{R}} = 1, \tag{4b}$$
$$c_1(1) = c_2(0) = 0. \tag{4c}$$

The planner chooses a feasible contract such that the marginal rate of substitution between the early and the late consumption is equal to the marginal rate of transformation of the production technology. Moreover, the fact that the degree of relative risk aversion is larger than or equal to 1 implies that:[3]

$$\beta\hat{R} = \frac{u'(c_1(0))}{u'(c_2(1))} \geq \frac{c_2(1)}{c_1(0)}, \tag{5}$$

hence $c_2(1) < \hat{R}c_1(0)$ since $\beta < 1$. Using this result in the feasibility constraint, we find that:

$$c_1(0) > \pi c_1(0) + (1-\pi)\frac{c_2(1)}{\hat{R}} = 1, \tag{6}$$

[3]The assumption about relative risk aversion is crucial for this result. Rewrite $-\frac{u''(c)c}{u'(c)} \geq 1$ as $-\frac{u''(c)}{u'(c)} \geq \frac{1}{c}$. This, in turn, means that $-(log[u'(c)])' \geq (log[c])'$. Integrate between z_1 and $z_2 > z_1$ so as to obtain $log[u'(z_1)] - log[u'(z_2)] \geq log[z_2] - log[z_1]$. Once taken the exponential, the last expression gives $\frac{u'(z_1)}{u'(z_2)} \geq \frac{z_2}{z_1}$. If $z_1 > z_2$, the inequality is reversed.

and therefore $c_2(1)/\hat{R} < 1$. This result states that, in the first best, the planner offers optimal insurance against the idiosyncratic shock by cross-subsidizing the impatient agents. For this purpose, the planner compresses the distribution of the ex post income profile of the agents: those who turn out to be impatient receive more than their endowment at date 1, while those who turn out to be patient receive less than their environment (in present value) at date 2.

3.2 The Hidden Market

I extend this environment by allowing the agents to engage in hidden trades at $t = 1$, after the idiosyncratic shock has been revealed to them. I model this feature of the economy as unobservable exchanges, through which individual depositors can anonymously borrow and lend an amount $b(\theta)$ of uncontingent bonds yielding a "hidden" interest rate R, to be determined in equilibrium. Notice two things. First, the fact that agents trade only uncontingent bonds is not an a priori restriction on the completeness of the market, but an endogenous feature of the environment, as I show in Appendix A. Second, the results proposed here hinge neither on the fact that the banks cannot access this market themselves nor on the date when the market opens, but only on the fact that the depositors can borrow and lend without being observed, while the activities of the banks are perfectly observable.

I formalize the problem faced by the agents in the hidden market in the following way. The agents take their decisions to borrow or lend at date 1. In doing so, they take as given the banking contract $C(\theta)$ that they signed with the representative bank in the previous period, the interest rate R (because they are price-takers), and the realization of their idiosyncratic types. Then, the problem in the hidden market reads:[4]

$$V(C(\theta), R, \theta) = \max_{x_1(\theta), x_2(\theta), \theta'(\theta), b(\theta)} U(x_1(\theta), x_2(\theta), \theta), \tag{7}$$

subject to:

$$x_1(\theta) = c_1(\theta'(\theta)) + b(\theta), \tag{8}$$
$$x_2(\theta) = c_2(\theta'(\theta)) - Rb(\theta). \tag{9}$$

Given the state variables, the agents decide which type $\theta'(\theta)$ to report to the banking sector (which will affect the amount of resources available for trade), the final consumption bundle $\{x_1(\theta), x_2(\theta)\}$ actually consumed in the two periods, and the amount $b(\theta)$ (which can be positive or negative) to borrow or lend in the hidden market, so as to maximize their welfare, subject to the budget constraints.

The environment described so far is a complex game of asymmetric information between

[4]To simplify the notation, I explicitly write the final consumption allocation, the reported types and the bond trades only as functions of the realization of the idiosyncratic types θ, but formally they also depend on the contract $C(\theta)$ and the equilibrium interest rate R.

the banks and their depositors. However, by the Revelation Principle, we can concentrate on truth-telling mechanisms, where the agents have incentives to reveal their true individual types to the banks. The incentive compatibility constraint can then be defined in the following way:

Definition 1. *A banking contract $C(\theta)$ is incentive-compatible if:*

$$V(C(\theta), R, \theta) \geq V(C(\theta'), R, \theta) \tag{10}$$

for any $\theta, \theta' \in \{0, 1\}$ with $\theta \neq \theta'$.

The incentive compatibility constraint states that each agent should get a higher welfare by reporting her true type than by reporting the other one and retrade but, given the presence of only two types, this can be simplified. Rewrite the problem as:

$$V(C(\theta), R, \theta) = \max_{x_1(\theta), x_2(\theta), \theta'(\theta)} U(x_1(\theta), x_2(\theta), \theta), \tag{11}$$

$$s.t. \qquad x_1(\theta) + \frac{x_2(\theta)}{R} = c_1(\theta'(\theta)) + \frac{c_2(\theta'(\theta))}{R}. \tag{12}$$

For type 0 and 1, the incentive compatibility reads, respectively:

$$V(C(0), R, 0) \geq V(C(1), R, 0), \tag{13}$$

$$V(C(1), R, 1) \geq V(C(0), R, 1), \tag{14}$$

which can be rewritten as:

$$u\left(c_1(0) + \frac{c_2(0)}{R}\right) \geq u\left(c_1(1) + \frac{c_2(1)}{R}\right), \tag{15}$$

$$u(Rc_1(1) + c_2(1)) \geq u(Rc_1(0) + c_2(0)), \tag{16}$$

because $x_2(0) = x_1(1) = 0$. Thus, it is easy to see that a banking contract $C(\theta)$ is incentive-compatible if:

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R}. \tag{17}$$

Truth-telling requires the banking contract to entitle the depositors to the same present value of consumption, evaluated at the interest rate on the hidden bond, regardless of the realization of the idiosyncratic uncertainty.

3.3 The Banking Problem

In the following, I focus my attention on pure strategy symmetric equilibria, where the banks share the same investment strategy. Therefore, without loss of generality, I can restrict myself to the analysis of a representative bank. The problem of the representative bank, given the assumptions of perfect competition and free entry, is to maximize the expected welfare of its customers:

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}} \quad \pi V(C(0), R, 0) + \beta(1-\pi)V(C(1), R, 1), \tag{18}$$

subject to the intertemporal budget constraint:

$$\pi\left(c_1(0) + \frac{c_2(0)}{\hat{R}}\right) + (1-\pi)\left(c_1(1) + \frac{c_2(1)}{\hat{R}}\right) \leq 1, \tag{19}$$

and the incentive compatibility constraint (17). The definition of a competitive banking equilibrium is straightforward:

Definition 2. *Given an endowment $e = 1$ for each agent and a probability distribution π for the idiosyncratic shock, a competitive banking equilibrium with hidden markets is a contract $C(\theta) = \{c_1(\theta), c_2(\theta)\}$, a final consumption allocation $\{x_1(\theta), x_2(\theta)\}$, an interest rate R and an amount of hidden bonds $\{b(\theta)\}$ for every type $\theta \in \{0, 1\}$, such that:*

- *for a given interest rate and contract, the final consumption allocation solves the problem in the hidden market* (7)*, for every type;*
- *the contract solves the banking problem* (18)*;*
- *markets clear:*

$$\pi\left(c_1(0) + \frac{c_2(0)}{\hat{R}}\right) + (1-\pi)\left(c_1(1) + \frac{c_2(1)}{\hat{R}}\right) = 1, \tag{20}$$

$$\pi b(0) + (1-\pi)b(1) = 0. \tag{21}$$

3.4 Timing

Before proceeding, it is useful to summarize the timing of actions and events: at date 0, each agent deposits her endowment into the banks, hence the total deposits are equal to 1. The banks set up an incentive-compatible contract with their depositors, entitling them to an amount of real consumption at date 1 and 2, and decide the portfolio allocation between the short and the long asset. At $t = 1$, all uncertainty is resolved: each agent gets to know her private type and, according to the report that she makes to the bank, receives an amount $c_1(\theta)$ of the consumption good. After the withdrawal, the agents engage in side trades in the hidden market. Finally, at $t = 2$ the agents receive $c_2(\theta)$, and their return on the hidden investment.

3.5 Competitive Equilibrium

To characterize the equilibrium of this economy, I go by backward induction. In the hidden market, the equilibrium interest rate R must be equal to the marginal rate of transformation $\hat{R}$. The rationale for this result is the following: if $R < \hat{R}$, the long asset dominates the short asset, and the banks invest all their deposits in the first one only, and let the depositors trade in the hidden market; but the impatient agents would be willing to borrow, while nobody would be there to lend, thus the equilibrium return would go to infinity, which is a contradiction. Similar lines of reasoning rule out the possibility that $R > \hat{R}$: if that were the case, the short asset would dominate the long asset, and the banks would invest only in liquidity. Then, the patient agents would be willing to lend $c_1(1)$ in the hidden market, but they would not find any borrower, hence the interest rate R would go to 1, which is once more a contradiction.

With this result in hand, I can characterize the problem in the hidden market. A type-0 agent takes as given the banking contract $\{c_1(\theta), c_2(\theta)\}$ and the interest rate $R = \hat{R}$, and solves:

$$\max \ u(x_1(0)), \tag{22}$$

subject to the budget constraints:

$$x_1(0) = c_1(0) + b(0), \tag{23a}$$
$$x_2(0) = c_2(0) - \hat{R}b(0). \tag{23b}$$

Notice that I am implicitly saying that the banking contract is incentive-compatible, so every agent reports her correct type: $\theta'(0) = 0$, and $\theta'(1) = 1$. A type-0 agent chooses $x_2(0) = 0$, because she does not enjoy utility from consuming at date 2, so it must be the case that $b(0) = c_2(0)/\hat{R}$, and $x_1(0) = c_1(0) + c_2(0)/\hat{R}$. In a similar way, a type-1 agent solves:

$$\max \ \beta u(x_2(1)), \tag{24}$$

subject to the budget constraints:

$$x_1(1) = c_1(1) + b(1), \tag{25a}$$
$$x_2(1) = c_2(1) - \hat{R}b(1). \tag{25b}$$

Thus, she chooses $x_1(1) = 0$, so that $b(1) = -c_1(1)$, and $x_2(1) = \hat{R}c_1(1) + c_2(1)$. These results imply that the hidden market clears if the total supply of bonds from the π impatient agents is equal to the total demand of bonds from the $(1 - \pi)$ patient agents, or:

$$\pi \frac{c_2(0)}{\hat{R}} = (1 - \pi)c_1(1). \tag{26}$$

The representative bank at date 0 takes into account what the depositors choose at date 1, and solves:

$$\max \ \pi u\left(c_1(0) + \frac{c_2(0)}{\hat{R}}\right) + \beta(1 - \pi)u(\hat{R}c_1(1) + c_2(1)), \tag{27}$$

subject to the intertemporal budget constraint (19) and the incentive compatibility constraint (17). Define the incentive-compatible present value of consumption as I. Using the incentive compatibility constraint in the objective function and in the budget constraint, the problem now reads:

$$\max_{I} \; \pi u(I) + \beta(1-\pi)u(\hat{R}I), \tag{28}$$

subject to $I \leq 1$. Attach the Lagrange multiplier λ to the budget constraint. The first-order condition with respect to I gives the equilibrium value of the Lagrange multiplier:

$$\lambda = \pi u'(I) + \beta\hat{R}(1-\pi)u'(\hat{R}I), \tag{29}$$

which is strictly positive because the felicity function $u(c)$ is increasing and satisfies the Inada conditions. Therefore, by complementary slackness, the budget constraint is binding, which implies:

$$c_1(0) + \frac{c_2(0)}{\hat{R}} = c_1(1) + \frac{c_2(1)}{\hat{R}} = 1. \tag{30}$$

The system of equations consisting of (26) and (30) characterizes the solution to the banking problem: the equilibrium contract is undetermined, since this is a system of three equations in four unknowns.[5] Nevertheless, the final consumption bundle is determined by the type-0 and type-1 problems in (22) and (24), respectively:

$$x_1(0) = c_1(0) + \frac{c_2(0)}{\hat{R}} = 1, \tag{31a}$$

$$x_2(1) = \hat{R}c_1(1) + c_2(1) = \hat{R}, \tag{31b}$$

$$x_2(0) = x_1(1) = 0. \tag{31c}$$

This last result, together with the market clearing condition (26) for the hidden market, allows me to further characterize the bank portfolio allocation between the short and long assets, since:

$$X = \pi c_1(0) + (1-\pi)c_1(1) = \pi c_1(0) + \pi\frac{c_2(0)}{\hat{R}} = \pi, \tag{32}$$

and $Y = 1 - X = 1 - \pi$. I summarize the results in the following proposition:

Proposition 1. *The competitive banking equilibrium in a closed economy is characterized by the final consumption allocation:*

$$x_1(0) = 1, \qquad x_2(1) = \hat{R}, \tag{33a}$$

$$x_2(0) = 0, \qquad x_1(1) = 0; \tag{33b}$$

[5] The solution proposed, among the others, by Farhi et al. (2009), where $c_1(1) = c_2(0) = 0$, $c_1(0) = 1$ and $c_2(1) = \hat{R}$, is one of the many possible solutions. In particular, it is the "no-retrade" contract, where the depositors directly get the incentive-compatible outcome of the hidden trades from the banks.

the return on the hidden bond $R = \hat{R}$, and the bond trading:

$$b(0) = \frac{c_2(0)}{R}, \tag{34}$$
$$b(1) = -c_1(1). \tag{35}$$

The banking contract must satisfy:

$$c_1(0) + \frac{c_2(0)}{\hat{R}} = c_1(1) + \frac{c_2(1)}{\hat{R}} = 1, \tag{36}$$
$$\pi\frac{c_2(0)}{\hat{R}} = (1-\pi)c_1(1), \tag{37}$$

and is undetermined. The bank portfolio allocation is $X = \pi$ and $Y = 1 - \pi$.

The intuition for this result is the following: in equilibrium, the banks must be ex ante indifferent between the "official" banking channel and the hidden market. Therefore, the interest rate on the hidden bond R is equal to the return on the long asset $\hat{R}$, and the bank asset portfolio is undetermined. Nevertheless, this result, together with the incentive compatibility constraint and the binding intertemporal budget constraint, obliges them to set up a contract such that the present value of the consumption bundle that each agent receives (evaluated at the marginal rate of transformation) is independent of the realization of the idiosyncratic type, and equal to the initial endowment. As a consequence, the share of deposits X invested in the short asset is exactly equal to the fraction of depositors π in liquidity need.

4 A Two-Country Economy

In order to study how the presence of hidden channels for the circulation of liquidity affects the process of financial liberalization, in this section I extend the previous environment to a two-country world.

The economy is divided into two countries of equal dimension, that I label Home (H) and Foreign (F). The agents in the two countries are all ex ante equal, in the sense that they all have the same endowment and face the same probability π of being an early consumer. However, the banks have access to two different sets of technologies to hedge against the idiosyncratic risk: the short assets in both countries yield 1 unit of consumption at $t+1$ for each unit invested in t, but the two long assets deliver a country-specific amount of consumption $\hat{R}^i$ in $t = 2$ for each unit invested in $t = 0$, with $\hat{R}^F > \hat{R}^H > 1$. A way of rationalizing this assumption is to think that the two countries face different regulatory environments, or have access to different production technologies. In this respect, we can interpret the returns on the long assets as the opportunity costs of holding liquidity, thus $\hat{R}^F > \hat{R}^H$ means that Home has a comparative advantage in the liquid technology, while Foreign has a comparative advantage

Table 1: The Different Levels of Market Integration

	Domestic HM	International HM
Autarkic BS	Case #1	Case #4
Integrated BS	Case #2	Case #3

in the long asset.

In what follows, I compare four different environments, which reflect all possible combinations of the different levels of integration in the banking system (BS) and the hidden market (HM), that I take as exogenous: I allow the agents to trade either in a domestic or in an international hidden market and, similarly, the two banking systems to be integrated or not. The different levels of integration in the banking system reflect the ability of the banks in each country to invest in the technology of the other country.

Clearly, the case #1 (autarkic banking systems and domestic hidden markets) is the two-country version of the closed economy that I analyzed in the previous section. In the three remaining cases, instead, the two countries integrate, in the sense that the interest rates are equalized across countries. However, the channel through which this equalization is achieved matters in terms of welfare and, in turn, for the level of integration that the two countries are able to coordinate.

4.1 Case #2: Integrated BS and Domestic HM

In this environment, the two hidden bond markets are local, in the sense that the individual depositors cannot borrow and lend among themselves across the border, but only with other agents in the same country. However, the two banking systems are integrated, which means that, given my assumption that $\hat{R}^H < \hat{R}^F$, the representative bank in Home can also invest in the long asset of Foreign. The definition of the competitive banking equilibrium changes accordingly, and takes into account the different ways in which the markets must clear:

Definition 3. *Given an endowment $e = 1$ for each agent and a probability distribution π for the idiosyncratic shock, a competitive banking equilibrium with integrated banking systems and domestic hidden markets is a contract $C^i(\theta) = \{c_1^i(\theta), c_2^i(\theta)\}$, a final consumption allocation $\{x_1^i(\theta), x_2^i(\theta)\}$, an interest rate R^i and an amount of hidden bonds $b^i(\theta)$ for every type $\theta \in \{0, 1\}$ and country $i = H, F$, such that:*

- *for a given interest rate and contract, the final consumption allocation solves the problem in the hidden market* (7)*, for every type in every country;*
- *the contract solves the banking problem* (18)*, in every country;*

- *markets clear in every country:*

$$\pi\left(c_1^i(0)+\frac{c_2^i(0)}{\hat{R}^i}\right)+(1-\pi)\left(c_1^i(1)+\frac{c_2^i(1)}{\hat{R}^i}\right)=1; \tag{38}$$

- *the bond market clears in every country:*

$$\pi b^i(0)+(1-\pi)b^i(1)=0. \tag{39}$$

Following the same logic that I exploited in the closed-economy case, I argue that, in equilibrium, the country-specific interest rate R^i must be equal, in both countries, to the marginal rate of transformation $\hat{R}^F$ of the technology of Foreign, so that the representative banks are ex ante indifferent between investing in the short asset and in the long asset of that country.

This result makes the case #2 similar to the closed economy, with the only exception of the availability of the foreign technology for the Home country. In equilibrium, the present value of the consumption bundle that each type in each country receives, evaluated at the marginal rate of transformation, must be the same, and equal to the initial deposit. Moreover, the amount of resources invested in the liquid asset is again equal to the fraction of depositors who turn out to be impatient, or $X^i=\pi$. I summarize the equilibrium in the following proposition:

Proposition 2. *The competitive banking equilibrium with integrated banking systems and domestic hidden markets is characterized by the final consumption bundles:*

$$x_1^i(0)=1, \qquad x_2^i(1)=\hat{R}^F, \tag{40a}$$
$$x_2^i(0)=0, \qquad x_1^i(1)=0; \tag{40b}$$

the interest rate on the hidden bonds $R^i=\hat{R}^F$ *and the bond trading:*

$$b^i(0)=\frac{c_2^i(0)}{R^i}, \qquad b^i(1)=-c_1^i(1), \tag{41}$$

in every country $i=H,F$. *The banking contracts must satisfy:*

$$c_1^i(0)+\frac{c_2^i(0)}{\hat{R}^F}=c_1^i(1)+\frac{c_2^i(1)}{\hat{R}^F}=1, \tag{42}$$
$$\pi\frac{c_2^i(0)}{\hat{R}^F}=(1-\pi)c_1^i(1), \tag{43}$$

in every country, and is undetermined. The bank portfolio allocations are $X^i=\pi$ *and* $Y^i=1-\pi$ *in every country* $i=H,F$.

4.2 Case #3: Integrated BS and International HM

In this third case, both the banking systems and the hidden bond markets of the two countries are integrated. This means that, as before, the banks in Home can invest in the high-yield technology of Foreign, while the depositors can also exchange resources across the border without being observed. Thus, the only change in the definition of the equilibrium from the previous case lies in the clearing condition in the bond market, which now reads:

$$\sum_{i=H,F} \left[\pi b^i(0) + (1-\pi) b^i(1)\right] = 0. \tag{44}$$

Following the same reasoning as in the previous sections, in equilibrium the (unique) interest rate R on the hidden bond must be equal to the marginal rate of transformation of the integrated economy, $\hat{R}^F$. The problem in the hidden markets also yields the same results as before, so for a type-0 agent $x_2^i(0) = 0$, $b^i(0) = c_2^i(0)/R$, and $x_1^i(0) = c_1^i(0) + c_2^i(0)/\hat{R}^F$, while for a type-1 agent $x_1^i(1) = 0$, $b^i(1) = -c_1^i(1)$, and $x_2^i(1) = \hat{R}^F c_1^i(1) + c_2^i(1)$. This means that the final consumption bundle will be the same as in case #2, i.e. such that the present value that each type in each country receives, evaluated at the marginal rate of transformation, is the same, and equal to the initial deposit. However, the clearing condition in the bond market does not characterize the bank portfolio allocation in each country, but only the total amount of resources invested in liquidity in the whole economy. To see this, notice that (44) can be rewritten as:

$$(1-\pi)c_1^H(1) + (1-\pi)c_1^F(1) = \pi \frac{c_2^H(0)}{\hat{R}^F} + \pi \frac{c_2^H(0)}{\hat{R}^F}. \tag{45}$$

Thus, we can derive the equilibrium world amount of short assets as:

$$\begin{aligned} X^W &= \pi c_1^H(0) + \pi c_1^F(0) + (1-\pi)c_1^H(1) + (1-\pi)c_1^F(1) = \\ &= \pi\left[c_1^H(0) + \frac{c_2^H(0)}{\hat{R}^F}\right] + \pi\left[c_1^F(0) + \frac{c_2^F(0)}{\hat{R}^F}\right] = 2\pi. \end{aligned} \tag{46}$$

Proposition 3. *The competitive banking equilibrium with integrated banking systems and international hidden markets is characterized by the final consumption bundles:*

$$x_1^i(0) = 1, \qquad x_2^i(1) = \hat{R}^F, \tag{47a}$$
$$x_2^i(0) = 0, \qquad x_1^i(1) = 0; \tag{47b}$$

the interest rate on the hidden bond $R = \hat{R}^F$, and the bond trading:

$$b^i(0) = \frac{c_2^i(0)}{R^i}, \qquad b^i(1) = -c_1^i(1). \tag{48}$$

The banking contracts must satisfy:

$$c_1^i(0) + \frac{c_2^i(0)}{\hat{R}^F} = c_1^i(1) + \frac{c_2^i(1)}{\hat{R}^F} = 1, \tag{49}$$

in every country, and the clearing condition in the hidden market:

$$(1-\pi)c_1^H(1) + (1-\pi)c_1^F(1) = \pi\frac{c_2^H(0)}{\hat{R}^F} + \pi\frac{c_2^H(0)}{\hat{R}^F}, \tag{50}$$

and are undetermined. The bank portfolio allocations in the two countries are also undetermined, but the total amount of short and long assets held in the world economy are $X^W = 2\pi$ *and* $Y^W = 2(1-\pi)$*, respectively.*

4.3 Case #4: Autarkic BS and International HM

The last case, where the banks can only invest in their own domestic technologies, but the depositors can borrow and lend among themselves in an international hidden market, deserves some deeper thoughts. The competitive banking equilibrium is defined as follows:

Definition 4. *Given an endowment* $e = 1$ *for each agent and a probability distribution* π *for the idiosyncratic shock, a competitive banking equilibrium with autarkic banking systems and international hidden markets is a contract* $C^i(\theta) = \{c_1^i(\theta), c_2^i(\theta)\}$*, a final consumption allocation* $\{x_1^i(\theta), x_2^i(\theta)\}$*, an interest rate on the hidden bond* R *and an amount of hidden bonds* $\{b^i(\theta)\}$ *for every type* $\theta \in \{0,1\}$ *and country* $i = H, F$*, such that:*

- *for a given interest rate and contract, the final consumption allocation solves the problem in the hidden market* (7)*, for every type in every country;*
- *the contract solves the banking problem* (18)*, in every country;*
- *markets clear in every country* $i = H, F$*:*

$$\pi\left(c_1^i(0) + \frac{c_2^i(0)}{\hat{R}^i}\right) + (1-\pi)\left(c_1^i(1) + \frac{c_2^i(1)}{\hat{R}^i}\right) = 1; \tag{51}$$

- *the bond market clears internationally:*

$$\sum_{i=H,F}\left[\pi b^i(0) + (1-\pi)b^i(1)\right] = 0. \tag{52}$$

Following the same logic that I exploited in the closed-economy case, I argue that the equilibrium interest rate on the hidden bond must lie between $\hat{R}^H$ and $\hat{R}^F$. In fact, if $R > \hat{R}^F$, the short assets would dominate the long assets in both countries. Thus, the patient depositors in both countries would be willing to lend but, finding no borrowers on the hidden market, the equilibrium interest rate R would go to 1, which is a contradiction. Similarly, if $R < \hat{R}^H$, the long assets would dominate the short assets, and the impatient depositors would want to borrow in the hidden market, driving the equilibrium interest rate to infinity, which is again a contradiction. Thus, the only case left is $\hat{R}^H \leq R \leq \hat{R}^F$.

Once more, a type-0 agent in country i solves the problem in the hidden market in (22), so the solution yields $x_2^i(0) = 0$, $b^i(0) = c_2^i(0)/R$, and $x_1^i(0) = c_1^i(0) + c_2^i(0)/R$. Similarly, for a type-1 agent in country i, we follow the problem in (24) and obtain $x_1^i(1) = 0$, $b^i(1) = -c_1^i(1)$, and $x_2^i(1) = Rc_1^i(1) + c_2^i(1)$. The market clearing condition for the hidden bonds then reads:

$$\pi \frac{c_2^H(0)}{R} + \pi \frac{c_2^F(0)}{R} - (1-\pi)c_1^H(1) - (1-\pi)c_1^F(1) = 0. \tag{53}$$

The banks in each country solve the problem (27), subject to the budget constraint (19) and the incentive compatibility constraint (17). However, since $\hat{R}^H \leq R \leq \hat{R}^F$, the representative bank in Home only invests in the short asset, and the representative bank in Foreign only in the its local long asset:

$$X^H = Y^F = 1, \tag{54}$$
$$Y^H = X^F = 0. \tag{55}$$

This also means that we can simplify the expressions for the banking problems in the two countries, since:

$$c_2^H(0) = c_2^H(1) = 0, \tag{56}$$
$$c_1^F(0) = c_1^F(1) = 0. \tag{57}$$

The banking problem in Home becomes:

$$\max \ \pi u(c_1^H(0)) + \beta(1-\pi)u(Rc_1^H(1)), \tag{58}$$

subject to:

$$\pi c_1^H(0) + (1-\pi)c_1^H(1) \leq 1, \tag{59}$$

and the incentive compatibility constraint, which now simplifies to $c_1^H(0) = c_1^H(1)$. This transforms the bank budget constraint in $c_1^H(0) \leq 1$. From the first-order condition, it is easy to see that, as in the closed-economy case, the Lagrange multiplier on the budget constraint:

$$\lambda = \pi u'(c_1^H(0)) + \beta R(1-\pi)u'(Rc_1^H(0)) \tag{60}$$

is strictly positive in equilibrium, because the felicity function $u(c)$ is increasing and satisfies the Inada conditions. Hence, by complementary slackness, the budget constraint holds with equality, and we derive the equilibrium contract $c_1^H(0) = c_1^H(1) = 1$. In a similar way, $c_1^F(0) =$

$c_1^F(1) = 0$ modifies the banking problem in Foreign as:

$$\max\ \pi u\left(\frac{c_2^F(0)}{R}\right) + \beta(1-\pi)u(c_2^F(1)), \tag{61}$$

subject to:

$$\pi\frac{c_2^F(0)}{\hat{R}^F} + (1-\pi)\frac{c_2^F(1)}{\hat{R}^F} \leq 1, \tag{62}$$

and the incentive compatibility constraint $c_2^F(0) = c_2^F(1)$. The same lines of reasoning as before lead us to an equilibrium where the budget constraint holds with equality, so we find that $c_2^F(0) = c_2^F(1) = \hat{R}^F$.

Once we have pinned down the equilibrium banking contracts, the clearing condition in the bond market (53) gives the equilibrium interest rate R as:

$$R = \frac{\pi}{1-\pi}\frac{c_2^F(0)}{c_1^H(1)} = \frac{\pi}{1-\pi}\hat{R}^F, \tag{63}$$

which is an equilibrium if it lies between $\hat{R}^H$ and $\hat{R}^F$. I summarize the results in the following proposition:

Proposition 4. *Assume that* $\frac{\hat{R}^H}{\hat{R}^F} < \frac{\pi}{1-\pi} < 1$. *The competitive banking equilibrium with autarkic banking systems and international hidden markets is characterized by the banking contracts:*

$$c_2^H(0) = c_2^H(1) = 0, \tag{64a}$$
$$c_1^H(0) = c_1^H(1) = 1, \tag{64b}$$
$$c_1^F(0) = c_1^F(1) = 0, \tag{64c}$$
$$c_2^F(0) = c_2^F(1) = \hat{R}^F; \tag{64d}$$

the interest rate on the hidden bond $R = \frac{\pi}{1-\pi}R^F$*; the bond trading:*

$$b^H(0) = b^F(1) = 0, \tag{65a}$$
$$b^H(1) = -1, \tag{65b}$$
$$b^F(0) = \frac{1-\pi}{\pi}; \tag{65c}$$

and the final consumption bundles:

$$x_1^H(0) = 1, \qquad x_2^H(1) = R, \tag{66a}$$
$$x_1^F(0) = \frac{1-\pi}{\pi}, \qquad x_2^F(1) = \hat{R}^F. \tag{66b}$$

The bank portfolio allocations are:

$$X^H = Y^F = 1, \tag{67}$$
$$Y^H = X^F = 0. \tag{68}$$

The intuition for this result is the following. At the equilibrium interest rate, the banks in both countries, which cannot make direct contact with each other, set up a contract such that their depositors enjoy the gains from "hidden" financial integration. The banks in Foreign, which hold a comparative advantage in the long asset, invest their entire endowment in it, and let their impatient depositors borrow liquidity in the hidden market by issuing a total amount of bonds equal to $\pi \hat{R}^F / R$, which is a decreasing function of the interest rate R. Conversely, the banks in Home, which hold a comparative advantage in the short asset, invest their entire endowment in it, and let their patient depositors lend a total amount of liquidity equal to $(1 - \pi)$ in the hidden market, which is completely inelastic to changes in the interest rate R.[6] In that way, the hidden market, in contrast to what happens in the closed economy, is not just a constraint on the portfolio allocation of the banks, but a channel that they can exploit in the absence of an integrated banking system. These considerations are going to be key for the analysis of policy coordination and of the efficiency of the banking equilibrium.

4.4 Hidden Trades and Policy Coordination

I summarize the final consumption allocations $\{x_1^i(0), x_2^i(1)\}$ of the four different equilibria in table 2, and use them to analyze how the presence of hidden channels for the circulation of liquidity affects the process of financial liberalization in the whole economy.

Assume that the two countries coordinate the level of integration that they are willing to implement, based on the expected welfare gains that they can achieve from the policy change. Therefore, with a slight change of notation, I say that the level of integration $j = 1, 2, 3, 4$ (corresponding to the four cases analyzed above) is preferred over another level j' if no country is worse off with j than with j', or:

$$\pi u(x_1^i(0, j)) + \beta(1 - \pi) u(x_2^i(1, j)) \geq \pi u(x_1^i(0, j')) + \beta(1 - \pi) u(x_2^i(1, j')), \tag{69}$$

[6] The assumption on the probability distribution of the idiosyncratic shock is necessary to obtain an interior solution. The results become more complex if we assume that $\pi/(1 - \pi)$ is outside the assumed bounds. If $\pi/(1 - \pi) \geq 1$, the interest rate hits its upper bound, or $R = \hat{R}^F > \hat{R}^H$: the banks in Home still invest their entire endowment in the short asset, but the banks in Foreign are indifferent between the short and the long asset, and need to adjust their portfolio allocation for the hidden market to clear. In particular, the banks in Foreign need to invest more in the short asset, and ensure that such an amount is, in turn, lent in the hidden market. They achieve this by offering early consumption only to the patient depositors. Thus, differently from the basic case, $c_1^F(1) > 0$ and $Y^F < 1$. Similarly, if $\pi/(1 - \pi) \leq \hat{R}^H / \hat{R}^F$, the interest rate hits its lower bound, or $R = \hat{R}^H < \hat{R}^F$: the banks in Foreign still invest their entire endowment in the long asset, but the banks in Home are indifferent between the short and the long asset, and need to adjust their portfolio allocation for the hidden market to clear. In particular, the banks in Home need to invest more in the long asset, and ensure that such an amount is, in turn, used to issue bonds in the hidden market. They achieve this by offering late consumption only to the impatient depositors, thus $c_2^H(0) > 0$ and $X^H < 1$.

Table 2: Competitive Banking Equilibria at Different Levels of Market Integration

	Domestic HM	International HM
Autarkic BS	$x_1^i(0) = 1$, $x_2^i(1) = \hat{R}^i$, $R^i = \hat{R}^i$.	$x_1^H(0) = 1$, $x_2^H(1) = R$, $x_1^F(0) = \frac{1-\pi}{\pi}$, $x_2^F(1) = \hat{R}^F$, $R = \frac{\pi}{1-\pi}\hat{R}^F$.
Integrated BS	$x_1^i(0) = 1$, $x_2^i(1) = \hat{R}^F$, $R^i = \hat{R}^F$.	$x_1^i(0) = 1$, $x_2^i(1) = \hat{R}^F$, $R = \hat{R}^F$.

for every country $i = H, F$.

When the hidden trades are completely forbidden, financial liberalization only pertains to the banking system, and it is easy to argue that is always dominant with respect to autarky: the welfare in Foreign is unaffected, while Home is better off, as its banks can access the high-yield technology of the other country.

In the presence of hidden trades, the governments of the two countries do not have more information than the banks, and are not able to observe the trades in the hidden markets. However, they can determine the level of cross-country integration by deciding who is allowed to trade internationally. Thus, the process of financial liberalization combines two distinct dimensions: the banking system, and the hidden market.

Assume that the two countries start in case #1: autarkic banking systems, and domestic hidden markets. A comprehensive process of financial liberalization, which includes both the integration of the banking systems and of the hidden markets, is equivalent to a move to case #3, and is always approved since:

$$\pi u(1) + \beta(1-\pi)u(\hat{R}^i) \leq \pi u(1) + \beta(1-\pi)u(\hat{R}^F) \tag{70}$$

for every $i = H, F$. This thorough integration allows the banks in Home to invest in the long-term foreign technology, so the patient agents in Home improve their welfare ex post, and all depositors improve their expected welfare ex ante. At the same time, the welfare of the agents in Foreign is unaffected, hence the final allocation in case #3 dominates that in case #1.

The move from case #1 to case #2 or #4 and then to case #3 instead represents a process of sequential liberalization, where either the two banking systems or the two hidden markets are integrated before the others. It is easily seen that, while keeping the hidden markets local, the integration of the two banking systems (moving from case #1 to case #2) is, for the same reasons as for the move to case #3, welfare improving. The opening of the hidden market, while keeping the two banking systems separated (moving from case #1 to case #4), is also welfare improving for both Home and Foreign, as:

$$\text{H}: \quad \pi u(1) + \beta(1-\pi)u(\hat{R}^H) < \pi u(1) + \beta(1-\pi)u\left(\frac{\pi}{1-\pi}\hat{R}^F\right), \tag{71}$$

$$\text{F}: \quad \pi u(1) + \beta(1-\pi)u(\hat{R}^F) < \pi u\left(\frac{1-\pi}{\pi}\right) + \beta(1-\pi)u(\hat{R}^F). \tag{72}$$

The rationale for this result is the following. At the equilibrium interest rate, the banks specialize in the asset in which they hold the comparative advantage (Home's banks in the short asset, and Foreign's banks in the long asset), and let their customers trade unobservably. The patient agents in Home can lend liquidity in the hidden market at a higher rate than in autarky, since $R \geq \hat{R}^H$, and, at the same time, the impatient agents in Foreign can borrow liquidity at a lower rate, since $R \leq \hat{R}^F$. In other words, despite forcing the banks to satisfy incentive-compatibility, the hidden market operates as a channel through which the two countries can exploit their comparative advantages, and enjoy the gains from "hidden" financial integration, so it is always welfare-improving with respect to autarky.

The move from case #2 to case #3, i.e. the opening of the hidden market to cross-country trades in the presence of an already-integrated banking system, is welfare-neutral, as the equilibrium interest rate R is unaffected. More interesting is to see what happens at the integration of the two banking systems, when the hidden markets are already integrated (the move from case #4 to case #3):

$$\text{H}: \quad \pi u(1) + \beta(1-\pi)u\left(\frac{\pi}{1-\pi}\hat{R}^F\right) < \pi u(1) + \beta(1-\pi)u(\hat{R}^F), \tag{73}$$

$$\text{F}: \quad \pi u\left(\frac{1-\pi}{\pi}\right) + \beta(1-\pi)u(\hat{R}^F) > \pi u(1) + \beta(1-\pi)u(\hat{R}^F). \tag{74}$$

While, in case #4, the interest rate R that clears the market must lie between the two marginal rates of transformation $\hat{R}^H$ and $\hat{R}^F$, when the two systems become integrated, the only possible equilibrium rate is that which makes the banks ex ante indifferent between the short asset and the long asset of Foreign, that is, $R = \hat{R}^F$. The increase in the equilibrium interest rate creates winners and losers from financial integration: the patient agents in Home (who are the lenders of case #4) are better off at integration, because their intertemporal terms of trade improve when moving to case #3; in contrast, the impatient agents in Foreign (who are the borrowers of case #4) are worse off, because they borrow at a higher rate. This means that Home is in favor of integration, and Foreign is not. Thus, the necessary mutual agreement to implement the policy reform is broken. In other words, in an environment with hidden trades, the countries can only coordinate a partial level of financial integration.

5 Planner Problem with Hidden Trades

In this section, I characterize the solution to the social planner problem in the presence of hidden trades. This is a necessary step to analyze whether there is space for a regulatory intervention by the government to improve the allocation of the decentralized environment. As in the previous section, in order to highlight the main features of the equilibrium, I first solve the problem in the closed economy, and then extend the results to the two-country world.

5.1 Closed Economy

In a closed economy, the planner chooses an allocation that maximizes the ex ante welfare of the agents:

$$\pi U(c_1(0), c_2(0), 0) + \beta(1-\pi)U(c_1(1), c_2(1), 1), \tag{75}$$

subject to the feasibility constraint (1). Moreover, by the Revelation Principle, the planner imposes a "no-retrade constraint", i.e. the contract must be such that the utility that each type receives must be larger than or equal to the one they would get by retrading, in order to induce truth-telling:

$$U(c_1(\theta), c_2(\theta), \theta) \geq V(C(\theta), R, \theta), \tag{76}$$

for every $\theta \in \{0, 1\}$.

Farhi et al. (2009) show that this problem is equivalent to one where the planner chooses a present value of consumption $\mathcal{I}$ that is equal for all types (so that the incentive compatibility constraint is satisfied and no agent retrades) and the interest rate R on the hidden bond. Intuitively, this is because the planner is not constrained by the no-arbitrage condition between the banking system and the hidden market. Thus, she does not take the return on the hidden bond as given, but is able to pick the optimal one by manipulating the allocation of the aggregate available resources between date 1 and date 2.

Technically, the objective function of the planner:

$$\pi u(\mathcal{I}) + \beta(1-\pi)u(R\mathcal{I}), \tag{77}$$

where $\mathcal{I} = c_1(\theta) + \frac{c_2(\theta)}{R}$, turns out to be similar to the one of the bank in (28), but with the key difference that the interest rate R is now a choice variable. The definition of the equilibrium is the following:

Definition 5. *Given an endowment $e = 1$ for each agent and a probability distribution π for the idiosyncratic shock, a constrained efficient allocation is a present value of consumption $\mathcal{I}$*

and an interest rate R on the hidden bond that solve:

$$\max_{\mathcal{I},R} \; \pi u(\mathcal{I}) + \beta(1-\pi)u(R\mathcal{I}), \tag{78}$$

subject to the resource constraint:

$$\pi\mathcal{I} + (1-\pi)\frac{R\mathcal{I}}{\hat{R}} \leq 1. \tag{79}$$

The first-order conditions of the program (78) with respect to $\mathcal{I}$ and R read:

$$\pi u'(\mathcal{I}) + \beta(1-\pi)Ru'(R\mathcal{I}) = \lambda\left[\pi + (1-\pi)\frac{R}{\hat{R}}\right], \tag{80}$$

$$\beta\mathcal{I}(1-\pi)u'(R\mathcal{I}) = \lambda(1-\pi)\frac{\mathcal{I}}{\hat{R}}, \tag{81}$$

where λ is the Lagrange multiplier attached to the resource constraint (79). Using (81) to simplify (80), we find that $\lambda = u'(\mathcal{I})$, which is strictly positive because the felicity function $u(c)$ is increasing and satisfies the Inada conditions. Thus, by complementary slackness, the resource constraint holds with equality, and the constrained efficient allocation is characterized in the following proposition:

Proposition 5. *The constrained efficient allocation in a closed economy is characterized by the system of equations:*

$$u'(\mathcal{I}^*) = \beta\hat{R}u'(R^*\mathcal{I}^*), \tag{82}$$

$$\pi\mathcal{I}^* + (1-\pi)\frac{R^*\mathcal{I}^*}{\hat{R}} = 1. \tag{83}$$

The constrained-efficient equilibrium interest rate on the hidden bonds R^* is always strictly lower than $\hat{R}$.

The expression in (82) states that, in equilibrium, the planner chooses a contract satisfying an Euler equation, that is, the planner offers a contract equivalent to the one that she would offer in the absence of information asymmetries (i.e. the first best), where the marginal rate of substitution between early and late consumption is equal to the marginal rate of transformation $\hat{R}$.

From the equilibrium condition (82), I can derive the upper bound for the efficient interest rate R^*, as:

$$\hat{R} > \beta\hat{R} = \frac{u'(\mathcal{I}^*)}{u'(R^*\mathcal{I}^*)} \geq R^*, \tag{84}$$

where the first inequality comes from β being less than 1, and the second is a consequence of the fact that the coefficient of relative risk aversion is larger than or equal to 1.[7] This

[7]See note 3.

result states that, in order to make the first best incentive-compatible, the planner imposes a wedge between the marginal rate of transformation $\hat{R}$ and the interest rate on the hidden bond R^*. This is because, as already mentioned in section 3, in the first best the planner compresses the ex post income profile of the agents to cross-subsidize the impatient ones, that is, $\mathcal{I}^* > 1$ and $R^*\mathcal{I}^* < \hat{R}$. However, in the presence of hidden trades, this would not be incentive-compatible, because the patient agents would rather misreport their types to get the higher early consumption $\mathcal{I}^*$ and retrade. The planner ensures that this is not accomplished by reducing the equilibrium interest rate below the marginal rate of transformation.

5.2 Two-Country Economy

I now extend the concepts on the behavior of the planner in a closed economy to the two-country economy of section 4. The planner maximizes the sum of the expected utilities of the two countries:

$$\sum_{i=H,F} \left[\pi U(c_1^i(0), c_2^i(0), 0) + \beta(1-\pi)U(c_1^i(1), c_2^i(1), 1)\right], \tag{85}$$

subject to the resource constraints and the no-retrading constraint. However, she takes as given the exogenous institutional environment, which might constrain her from accessing the technologies of the two countries. In other words, when the two banking systems are autarkic (cases #1 and #4), the planner can only invest the endowment of each country in the available domestic technologies, and when they are integrated (cases #2 and #3) she can instead make the long-term investment in the long asset of Foreign in both countries.

In the case where the banking systems are autarkic and the hidden markets domestic (case #1), the constrained efficient allocation in both countries is, as in the corresponding competitive equilibrium, the same as in the closed economy, and comes as the solution to the system of equations:

$$u'(\mathcal{I}^{i*}) = \beta\hat{R}^i u'(R^{i*}\mathcal{I}^{i*}), \tag{86}$$

$$\pi\mathcal{I}^{i*} + (1-\pi)\frac{R^{i*}\mathcal{I}^{i*}}{\hat{R}^i} = 1, \tag{87}$$

which must hold for every country $i = H, F$. The Euler equation (86) and the resource constraint (87) characterize the country-specific present values of consumption $\mathcal{I}^i$ and interest rates in the hidden markets R^{i*}. As in the closed economy, the equilibrium allocation is equivalent to the first best, *conditional on the institutional environment*: the planner provides an allocation of resources equivalent to the one with no information asymmetries, i.e. such that the marginal rate of substitution between early and late consumption is equal to the marginal rate of transformation $\hat{R}^i$. This means that, in order to provide insurance against the probability of being impatient, the planner compresses the ex post income profiles of the agents by cross-subsidizing early consumption, so $\mathcal{I}^i > 1$ and $R^{i*}\mathcal{I}^{i*} < \hat{R}^i$ in both countries.

Then, in order to make this allocation incentive-compatible, the planner imposes a country-specific wedge between the interest rate R^i and the marginal rate of transformation $\hat{R}^i$, so that no one has incentives to access the hidden market and retrade. For example, with a CRRA felicity function of the form $u(c) = c^{1-\sigma}/(1-\sigma)$, the Euler equation (86) gives $R^{i*} = (\beta \hat{R}^i)^{\frac{1}{\sigma}} < \hat{R}^i$, and $R^{H*} < R^{F*}$ since $\hat{R}^H < \hat{R}^F$.

In the case of integrated banking systems and domestic hidden markets (case #2), the planner can use the high-yield technology of Foreign in both countries, but must set up a contract such that the agents have no incentives to retrade in their domestic hidden markets. The definition of the equilibrium is the following:

Definition 6. *Given an endowment $e = 1$ for each agent and a probability distribution π for the idiosyncratic shock, a constrained efficient allocation with integrated banking systems and domestic hidden markets is a present value of consumption $\mathcal{I}^i$ and an interest rate on the hidden bond R^i for each country $i = H, F$ that solve:*

$$\max_{\mathcal{I}^i, R^i} \sum_{i=H,F} \left[\pi u(\mathcal{I}^i) + \beta(1-\pi) u(R^i \mathcal{I}^i) \right], \tag{88}$$

subject to the resource constraints:

$$\pi \mathcal{I}^i + (1-\pi) \frac{R^i \mathcal{I}^i}{\hat{R}^F} \leq 1, \tag{89}$$

for every $i = H, F$, where $\mathcal{I}^i = c_1^i(\theta) + c_2^i(\theta)/R^i$.

When instead both the banking systems and the hidden markets of the two countries are integrated (case #3), the definition of the planner's problem is different from the previous case only because the planner has to solve for the unique interest rate R that rules out retrading in the international hidden market. More formally, the planner solves:

$$\max_{\mathcal{I}^i, R} \sum_{i=H,F} \left[\pi u(\mathcal{I}^i) + \beta(1-\pi) u(R \mathcal{I}^i) \right], \tag{90}$$

subject to the resource constraints:

$$\pi \mathcal{I}^i + (1-\pi) \frac{R \mathcal{I}^i}{\hat{R}^F} \leq 1, \tag{91}$$

which must hold for every country $i = H, F$.

Essentially, the difference between these two problems lies in the fact that, when the hidden markets are domestic (case #2), the planner can fix a country-specific wedge between the interest rate in the hidden market R^{i*} and the marginal rate of transformation $\hat{R}^F$ while, when the hidden markets are integrated (case #3), there can only be one wedge, to ensure that market clearing is satisfied. However, since the two countries are exactly symmetric (with respect to the distribution of the initial endowment and of the idiosyncratic shock) and have access to the same technologies (because of the exogenous level of integration in the banking system), there is no reason why the two equilibria should be different. Hence, the constrained efficient interest rates in case #2, R^{H**} and R^{F**}, are actually the same, and equal to the constrained efficient interest rate R^{**} of case #3. Thus, the solution to both case #2 and #3 comes from the system of equations:

$$u'(\mathcal{I}^{**}) = \beta \hat{R}^F u'(R^{**}\mathcal{I}^{**}), \tag{92}$$

$$\pi \mathcal{I}^{**} + (1-\pi)\frac{R^{**}\mathcal{I}^{**}}{\hat{R}^F} = 1, \tag{93}$$

which must hold for every country $i = H, F$.

A completely different result comes from the planner's problem when the two banking systems are autarkic, but Home and Foreign share an international hidden market (case #4). Here, the institutional environment forbids the planner to use the high-yield long asset of Foreign for the long-term investment in Home. Thus, while in the other three cases the planner, as in the closed economy, chooses an allocation such that no agent has incentives to retrade, here she is going to exploit the hidden market to achieve the gains from hidden financial integration. For the sake of clarity, here I rewrite the problem in case #4. The planner solves:

$$\max \sum_{i=H,F} \left[\pi u\left(c_1^i(0) + \frac{c_2^i(0)}{R}\right) + \beta(1-\pi)u(Rc_1^i(1) + c_2^i(1))\right], \tag{94}$$

subject to the country-specific resource constraint:

$$\pi\left(c_1^i(0) + \frac{c_2^i(0)}{\hat{R}^i}\right) + (1-\pi)\left(c_1^i(1) + \frac{c_2^i(1)}{\hat{R}^i}\right) \leq 1, \tag{95}$$

which must hold for both countries $i = H, F$, and the incentive compatibility constraint (17), which must still hold because the individual types are private information.

Remember that, in cases #1 to #3, the clearing condition in the hidden market is satisfied because the planner is able to provide the first-best allocation, and therefore no agent retrades because no one can improve the proposed allocation. Here, as in the corresponding competitive equilibrium, the planner instead sets up a contract such that the agents do retrade in the hidden market. This means that, for the hidden market to clear, the unique equilibrium interest rate R^{***} must be the one at which the hidden demand and supply of liquidity are equalized. Thus, the same lines of reasoning as in section 4 leads me to argue that the constrained-efficient interest rate R^{***} must lie between the two marginal rates of transformation $\hat{R}^H$ and $\hat{R}^F$: values lower than $\hat{R}^H$ or higher than $\hat{R}^F$ would push the planner to invest the endowments of both countries either completely in the long assets or completely in the short assets, which would preclude the market clearing.

As a consequence, the planner invests all endowment of Home in the short asset, and lets the patient agents lend liquidity in the hidden market, and all endowment of Foreign in the long asset, and lets the impatient agents borrow liquidity in the hidden market:

$$c_2^{H***}(0) = c_2^{H***}(1) = 0, \tag{96a}$$
$$c_1^{F***}(0) = c_1^{F***}(1) = 0. \tag{96b}$$

This, together with the incentive compatibility constraint in (17), also implies that:

$$c_1^H(0) = c_1^H(1), \tag{97a}$$
$$c_2^F(0) = c_2^F(1). \tag{97b}$$

Thus, I can simplify the planner's problem in case #4 as:

$$\max_{c_1^H(0), c_2^F(0)} \pi \left[u(c_1^H(0)) + u\left(\frac{c_2^F(0)}{R}\right)\right] + \beta(1-\pi)\big[u(Rc_1^H(0)) + u(c_2^F(0))\big], \tag{98}$$

subject to:

$$c_1^H(0) \leq 1, \tag{99a}$$
$$c_2^F(0) \leq \hat{R}^F. \tag{99b}$$

The first-order conditions of the program give strictly positive Lagrange multipliers on the two budget constraints, because the felicity function $u(c)$ is increasing and satisfies the Inada conditions. Thus, by complementary slackness, the budget constraints hold with equality and, in equilibrium, the planner chooses $c_1^{H***}(0) = c_1^{H***}(1) = 1$ and $c_2^{F***}(0) = c_2^{H***}(0) = \hat{R}^F$, exactly as the banks do in the competitive equilibrium. Hence, we can state the following:

Proposition 6. *The competitive banking equilibrium with autarkic banking systems and international hidden markets (case #4) is constrained efficient.*

The intuition for this result is the following. At all other levels of financial integration, the hidden trades represent a burden on the competitive equilibrium: in fact, were they forbidden, the banks would be able to offer the same constrained-efficient allocation provided by the planner. In contrast, here the hidden trades are necessary, because they are the only available channel for integrating the two countries. The planner picks an interest rate on the hidden bond that lies between the two marginal rates of transformation, because that is the only unique interest rate that clears the hidden market for positive trades. As a consequence, the specialization of the banks of each country in the asset in which they hold a comparative advantage (the banks in Home in the short asset, and the banks in Foreign in the long asset) is a constrained efficient portfolio strategy: there exists no other feasible allocation that, at the equilibrium interest rate R^{***}, satisfies the incentive compatibility constraint and yields a higher welfare than the competitive banking equilibrium.

This is a critical result for two connected reasons: first, because it disproves the classic result of Jacklin (1987) and Allen and Gale (2004), who showed that the possibility for the agents of trading in the market distorts the efficiency of the banking equilibrium in Diamond-Dybvig environments; second, because, in all the other three cases, the differences between the competitive banking equilibria and the corresponding constrained efficient allocations provide the rationale for the introduction of some kind of government intervention to decentralize the constrained efficient outcome. However, when the two banking systems are separated and cross-country hidden trades are allowed, there is no way through which the government can improve the outcome of the decentralized environment.

6 Optimal Regulation

The complete characterization of the planner solution, for the different levels of financial integration, allows me to compare the constrained efficient allocations of section 5 to those offered by the banks in the competitive equilibria of section 4. This comparison will highlight the available space for a regulatory intervention, to improve the decentralized outcomes, and provide the lead to what is the right regulation that we should impose on the system, depending on the level of integration in the banking systems and in the hidden markets. As in the previous sections, I start with the analysis of the closed economy, and then move to the two-country environment.

6.1 Closed Economy

From the comparison between the constrained efficient allocation and the decentralized solution, it is evident that the difference between the two is essentially due to the equilibrium interest rates R and R^*, as the first must be equal to the marginal rate of transformation $\hat{R}$, while the second is instead lower. As I said above, this is a consequence of the fact that, in the decentralized environment, the only possible equilibrium that clears the hidden market is the one where there are no arbitrage opportunities, i.e. the banks and the agents are ex ante indifferent between the investment in the banking system and the hidden market. In turn, the equality of returns pushes the banks to skew their asset portfolios towards the long asset, in order to ensure incentive compatibility, and this rules out the efficient cross-subsidization of the impatient depositors that the planner provides.

The obvious consequence of this observation would then be to directly regulate markets, for example through the imposition of taxes, so as to affect the equilibrium interest rate R. However, this is impossible, because trades are observable to neither the intermediaries nor the regulators. Therefore, what I propose here is a regulatory intervention such that the banks autonomously implement the constrained efficient allocation, which takes the form of a minimum liquidity requirement:

$$X \geq M. \tag{100}$$

The rationale of such a rule is the following. In the newly regulated equilibrium, the interest rate is going to be lower than the marginal rate of transformation. This means that the short asset is going to be dominated by the long asset, and no intermediary will hold liquidity. However, this cannot be an equilibrium, since clearing in the hidden market would be violated: the impatient consumers would like to borrow, but no one would lend to them. Thus, the only way in which the banking system can sustain a competitive banking equilibrium where the interest rate is lower than the return on the long asset is via the introduction of a minimum liquidity requirement, so that the banks are forced to hold enough resources to finance early consumption. In other words, by picking the right requirement, the regulator manipulates the bank portfolios *directly*, and the interest rate *indirectly*, by reshuffling the resources across time in an efficient way.

More formally, I extend the problem in (27) with the imposition of the constraint in (100). To solve this problem, I apply the following change of variables:

$$\mathbf{I} = c_1(0) + \frac{c_2(0)}{R}, \tag{101}$$

$$\mathbf{H} = \pi c_2(0) + (1-\pi)c_2(1), \tag{102}$$

where $\mathbf{I}$ is the present value of the consumption bundle that an impatient depositor gets from the contract (that, by incentive compatibility, must be equal to the present value that a patient depositor gets), and $\mathbf{H}$ is the total amount of consumption good that the bank has to pay at $t = 2$. The minimum liquidity requirement can be expressed in the following way:

$$\begin{aligned} X &= \pi c_1(0) + (1-\pi)c_1(1) = \\ &= c_1(0) + (1-\pi)(c_1(1) - c_1(0)) = \\ &= c_1(0) + (1-\pi)\frac{c_2(0) - c_2(1)}{R} = \\ &= \left(c_1(0) + \frac{c_2(0)}{R}\right) - \frac{\pi c_2(0) + (1-\pi)c_2(1)}{R} = \\ &= \mathbf{I} - \frac{\mathbf{H}}{R} \geq F, \end{aligned} \tag{103}$$

where I used the incentive compatibility constraint in the third step. In a similar way, I rewrite the intertemporal budget constraint as:

$$\mathbf{I} - \mathbf{H}\left[\frac{1}{R} - \frac{1}{\hat{R}}\right] = 1. \tag{104}$$

Therefore, the banking problem now reads:

$$\max_{\mathbf{I},\mathbf{H}} \ \pi u(\mathbf{I}) + \beta(1-\pi)u(R\mathbf{I}), \tag{105}$$

subject to (103) and (104). I attach the multipliers μ and λ to (103) and (104), respectively, so that the first-order conditions of the program are:

$$\mathbf{I}: \qquad \pi u'(\mathbf{I}) + \beta R(1-\pi)u'(R\mathbf{I}) = \lambda - \mu, \tag{106a}$$

$$\mathbf{H}: \qquad \frac{\mu}{R} = \lambda\left[\frac{1}{R} - \frac{1}{\hat{R}}\right], \tag{106b}$$

which can be put together in:

$$\pi u'(\mathbf{I}) + \beta R(1-\pi)u'(R\mathbf{I}) = \lambda\frac{R}{\hat{R}}. \tag{107}$$

For some positive multipliers μ and λ, the constrained efficient allocation satisfies the optimality conditions: we can see this by substituting the interest rate R, the present value $\mathbf{I}$ and the amount of late consumption $\mathbf{H}$ with the corresponding values in the constrained efficient allocation $R^* < \hat{R}$, $\mathcal{I}^*$ and $\mathcal{H}^* = (1-\pi)R^*\mathcal{I}^*$. Hence, the minimum liquidity requirement in equilibrium holds with equality, and implements the planner solution as the outcome of the decentralized environment.

Proposition 7. *The minimum liquidity requirement:*

$$M = \pi\mathcal{I}^*, \tag{108}$$

where $\mathcal{I}^*$ comes from the solution to the planner problem, implements the planner solution in the competitive banking equilibrium.

Two things are worth noticing. First, since in the regulated equilibrium $R^* < \hat{R}$, no agent has incentives to retrade in the hidden market, the market clearing condition of the definition 2 is satisfied. Second, since $\mathcal{I}^* > 1$ as showed in section 3, the minimum liquidity requirement M is larger than π: the optimal regulation pushes the banks to hold more liquidity than in the unregulated equilibrium. Intuitively, this works because the minimum liquidity requirement reallocates the aggregate resources of the economy from date 2 to date 1, thus lowering the interest rate.

6.2 Two-Country Economy

The lesson that we learn from the closed-economy case is that, despite the fact that the origin of the inefficiency of the competitive banking equilibrium lies in the pricing system in the hidden market, we can solve the resulting misallocation of resources by instead imposing the right minimum liquidity requirement on the banking system. The aim of this section is to replicate the analysis in the two-country case, and characterize the optimal regulatory intervention as a function of the different levels of integration in the banking systems and the hidden markets.

The optimal regulation when the banking systems of the two countries are autarkic and each country has a domestic hidden market (case #1) is a replica of that in a closed economy: in each country, the equilibrium interest rate $R^i = \hat{R}^i$ is above its efficient level R^{i*}, and a country-specific minimum liquidity requirement of the form $M^i = \pi \mathcal{I}^{i*}$ decentralizes the constrained efficient allocation by raising the amount of short assets that the banks hold in portfolio. Two things are worth-noticing: first, the heterogeneity of the optimal regulation across countries is not a consequence of different levels of idiosyncratic risk, but of different investment technologies. Second, for a standard CRRA felicity function, the minimum liquidity requirement is looser (i.e., lower) in the country that holds a comparative advantage in liquidity (in this case, Home) than in the other. To see this, notice that the efficient amount of early consumption $\mathcal{I}^{i*}$ that the regulated banks provide is a decreasing function of the country-specific ratio $R^i/\hat{R}^i$ between the interest rate and the marginal rate of transformation. From the Euler equation (86):

$$\frac{R^i}{\hat{R}^i} = \beta^{\frac{1}{\sigma}} \hat{R}^{i\,\frac{1}{\sigma}-1}. \tag{109}$$

Since the coefficient of relative risk aversion σ is larger than or equal to 1 by assumption, we have that $R^H/\hat{R}^H \geq R^F/\hat{R}^F$, because $\hat{R}^H < \hat{R}^F$. Therefore $\mathcal{I}^{H*} \leq \mathcal{I}^{F*}$, and $M^H \leq M^F$.

In a similar way, when the banking systems are integrated (cases #2 and #3), and the banks in Home gain access to the long asset of Foreign, the interest rates on the hidden bonds $R^i = \hat{R}^F$ are above their economy-wide efficient levels, notwithstanding if the hidden markets are integrated or not. Therefore, the optimal regulation is an economy-wide minimum liquidity requirement $M = \pi \mathcal{I}^{**}$. This means that a process of financial liberalization that moves from the complete separation of the two countries to an integrated banking system (from case #1 to case #2 or #3) should also be accompanied by a regulatory change: an increase in the minimum liquidity requirement of the country that gains the most from integration, i.e. whose intertemporal terms of trade have improved (in this case, Home). However, there is a key difference between case #2 and case #3. In case #2 (integrated banking systems and domestic hidden markets), the market clearing conditions on the local hidden markets allow us to completely characterize the equilibrium share of initial wealth invested in the short asset by the banks in each country (see proposition 2). Hence, the imposition of the minimum liquidity requirements increases the equilibrium amount of short assets in both countries and, as a consequence, lowers the interest rates. Conversely, in case #3 (integrated banking systems and international hidden markets), we are only able to characterize the equilibrium total amount of short assets held by the banking system in the whole economy (see proposition

Table 3: Optimal Regulation at Different Levels of Market Integration

	Domestic HM	International HM
Autarkic BS	$M^H = \pi\mathcal{I}^{H*}$ $M^F = \pi\mathcal{I}^{F*}$	No Regulation
Integrated BS	$M^H = \pi\mathcal{I}^{**}$ $M^F = \pi\mathcal{I}^{**}$	$M^H = \pi\mathcal{I}^{**}$ $M^F = \pi\mathcal{I}^{**}$

3). Therefore, the imposition of the minimum liquidity requirements increases the *economy-wide* amount of short assets held by the banking system, but we cannot specify whether the amount of short assets held by the banks in each country increases or not.

Finally, we are left with the last, and most interesting, result of section 5: the competitive banking equilibrium with autarkic banking systems and international hidden markets (case #4) is constrained efficient. This means that there exists no other feasible allocation that satisfies the incentive compatibility constraint and dominates the market outcome and, therefore, no government intervention can be imposed without negatively altering the competitive equilibrium. As a consequence, a process of financial liberalization that opens the hidden markets to international trades, while keeping the two banking systems separate (a move from case #1 to case #4) – which we saw is the maximum level of coordination that Home and Foreign can achieve – should be accompanied by the elimination of the country-specific minimum liquidity requirements that the two countries need in financial autarky. If that were not the case, the equilibrium portfolio strategy of the banks in Home would not change (at the equilibrium interest rate, they would still invest all their endowment in the short asset and the minimum liquidity requirement would be slack), but the banks in Foreign would be forced to invest an inefficient amount of deposits in the short asset, thus lowering the interest rate below its constrained efficient level.

7 Concluding Remarks

In the present work, I propose a mechanism to rationalize the observation that the process of financial integration around the world has come to a halt in the last 10-15 years: financial integration affects the equilibrium prices of all those market-based unregulated channels for the circulation of liquidity that have developed as a consequence of financial liberalization and capital mobility, thus creating winners and losers from integration and hindering further expansions. To formalize this idea, I construct a two-country model of banking, where the banks have access to country-specific investment technologies, and the depositors can borrow and lend among themselves in a hidden market.

The main lesson that we learn from this exercise is that the order in which the markets are integrated is of importance for how deep the integration process can go. This also means that further waves of financial liberalization might come at a lower pace, and eventually among countries that are economically and financially homogeneous. Moreover, it is not clear whether a tighter international connection among countries can lead to a deeper financial integration than that achievable through coordination: in the present environment, for example, a political union (i.e. a supranational authority maximizing the sum of the total welfare of the two countries) would prefer a complete integration of the financial systems of the two countries (case #3) to a system where only the hidden markets are integrated (case #4) only if the expected welfare gains of the patient agents in Home are higher than the expected welfare losses of the impatient agents in Foreign (which is not always true here).

Finally, the analysis of the constrained efficiency of the different competitive equilibria suggests that financial regulation should adapt to the level of integration of the international financial system and, in general, become stricter for those countries that gain more from integration. However, regulation is not always necessary: there exist environments where the presence of unregulated trading opportunities does not limit the constrained efficiency of the competitive equilibrium, and therefore does not justify the introduction of minimum liquidity requirements.

The environment developed in the present work is a source of further interest because of the possible extensions that we can make from it: in particular, it would be interesting to study the robustness of the results to financial crises, in the form of bank runs or unexpected spikes in the probability of the idiosyncratic shock, and how the resilience of the system changes with the level of financial integration in the banking system (as in Allen and Gale, 2000), as well as in the hidden market. I leave these issues for future research.

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Appendices

A Why Only Bonds in the Hidden Market?

Remember that, when borrowing and lending, the individual types are still private information. In order to complete the set of traded securities, we may then add claims paying 1 unit of the consumption good conditional on reporting type θ. Define the price of those securities as $Q(\theta)$. The price of a risk-free bond delivering one unit of consumption in the following period for each unit invested today is $1/R$. I can prove the following:

Lemma 1. $Q(\theta) \geq \frac{1}{R}$ *for every type* $\theta \in \{0, 1\}$.

Proof. I prove the lemma by contradiction. Assume that $Q(\theta') < \frac{1}{R}$ for some θ'. That would give rise to arbitrage opportunities: agents would buy an infinite number of securities, sell uncontingent bonds of the same amount, then report exactly type θ', and enjoy infinite utility. That cannot be an equilibrium. ■

Given that $Q(\theta) \geq \frac{1}{R}$, no type-contingent claims will be traded: the agents will never exchange securities which yield one unit of consumption if a specific type is reported, when they have the opportunity to trade a cheaper bond which yields one unit of consumption whatever type is reported.

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