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FINANCIAL CRISIS: A NEW MEASURE FOR RISK OF PENSION FUNDS ASSETS

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Financial crisis: a new measure for risk of pension funds assets

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Abstract

It has been debated that pension funds should have limitations on their asset allocation, based on the risk profile of the different financial instruments available on the financial markets. This issue proves to be highly relevant at times of market crisis, when a regulation establishing limits to risk taking for pension funds could prevent defaults. In this paper we present a framework for evaluating the risk level of a single financial instrument or a portfolio. By assuming that asset returns can be described by a multifractional Brownian motion, we evaluate the risk using the time dependent Hurst parameter H(t) which models volatility. To provide a measure of the risk, we model the Hurst parameter with a random variable with beta distribution. We prove the efficacy of the methodology by implementing it on different risk level financial instruments and portfolios.

Keywords: Pension Funds, risk control, multifractional Brownian motion.

JEL Classification: C22; G11; G23.

1 Introduction

The performance of pension funds is usually measured in terms of returns rather than risk. One of the consequences of the 2008 financial crisis was the default of some of the biggest pension funds worldwide. The case of the California fund CalPERS is emblematic: by focusing on high rate-ofinvestment-returns whilst overlooking risk levels they now carry huge unfunded liabilities. Measuring returns rather than risk has therefore proved to be inadequate during market crises.

Earlier studies (Ryan and Fabozzi [24]) show that post 2001 bankruptcies of US pension funds had their roots in the actuarial evaluation techniques rather than in asset losses, if long-term stock return is considered. According to Bader [5] and McClurken [17], post retirement benefit plans, pertaining to the 'first pillar' of a pension system, should not invest in high-risk financial instruments because this would lead to problems related to moral hazard and to the evaluation of 'superfluous risk'.

In [26], Stewart analyzes the increasing tendency of pension funds to invest in hedge funds. He observes that in many cases the real risk is not correctly perceived. This is due to an inefficient regulating system and, in several countries, the absence of risk monitoring instrument. In many cases the rules on pension funds investments are derived from the same laws that regulate investment companies, considering their speculative function. These regulations often indicate a qualitative restriction without limiting the quantitative measurement of the risk.

In [13], Halim analyzes how funds manage investments and show that funds that manage both active and surplus risk have generated better riskreward trade offs.

In Italy the regulating system for pension funds establishes non restrictive rules in the investment portfolio composition. Pension funds can invest in liquid assets, stocks, share of common investment funds. There are some restrictions about investments in equity and bonds traded in the over the counter markets and/or in non OECD countries.

The aim of our paper is to model the investment risk and our framework is based on that of Bianchi and Trudda (see [8]). We define the risk as the roughness of the returns of the price series and we model the returns of a price series by a Multifractional Brownian Motion with random exponent. The roughness of the process is then represented by the Hölder exponent. Our main contribution consists in interpreting the Hölder exponent as a random variable H and in deriving the salient characteristics of its distribution to provide a measure of the risk. In the application, two investment portfolios are simulated to show how the levels of risk obtained can be very different.

2 Time series and volatility modelling

To describe the price dynamics we use a multifractional process with random exponent (MPRE). Let us first introduce the fractional Brownian motion (fBm)(see paper by Mandelbrot and Van Ness [16]).

2.1 Fractional and Multifractional Brownian Motion for time series and volatility modelling

The fBm is characterized by a slowly decaying autocorrelation function depending on the *Hurst* exponent $H \in (0, 1]$. Following the definition that can be found in [9], the process has moving average representation

$$B_H(t) = C\{\pi K(2H)\}^{1/2} \int_R f_t(s) dB(s)$$
(1)

with

$$f_t(s) = \frac{1}{\Gamma\left(H + \frac{1}{2}\right)} \left\{ |t - s|^{H - \frac{1}{2}} \mathbf{1}_{] - \infty, t]}(s) - |s|^{H - \frac{1}{2}} \mathbf{1}_{] - \infty, 0]}(s) \right\}$$

where $B(\cdot)$ stands for the ordinary Brownian motion, C is a positive constant and K is the function defined on]0,2[as $K(\alpha) = \Gamma(\alpha+1)\frac{\sin\frac{\alpha\pi}{2}}{\pi}$. The process is self-similar of parameter H and has stationary increments. Its covariance function reads as

$$E(B_H(t)B_H(s)) = \frac{c^2}{2} \left(|t|^{2H} + |s|^{2H} - |t-s|^{2H} \right)$$
(2)

The multifractional Brownian motion (mBm, see [20], [15], [3]) is a generalization of the fBm obtained by allowing H to vary over time and has the following representation

$$M_{H(t)}(t) = C\{\pi K(2H(t))\}^{1/2} \int_{R} f_t(s) dB(s)$$
(3)

with

$$f_t(s) = \frac{1}{\Gamma\left(H(t) + \frac{1}{2}\right)} \left\{ |t - s|^{H(t) - \frac{1}{2}} \mathbf{1}_{]-\infty,t]}(s) - |s|^{H(t) - \frac{1}{2}} \mathbf{1}_{]-\infty,0]}(s) \right\}$$

where $H : [0, \infty) \to (0, 1]$ is required to be a Hölder function of order $0 < \eta \leq 1$ to ensure the continuity of the motion.

Notice that since H(t) is the punctual Hölder exponent of the mBm at point t, the process is locally asymptotically self-similar with index H(t) (see, e.g., Benassi et al.[6])) in the sense that, denoted by $Z(t, au) := M_{H(t+au)}(t+au) - M_{H(t)}(t)$ the increment process of the mBm at time t and lag au, it holds

$$\lim_{a \to 0^+} a^{-H(t)} Z(t, au) \stackrel{d}{=} B_{H(t)}(u), \ u \in R.$$
(4)

The above distributional equality indicates that at any point t there exists an fBm with parameter H(t) tangent to the mBm. Moreover, since $B_{H(t)}(u) \sim \mathcal{N}(0, C^2 u^{2H(t)})$, the infinitesimal increment of the mBm at time t, normalized by $a^{H(t)}$, normally distributes with mean 0 and variance $C^2 u^{2H(t)}$ $(u \in R, a \to 0^+)$.

The increments of the mBm are no longer stationary nor self-similar; despite this, the process is extremely versatile since the time dependency of H is useful to model phenomena whose punctual regularity is time changing.

In our context, H(t) can represent the degree of confidence the investors nourish in the past. High values of H(t) correspond to trends (or low volatility phases), i.e. to periods in which the past information weighs in the investors' trading decisions; low values of H(t) are associated to high volatility periods, in which prices display an anti-persistent or mean reverting behaviour because of the quick buy-and-sell activity that is typically induced by uncertainty. Standard financial theory is recovered when $H = \frac{1}{2}$, case in which the mBm reduces to the Brownian motion.

2.2 Our proposed model

By allowing H to be a stochastic process or a r.v, the mBm can be further generalized to the Multifractional Process with Random Exponent (see [4]):

Let: $H : [0,1] \to [a,b] \subset [0,1]$ be a random or stochastic process, $B_H : [0,1] \times [a,b] \subset (0,1)$ a Gaussian field and $f_1 : [0,1] \to [0,1] \times [a,b]$, $t \to (t, H(t,w))$ and $f_2 : [0,1] \times [a,b] \to \mathbb{R}$, $(t,H) \to B_H(t,w)$. Then the process is defined by:

$$Z(t,w) = f_2(f_1(t)) = B_{H(t,w)}(t,w)$$

Here follow the salient properties of the MPRE that inspired our model:

- 1 (Ayache and Taqqu [4]) The pointwise Hölder exponent of the MPRE is determined by H(t, w) a.s..
- 2 (Ayache, Taqqu [4]) If H is a random variable independent of the Brownian motion then the process is stationary.
- 3 (Bianchi [7]) H(t, w) can be interpreted as the confidence level investors have in past information.

Property 1 means that we can evaluate the roughness of the series by using H(t, w). Property 2 claims that stationarity is a necessary condition for H to be a random variable independent of the Brownian motion. A consequence of property 3 is that H is not symmetrical with respect to its central value 1/2 as it is easier (quicker) to loose confidence than to build it. It therefore follows that, provided we are dealing with stationary processes we can model H with a random variable and this will give us a measure of the roughness and so the risk of a price series. Given the asymmetry of H w.r.t. its central value of 1/2 and its range in the interval [0, 1], we choose a random variable with beta distribution. Indeed, the beta distribution presents some characteristics that mime very well those of H.

Our proposed methodology to evaluate the risk of a price series can be summarized in the following steps:

1) Establish the stationarity of the series of returns

To accomplish this we use the Dickey-Fuller roots of unity test.

- 2) Define the risk as the roughness of the series of returns
- 3) Model the series of returns by a MPRE and the roughness by the Hölder exponent

At this stage, we need to estimate the Hölder exponent from real data. To do so, we adopt a family of "moving-window" estimators of H(t) based on the k-th absolute moment of a Gaussian random variable of mean zero and given variance V_H (the variance of the unit lag increment of a mBm) as defined in [7]. Given a series of length N and a window of length δ , the estimator has the form

$$H_{\delta,N}^{k}(t) = \frac{\log\left(2^{k/2}\Gamma\left(\frac{k+1}{2}\right)V_{H}^{k/2}\right) - \log\left(\frac{\sqrt{\pi}}{\delta}\sum_{j=t-\delta}^{t-1}|X_{j+1,N} - X_{j,N}|^{k}\right)}{k\log(N-1)} (5)$$

for $j = t - \delta, ..., t - 1; t = \delta + 1, ..., N; k \ge 1.$

The quick rate of convergence $O\left(\delta^{-\frac{1}{2}}(\log N)^{-1}\right)$, the estimator is reliable even for very short $\delta's$. Toilsome computations show that when $H = \frac{1}{2}$ the variance of the estimator reduces to

$$Var(H^{k}_{\delta,N}(t)) = \frac{\sqrt{\pi}}{\delta k^{2} \ln^{2}(N-1) \left[\Gamma\left(\frac{k+1}{2}\right)\right]^{2}} \cdot \left(\Gamma\left(\frac{2k+1}{2}\right) - \frac{1}{\sqrt{\pi}} \left[\Gamma\left(\frac{k+1}{2}\right)\right]^{2}\right)$$
(6)

and the optimal value of k is deduced by minimizing the last relation. (6) reaches a minimum for k = 2, so this is the value we set in our experiments.

4) Model H with a random variable with beta distribution

We found that the *Beta* distribution can approximate quite well the Hölder exponent.

The probability density function of the *beta* distribution is given by:

$$y = f(x, a, b) = \frac{1}{B(a, b)} x^{a-1} (1-x)^{b-1}$$

where $B(a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$ is the beta function. Its mean and variance are given by:

$$\mu = \frac{a}{a+b}$$
 $\operatorname{Var} = \frac{ab}{(a+b+1)(a+b)^2}$

5) Use the parameter of the distribution as a risk measure

The parameters a and b of beta provide us with a measure of the risk of the price series. In particular we can use b as a direct measure: the higher the value of b the smaller the risk and vice versa.

3 Experimental Evaluation

3.1 *beta* goodness of fit

We first need to establish how well the *beta* distribution approximates the Hölder exponents that we derive from real data. In order to do this we

Table 1: beta's parameter error estimation

| Parameter | Average Error | Maximum Error |
|-----------|---------------|---------------|
| a | 1.1032 | 1.4329 |
| b | 1.0035 | 1.3562 |

derived H from 20 daily price series over a time span of 10 years. This included market indexes (Bovespa, Athen Index Compos, Hang Seng Index, Nasdaq, Dow Jones, FTSE) and various shares listed in those indexes. We then fitted the *beta* distribution and derived the parameters a and b, keeping track of the standard error estimation and of the log likelihood. The mean log likelihood of the fit was 3572. The parameters estimation errors are summarized in table 1. Figure 1 shows the fitting of the distribution on four different return series: HK, Arca Bond Emergenti, Telecom, Bovespa. We can see how the beta distribution fits the asymmetry of the density distribution of the return series.



Figure 1: Beta distribution fit to four different returns series

3.2 Application at time of financial crisis

To evaluate the performance of the method at times of financial crisis, we simulated two portfolios, made of the following assets: four indexes, Athen, Bovespa, Ibex,HSI; 4 shares listed in the previous indexes, HK, HK bank of China, Brasil Petroleo, Telecom, and a fund made of bonds from emerging markets, Arca Bond Emergenti. The assets were daily time series from January 2006 to December 2011, which included the 2007-2008 market crisis.

The first portfolio consisted of all the previously listed assets, and being made of 90% shares and 10% bond we label it as a high risk portfolio (HRP). The second portfolio was made of the Arca Bond Emergenti for the 90% and for the 10% of a share (HK). We will refer to the second portfolio as to LRP (low risk portfolio).

For each asset in the portfolio we estimated H with a moving window of 20 days lag. Figure 2 shows the results: the green curve is relative to the bond, which proves to be less risky than all the shares, as expected.



Figure 2: Estimated H for all assets in portfolios

To estimate H for the two portfolios, we summed up the return series of the portfolios and then calculated the value of H. Figure 3 (a) shows how the high risk portfolio has lower value of H and higher return variations, whereas in figure (b) we can see how the low risk portfolio has higher values of H and lower variations of returns.

We then fitted the *beta* distribution to find the parameters that give a quantitative measure of the risk. Figure 4 (a) shows the two fits, whereas



Figure 3: Estimated H for the portfolios HRP and LRP respectively



Figure 4: (a): beta fitting of H of portfolios HRP and LRP; (b): mean and variance of the beta distribution for the single assets and for the two portfolios.

in 4 (b) the mean and variance of the fits for the single assets and the two portfolios are plotted. As we can see, the low risk portfolio has a much higher mean compared to the high risk one, suggesting the mean of beta as the strongest value to take into account when calculating the risk. Despite the high mean, the variance is higher for the bonds based asset and the lower risk portfolio. Therefore, the variance information must be coupled with the mean one to give us a measure of risk. This will be the subject of our investigation in the coming future.

4 Conclusions and further developments

In 2008 a market crisis caused the failure of major pension funds worldwide. Several analysis show the funds tend to increase the portfolios risk in order to obtain higher values of the expected global asset return while many authors emphasize that pension funds have to maintain a prudent profile because the social function (in particular for the first pillar) prevails over the speculative function. In general, financial laws use mutual fund regulations to determine the limits of investments in risky financial instruments. Moreover, regulations are often qualitative and do not use quantitative methods. We proposed a method to quantitatively asses the risk of pension funds, which consists of modelling the return series with a MPRE and the risk with the Hölder exponent H of the process. In order to quantify the risk we model H with a random variable with beta distribution, which depends on two parameters that can vary to fit very well the asymmetry of H, at the same time providing us with different possible measure of the risk. For the time being, we propose to use the mean as a major parameter to evaluate the risk but we are investigating risk measures that depend on both the mean and the variance and more generally on functions of the two parameters a and b of beta.

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