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GLOBAL INDETERMINACY IN A TOURISM SECTOR MODEL

Oliviero Antonio Carboni
Paolo Russu

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CRENoS – CAGLIARI
VIA SAN GIORGIO 12, I-09100 CAGLIARI, ITALIA
TEL. +39-070-6756406; FAX +39-070- 6756402

CRENoS - SASSARI
VIA TORRE TONDA 34, I-07100 SASSARI, ITALIA
TEL. +39-079-2017301; FAX +39-079-2017312

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Via Is Mirrionis, 1
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Global Indeterminacy in a Tourism Sector Model

Oliviero A. Carboni
University of Sassari and CRENoS
Paolo Russu
University of Sassari

Abstract
This article develops a dynamic optimising macro model of a open economy specialised in tourism based on natural resources. Environmental externalities are explicitly introduced in the production function. Global dynamic analysis shows that, under some conditions on the parameters, if the initial values of the state variables are close enough to the coordinates of P_A, then there exists a continuum of equilibrium trajectories approaching P_A and one trajectory approaching P_B. Therefore, the model exhibits global indeterminacy, since either P_A or P_B can be selected according to agent expectations.

Keywords: global and local indeterminacy, environmental externalities, history versus expectations, Hopf bifurcation.

JEL Classification: C62, O13, O41, Q22

Corresponding authors: Email: ocarboni@uniss.it; russu@uniss.it

1 Introduction

As [9] and [10] pointed out in their seminal papers,equilibrium selection in dynamic optimization models with externalities may depend on expectations; that is, given the initial values of the state variables (history), the path followed by the economy may be determined by the choice of the initial values of the jumping variables. This implies that expectations play a key role in equilibrium selection and global indeterminacy may occur. Starting from the same initial values of the state variables, different equilibrium paths can approach different ω-limit sets (for example, different stationary states).

In this paper, indeterminacy is generated by negative environmental externalities of production activity affecting the natural resource, as in [3] and in [4], where negative externalities may generate indeterminacy in an economy where private goods can be consumed as substitutes for free access environmental goods.

Following [2], we take into consideration the case where two steady states exist, $P_A^* = (E_A^*, K_A^*, C^*)$ (sink) and $P_B^* = (E_B^*, K_B^*, C^*)$ (saddle with two-dimensional stable manifold), with $E_A^* < E_B^* <,\ K_A^* > K_B^* <,\ C^{å} = (\frac{2r}{\sigma_2})^2$. So, if we obtain, through the numerical simulation the shape of the stable manifold, that in three-dimensional is separatrices between different regimes of the trajectories, it is possible to modify the initial choice of consumption in such a way that the economy approaches either P_A^* or P_B^*. (see Figure 3 in Section 4).

2 Formulation of the model

Nature based destinations utilize natural resources and physical capital in the production of tourist services. The objective is to maximize welfare to the local community from the production of tourist services. This involves determining the optimal trajectories of consumption in the local area and the use of the environmental resource. Initially, we consider that the production of tourist services depends on the tourist demand Q attended by the local economy. The tourists are attracted by the stock of physical capital offered K (hotels, transport infrastructure and services), the stock of natural resource E in the area and negatively affected by price p. Assuming as in [7] a linear tourism demand [1]

$$Q(t) = \sigma_1 E(t) + \sigma_2 K(t) - p(t) \qquad (1)$$

the tourism revenues correspond to the value of the economy's output and are given by

$$Y(t) = p(t)Q(t) = p(t)\big(\sigma_1 E(t) + \sigma_2 K(t) - p(t)\big) \quad (2)$$

We assume that the representative agent's instantaneous utility function depends on consumption *C(t)* of the output $Y(t)$

$$U[C(t)] = \frac{[C(t)]^{1-\sigma} - 1}{1-\sigma}$$

The parameter σ denotes the inverse of the intertemporal elasticity of substitution in consumption. The evolution of $K(t)$ (assuming, for simplicity, the depreciation of K to be zero) is represented by the differential equation

$$\dot{K} = pQ(E, K, p) - C$$

where $\dot{K}$ is the time derivative of K. In order to model the dynamics of E we start from the well-known logistic equation[2] and augment it by considering the negative impact due to the production process

$$\dot{E} = E(\overline{E} - E) - \alpha\overline{Q} \qquad (3)$$

where the parameter $\overline{E} > 0$ represents the carrying capacity of the natural resource, $\overline{Q}$ is the economy-wide average output and the parameter $\delta > 0$ measures the negative impact of $\overline{Q}$ on E. We assume that the representative agent chooses the functions $C(t)$ and $p(t)$ in order to solve the following problem

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[1] In [11] is analyzed a growth economic model with Perfect-Substitution Technologies, where is proved that indeterminacy does not occur.

[2] The logistic function has been extensively used as a growth function of renewable resources; see, for example, [5] and [8].

$$\underset{C,p}{MAX} \int_0^\infty \frac{[C]^{1-\sigma}-1}{1-\sigma} e^{-rt} dt \qquad (4)$$

subject to

$$\dot{K} = p(\sigma_1 E + \sigma_2 K - p) - C$$
$$\dot{E} = E(\overline{E} - E) - \alpha \overline{Q}$$

with $K(0)$ and $E(0)$ given, $K(t)$, $E(t)$, $C(t) \geq 0$ and $p(t) \geq 0$ for every $t \in 0, +\infty)$; $r > 0$ is the discount rate.

We assume that capital K is reversible, i.e., we allow for disinvestment ($\dot{K} < 0$) at some instants of time. Furthermore we assume that, in solving problem (4), the representative agent considers $\overline{Q}$ as exogenously determined since, being economic agents a continuum, the impact on $\overline{Q}$ of each one is null. However, since agents are identical, ex post $\overline{Q} = Q$ holds. This implies that the trajectories resulting from our model are not socially optimal but Nash equilibria, because no agent has an incentive to modify his choices if the others don't modify theirs.

3 Dynamics

The current value Hamiltonian function associated to problem (4) is (see [12])

$$H = \frac{[C]^{1-\sigma} - 1}{1-\sigma} + \Omega(p(\sigma_1 E + \sigma_2 K - p) - C)$$

where Ω is the co-state variable associated to K. By applying the Maximum Principle, the dynamics of the economy is described by the system

$$\dot{K} = \frac{\partial H}{\partial \Omega} = p(\sigma_1 E + \sigma_2 K - p) - C \qquad (5)$$
$$\dot{\Omega} = \theta\Omega - \frac{\partial H}{\partial K} = \Omega(r - \sigma_2 p)$$

with the constraint

$$\dot{E} = E(\overline{E} - E) - \alpha \overline{Q} \qquad (6)$$

where C and p satisfy the following conditions

$$\frac{\partial H}{\partial C} = C^{-\sigma} - \Omega = 0$$

$$\frac{\partial H}{\partial p} = (\sigma_1 E + \sigma_2 K - 2p)\Omega = 0 \Rightarrow p = \frac{\sigma_1 E + \sigma_2 K}{2}$$

The above conditions plus the limit transversality condition $\lim_{t \to +\infty} \Omega(t)K(t)e^{-rt} = 0$ are sufficient to solve problem (4). By replacing $\overline{Q}$ with $\sigma_1 E + \sigma_2 K + p$, the Maximum Principle conditions yield a dynamic system with two state variables, K and E, and one jumping variable, Ω. We can write the following system, equivalent to (5)-(6)

$$\dot{E} = E(\overline{E} - E) - \tfrac{\alpha}{2}(\sigma_1 E + \sigma_2 K) \quad (7)$$
$$\dot{K} = \frac{1}{4}(\sigma_1 E + \sigma_2 K)^2 - C$$
$$\dot{C} = \frac{C}{\sigma}\left[\frac{\sigma_2}{2}(\sigma_1 E + \sigma_2 K) - r\right]$$

with a non-negativity constraint on k: $K > \frac{p}{\sigma_2}$

In such a context, the jumping variable is C, instead of Ω. As a consequence, given the initial values of the state variables, K_0 and E_0, the representative agent has to choose the initial value C_0 of C.

4 Stationary states, stability and Hopf bifurcations

To express the next proposition, we have to define the following threshold values (see the proof of the proposition):

$$\overline{E}_T := 2\sqrt{\frac{r\alpha}{\sigma_2}}$$

$$\overline{E}_1(\sigma_1) := \frac{\alpha}{2}\sigma_1 + \frac{2r}{\sigma_1 \sigma_2}$$

$$\sigma_T := \sqrt{\frac{r}{\alpha\sigma_2}}$$

System (7) has:[3]

- zero stationary state if and only if (iff) $\overline{E} < \overline{E}_T$
- one stationary state $P_A = (E_A, K_A, C_A)$, iff $\overline{E} > \overline{E}_1$
- two stationary states $P_A = (E_A^*, K_A^*, C^*)$ and $P_B = (E_B^*, K_B^*, C^*)$ iff

$\overline{E}_T < \overline{E} < \overline{E}_1(\sigma_1)$ and $\sigma_1 < \sigma_T$ where,

$$E_{A,B}^* = \frac{\overline{E}}{2} \mp \frac{1}{2}\sqrt{\overline{E}^2 - \frac{4\alpha r}{\sigma_2}},\ K_{A,B}^* = -\frac{\sigma_1}{\sigma_2}E_{A,B}^* + \frac{2r}{\sigma_2^2},\ C^* = \frac{2r}{\sigma_2}$$

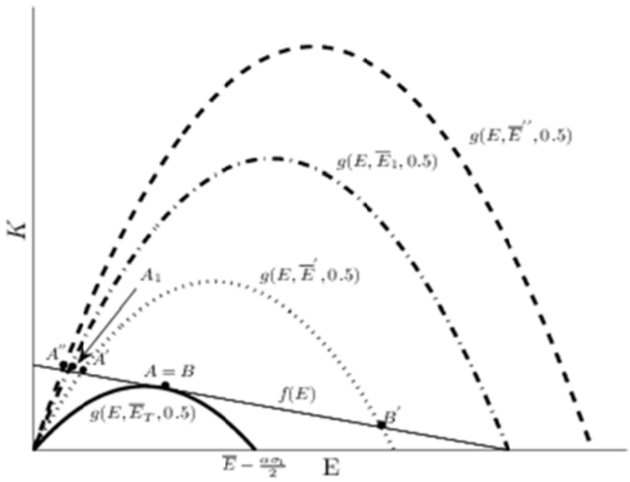

Figure 1: Determination of number of possible steady state for different values of $\overline{E}$

[3] For simplicity, in this classification, we do not take into account the "non robust" cases corresponding to an equality condition on parameter values (for example, the cases in which $\sigma = \sigma_T$ or $\overline{E} = \overline{E}_1$).

Proof. A stationary state $P^* = (E^*, K^*, C^*)$ of (7) must satisfy

$$K^* = f(E) := -\frac{\sigma_1}{\sigma 1} E^* + \frac{2r}{\sigma_2^2}$$
$$K^* = g(E) := \frac{2}{\alpha\sigma_2} E^* (\overline{E} - \frac{\alpha\sigma_1}{2} - E^*) \qquad (8)$$

We have tangency between the curves f and g, as $\overline{E} = 2\sqrt{\frac{\alpha r}{\sigma_2}} = \overline{E}_T$, therefore increasing $\overline{E}$, f remains stationary,while the curve g moves upward, then there are two points of intersection A e B. It possible increase the value of $\overline{E}$ until $E_B^* = \frac{\overline{E}}{2} + \frac{1}{2}\sqrt{\overline{E}^2 - \frac{4\alpha r}{\sigma_2}} < \overline{E} - \frac{\alpha\sigma_1}{2}$, condition for which is $\overline{E} = \overline{E}_1$. Moreover, the condition $\overline{E}_1 > 0$ i.e. $\sigma_1 < \frac{2r}{\sigma_2 E_T}$, must be satisfied, but since $E_T = \frac{\overline{E}_T}{2}$, we obtain the condition $\sigma_1 < \sigma_T = 2\sqrt{\frac{r}{\alpha\sigma_2}}$ (see Figure 1)

Now, let $P_i^* = (K_i^*, E_i^*, C_i^*)$, $i = A, B$ be a stationary state of (7) and consider the Jacobian matrix of system (7) evaluated at P_i^*

$$J^* = \begin{pmatrix} \overline{E} - 2E^* - \frac{\alpha\sigma_1}{2} & -\frac{\alpha\sigma_2}{2} & 0 \\ \frac{\alpha\sigma_1}{2} & r & -1 \\ \frac{r^2\sigma_1}{2\sigma_1\sigma} & \frac{r^2}{2\sigma} & 0 \end{pmatrix}$$

where we can obtain

$$tr(J^*) = \overline{E} - 2E^* + r - \frac{\alpha\sigma 1}{2}$$
$$M_2 = r(\overline{E} - 2E^* + \frac{r}{2\sigma})$$
$$det(J^*) = \frac{r^2}{2\sigma}(\overline{E} - 2E^*)$$

Briefly, we are able to derive the characteristic equation of the system, defined as

$$z^3 - tr(J)z^2 + M_2 z - det(J) = 0 \quad (9)$$

being z the auxiliary variable (the eigenvalue of the system).

Then we can conclude the following proposition As A exists, it may be either a repellor or a saddle with two-dimensional stable manifold, while if B exists, it may be either a sink or a saddle with one-dimensional stable manifold.

Proof. Steady state P_A^*

Since $E_A < \frac{\overline{E}}{2}$, $M_2 > 0$ and $D(J^*) > 0$, the eigenvalues of its Jacobian matrix are either all with positive real part or two with negative real part (see, [12]);

Proof. Steady state P_B^*

Since $D(J^*) < 0$, the eigenvalues of its Jacobian matrix are either all with negative real part or with two positive real part, i.e except for a one dimensional manifold of initial conditions (E_0, K_0), it is impossible to reach the steady state. (see, [12]).

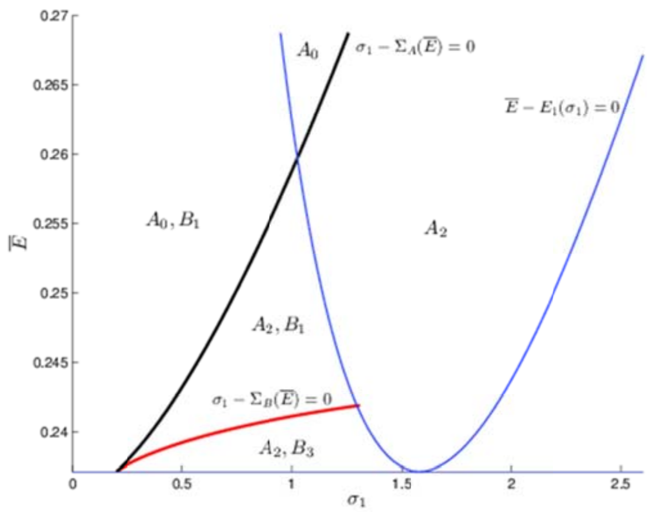

Figure 2: Characterization of the stability of steady states and bifurcation curves; parameter values : $r = 0.015$, $\alpha = 0.075$, $\sigma_2 = 0.08$, $\sigma = 0.1$

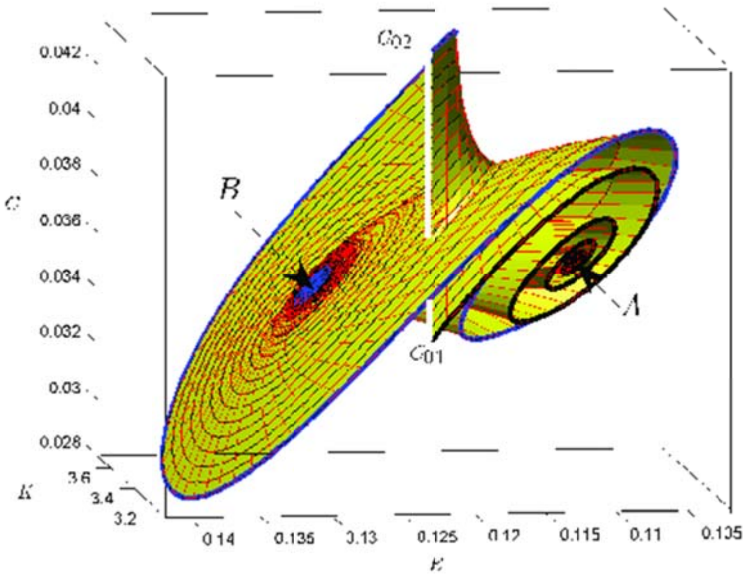

Figure 3: Global indeterminacy in the space (E, K, C); the black trajectory approaching $P_A^* = (E_A^*, K_A^*, C^* = 0.035156250)$ start of the point $(E_0 = 0.1090252, K_0 = 3.6067471, C_{01} = 0.0315816)$; the blue trajectory approaching $P_B^* = (E_B^*, K_B^*, C^* = 0.035156250)$ start of the point $(E_0 = 0.1090252, K_0 = 3.6067471, C_{02} = 0.0425518)$; parameter values : $\overline{E} = 0.238$, $r = 0.015$, $\alpha = 0.075$, $\sigma_1 = 0.8$, $\sigma_2 = 0.08$, $\sigma = 0.1$

Figure 2 illustrates the three dimension system with state variables E, K and jump variable C. There may realize at most two equilibria, A and B (where $E_A < E_B$, and $K_A > K_B$) which may have the following characteristics:

• Attractive, three eigenvalues with negative real part $(-, -, -)$;

• Saddle, with n-dimensional stable manifold (n representing the number of eigenvalues with negative real part, for instance, dimension 2 would be $(-,-,+)$, i.e. reachable,[4] while $(-,+,+)$ is non-reachable;

• Repulsive, with three eigenvalues with positive real domain $(+,+,+)$;

There can be identified five different areas of interest, depending on the type of equilibrium they admit, respectively: A_0, B_1; A_2, B_1, A_2, B_3; A_0; A_2. In particular, we are interested in areas (A_2, B_3), where global indeterminacy occurs (see, [6]). Furthermore, Figure 2 illustrates two bifurcation curves, $\sigma_1 - \Sigma_A = 0$ and $\sigma_1 - \Sigma_B = 0$, where easy computations lead to

$$\Sigma_i(\overline{E}) = \frac{2}{\alpha}\left(r + \frac{\overline{E}^2 - \frac{4\alpha r}{\sigma_2}}{\frac{r}{2\sigma} \pm \sqrt{\overline{E}^2 - \frac{4\alpha r}{\sigma_2}}}\right) \qquad i = A, B \qquad (10)$$

Figure 3 represents the global indeterminacy and shows the two equilibria A and B. A is a saddle with two-dimensional stable manifold, the black spiral trajectory, which has initial values $(E_0, K_0, C_0 = C_{01})$, converges to A. Increasing the initial value of C, $C_0 > C_{01}$ (moving along the white line while keeping E_0 and K_0 fixed) the trajectory converges towards B tracking the yellow area. For the initial value of $C = C_{02}$ the trajectory also converges towards B following the blue curve. Beyond C_{02} no convergence realizes. Hence, given the initial condition of E and K, the state variables (i.e. the "history" of the economy), there is a unique initial jumping value C_{01}, for which the system converges towards the equilibrium A, while there is a continuum of initial values of $C \in (C_{01}, C_{02})$ which drives the system in B. This implies that, in order to determine the long run equilibrium, beside the "history" of the economy, also agents' expectations (about consumption) play a crucial role.

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[4] A steady state with n-dimensional stable manifold, if $n > 1$ then the economy, starting from state variables (E_0, K_0), reach the steady state, thus, with abuse of notation, we say that steady state is reachable.

References

[1] A. Antoci , M. Galeotti and P. Russu. Global Analysis and Indeterminacy in a Two-sector Growth Model with Human Capital, DiMaD Working Paper N. 2012-14, Dipartimento di Matematica per le Decisioni, Firenze (2012).

[2] A. Antoci, M. Galeotti and P. Russu. Poverty Trap and Global Indeterminacy in a Growth Model with Open Access Natural Resources Growth Model, *Journal of Economic Theory*, **146**(2) (2011), 569--591.

[3] A. Antoci, M. Galeotti and P. Russu. Consumption of private goods as substitutes for environmental goods in an economic growth model, *Nonlinear Analysis: Modelling and Control* **10** (2005), 3--34.

[4] A. Antoci, M. Sodini. Indeterminacy, bifurcations and chaos in an overlapping generations model with negative environmental externalities, *Chaos, Solitons and Fractals* **42** (2009), 1439--1450.

[5] G.,M. Brown. Renewable natural resource management and use without markets, *Journal of Economic Literature* **38** (2000), 875--914.

[6] O.A. Carboni and P. Russu. Linear Production Function, Externalities and Indeterminacy in a Capital-Resource Growth Model, mimeo University of Sassari, 2012.

[7] R. Damania, J. Hatch. Protecting Eden: markets or government?, *Ecological Economics* **53** (2005), 339--351.

[8] L. Eliasson and S. Turnovsky. Renewable resources in an endogenously growing economy: balanced growth and transitional dynamics, *Journal of Environmental Economics and Management* **48** (2004), 1018--1049.

[9] P. Krugman, History versus expectations, *Quartely Journal of Economics* **106** (1991), 651--667.

[10] K. Matsuyama, Increasing returns, industrialization, and indeterminacy of equilibrium, *Quartely Journal of Economics* **106** (1991), 617--650.

[11] P. Russu. Global Analysis in a Growth Economic Model with Perfect-Substitution Technologies, mimeo University of Sassari 2012.

[12] F. Wirl. Stability and limit cycles in one-dimensional dynamic optimizations of competitive agents with a market externality, *Journal of Evolutionary Economics* **7** (1997), 73--89.

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