



**RISK-TAKING AND ASSET-SIDE CONTAGION IN AN  
ORIGINATE-TO-DISTRIBUTE BANKING MODEL**

**Andrea Pinna**

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**CRENoS – CAGLIARI  
VIA SAN GIORGIO 12, I-09100 CAGLIARI, ITALIA  
TEL. +39-070-6756406; FAX +39-070- 6756402**

**CRENoS - SASSARI  
VIA TORRE TONDA 34, I-07100 SASSARI, ITALIA  
TEL. +39-079-2017301; FAX +39-079-2017312**

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# Risk-Taking and Asset-Side Contagion in an Originate-to-Distribute Banking Model\*

Andrea Pinna

*European Central Bank  
and CRENoS*

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## Abstract

In a model of originate-to-distribute (OTD) banking, I show that contagion may spread before any preference shock, fire sale, or change in lending margins takes place. The drivers of contagion are opaqueness of collateral and roll-over frequency. Complexity of structured finance and poor screening of noninstitutional borrowers induce both originators and investors at different stages of the OTD chain to develop heterogeneous expectations on the future value of securitized debt. When new information on the value of collateral is sufficiently bad, overleveraged banks are unable to repay loans and the supply of liquidity by lenders in the money market shrinks in the short term. Banks with accurate pricing models are unable to roll over their collateralized loans and go bankrupt for lack of liquidity. Under some conditions the industry is able to prevent contagion to solvent institutions. In the scenario where that is not feasible, policy makers shall limit their intervention to ensuring orderly resolution of insolvent banks' default.

*Keywords:* Contagion, Originate to distribute, Crisis resolution.

*JEL:* G01, G32

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*Email address:* andrea.pinna@ecb.europa.eu (Andrea Pinna)

## 1. Introduction

The subprime crisis spread around the globe through a drop in the liquidity available to financial institutions and the real economy. An extensive body of literature suggests that the causes are to be found in tightening lending margins and fire sales (see for instance Gorton and Metrick (2011), Acharya et al. (2011), Diamond and Rajan (2011)). New evidence shows that the role of haircuts and repurchase agreements has been overplayed (see Comotto (2012) and Krishnamurthy et al. (2014)). Moreover, fire sales worsened contagion only after the crisis was underway, when wholesale liquidity providers sold the collateral posted by their borrowers.<sup>2</sup>

The aim of this paper is to gain new insights on contagion and crisis resolution in financial markets. I try to do so by filling a gap between theoretical accounts of the subprime crisis and recent stylized facts. In particular, I develop a model that accounts for financial contagion regardless of preference shocks, margin adjustments, balance sheets interconnection, or fire sales. The major policy implication is that an orderly resolution of insolvent banks in the initial phase of a crisis is key to avoid indirect contagion. I show that the burden of crisis resolution can be borne to various extents by solvent institutions in their own interest.

Even positive news on the value of collateral may endanger solvent banks. That happens when opaqueness of collateral and the use of arbitrary pricing models induce borrowers in the short-term money market to undertake sufficiently heterogeneous portfolio allocations. When new information on the future value of collateral arrives, banks which expected a better-than-realized signal are unable to repay their loans to wholesale liquidity providers. Hence, the latter have to lower their supply of liquidity in the short term.

Commercial paper originated in the eve of the crisis had little use case outside that of collateral in loans. Thus, as the supply of liquidity in the money market drops for the given amount of collateral, the latter

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<sup>2</sup>The main business newspapers on 6/21/2007 report that fire sales begun just one day earlier, when two Bear Stearns' vehicles defaulted on loans collateralized by subprime loans. See for instance The Financial Times or The Wall Street Journal.

trades at cash-in-the-market prices in its principal market (see Allen and Gale (1994)). Even banks with lower exposure to the secured loans market are then unable to fulfill commitments with their lenders, who have to lower liquidity supply even further. Some banks may default for lack of liquidity although their initial expectation on the intrinsic value of collateral was correct or even conservative.

The liquidity crisis may endanger conservative banks if both rollover frequency and the number of defaulting peers are high enough. Rollover frequency is an obstacle to the injection of new liquidity in a mispricing market, whereas the amount of losses borne by wholesale liquidity providers determines the size of the price distortion.

Fire sales and tightening margins may worsen the situation, but they are unwarranted as long as the number of defaulting banks is sufficiently low. Hence, this paper focuses on the first phase of contagion and models it to assess the tools available to ensure financial stability before additional contagion mechanisms take place. Such approach goes along the lines of the strategy recently implemented by the European Union with the creation of the Single Resolution Board. It recognizes the importance of resolving adverse shocks to financial stability before these escalate and with the least possible involvement of taxpayers' money.

In section 2 I motivate the present paper by giving an account of early stages of the subprime crisis. In section 3, the model of indirect contagion is introduced and banks' optimal portfolio allocation is characterized. Section 4 describes the impact interim information has on the banking industry, showing how the misvaluation by some banks propagates as a liquidity crisis. Section 5 discusses policy implications and section 6 concludes.

## **2. Modelling the subprime crisis**

A key feature of modern banking is the link between traditional banks and unregulated financial institutions constituting the diverse shadow banking system (see Pozsar et al. (2010)). That allowed traditional banks, subject to prudential regulation and blessed with more or less explicit government backstops, to adopt the originate-to-distribute

(OTD) business model and exploit profit opportunities outside regulatory umbrellas in the eve of the subprime crisis.

Loan brokers at the final retail stage of the OTD chain granted loans provided by warehouse lenders, with the sole purpose of transferring those risky assets to other financial institutions. Noninstitutional loans were then pooled and sold to a bank, who used special purpose vehicles (SPVs) to get rid of their risk weight on capital requirements and securitize them into mortgage-backed securities (MBSs) such as collateralized debt obligations (CDOs).<sup>3</sup>

The opaqueness of structured MBSs determined heterogeneity of pricing and portfolio allocations. Specialists of structured finance developed pricing models whose outcome depends on arbitrary parameters and limited data, whereas wholesale liquidity providers did not understand the complexity of the subprime chain and had to rely on credit ratings (see Gorton (2008)).

Banks created and sponsored SPVs mainly to attain leverage outside prudential regulation, smoothing liquidity before noninstitutional loans paid out. In order to allow risky SPVs to raise cheap liquidity, parent banks provided backstops against the possibility that cash flows delivered by the pool of loans be not sufficient to repay counterparties. Reputational credit lines commanded no capital charge before the crisis, whereas the difference between interest earned on noninstitutional loans and that required by wholesale lenders to the SPV yielded a positive net return. The OTD model succeeded until MBSs allowed to borrow liquidity under favourable terms in the money market, either through the issuance of asset backed commercial paper (ABCP) or in repurchase agreements (repos).

The delinquency rate of US subprime borrowers rose dramatically by mid 2007 and credit ratings assigned to many MBSs proved wrong.<sup>4</sup> Pricing models adopted by some SPVs had overvalued collateral, hence the liquidity that could be borrowed against it, and parent banks had to activate liquidity backstops that showed the economic integration of the two legally standalone institutions in one business entity.

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<sup>3</sup>Gorton and Metrick (2011) report that subprime mortgage origination was about \$1.2 trillion in 2005 and 2006, and 80% of it was securitized.

<sup>4</sup>"In the second half of 2007, Moody's downgraded more bonds that it had over the previous 19 years combined". The Financial Times, 18 October 2008.

Only after the \$62.5bn Primary Fund defaulted, in September 2008, the US government realized the destabilizing effect of shrinking wholesale liquidity providers on financial markets and it insured money market funds until September 2009. However, lenders were hit by write-downs of SPVs already in 2007. By June 2007, Bear Stearns had to shut two of its hedge funds and Wharton's fund Y2K Finance lost 25% of its value, amid rating downgrades on their MBSs. Funding conditions tightened and the SPVs Cheyne Finance and Rhinebridge defaulted on more than \$7bn of debt in October. The \$67bn worth Columbia Cash Reserves fund held almost \$1bn of Cheyne Finance's ABCP. In November, Bank of America had to shift \$600mn of assets to monetary market funds who risked breaking the buck and HSBC took \$45bn of MBSs on its own balance sheet to bail out one of its SPVs. Liquidity providers had good reasons to not rush in and replenish the troubled money market. To begin with, funds had limited possibilities to hold illiquid assets maturing in more than seven days (the limit was 10% of total assets when the crisis hit the US market). Moreover, amid economic uncertainty on the value of opaque securities and their holding by both traditional and shadow institutions, investors shunned funds exposed to MBSs.<sup>5</sup> The US insurance on money market funds was meant to re-establish confidence and liquidity supply, but it was provided more than one year later when the spiral of fire sales and margin calls had materialized.

Central banks and governments of major economies showed they are credibly committed to protect households' deposits against banks' failure.<sup>6</sup> Banks experienced difficulties before anything happened to liquidity demand, and they kept such information hidden as long as they could.<sup>7</sup> Episodes of bank runs were isolated, and investors with-

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<sup>5</sup>The stigma was so strong that the Legg Mason Fund, which previously held billions of dollars in SPVs holdings, advertised itself as "SPV-free" by February 2009.

<sup>6</sup>EU countries insure 100% of deposits up to euro 100,000. Switzerland covers up to CHF 100,000, whereas the US insure USD 100,000-250,000 depending on the account.

<sup>7</sup>Shin (2008) reports on the run to Northern Rock: "The Bank of England was informed [of Northern Rock's funding problems] on August 14th. From that time until the fateful announcement on September 14th that triggered the deposit run (i.e. for a full month), the Financial Service Authority and Bank of England sought to resolve the crisis behind the scenes, possibly arranging a takeover by another UK bank."

drew their liquidity only after the latter appeared to be clearly endangered. Thus, also preference shocks typical of the literature on “early diers” shall be kept off the model of contagion.

### *2.1. Mapping of stylized facts into the model*

Some banks adopted conservative pricing models and engaged only moderately in the OTD business before the crisis, whereas others originated a greater number of securities and attained more leverage. The amount of securities a bank originates in this model depends on the information it receives. Moreover, to account for the OTD business model that reached its peak before the crisis, each bank that collects depositors’ liquidity is recast as a complex business entity made of an investment bank and a SPV.<sup>8</sup>

At the initial date, banks receive deposits from customers exhibiting preference for smooth consumption and decide what part of such liquidity shall be lent to noninstitutional borrowers. Noninstitutional loans are securitized to be pledged as collateral with wholesale liquidity providers, which require the payment of interests at any rollover date. Each bank competes for deposits by committing to a periodic remuneration that it pays, for the sake of simplicity, in correspondence of rollover dates. Competition ensures the payment to depositors equals the whole liquidity a bank expects to hold at any date, net of the interest it pays to wholesale liquidity providers, given its initial expectation on the final repayment on noninstitutional loans.

Similarly to common market practice, each bank can borrow the current market price of collateral minus the margin wholesale liquidity providers require against market risk. Margins are based on the exogenous Value at Risk (VaR) constraint used by financial regulators to assess and limit risk exposure. Thus, the prevailing price of securitized loans determines the amount of cash a bank can raise to satisfy its commitments before loans pay out. At the interim date, banks discover the

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<sup>8</sup>To abstract from issues of moral hazard I assume the owner of a security is its originator, but the results of the model do not change if one adds more steps to the OTD chain and the owner bought its MBSs from other banks. Moreover, heterogeneity of investment strategies among banks at a stage of the chain relies on genuinely different valuations of the security.



true final repayment on loans. Such information is either a positive shock – for banks who acted relatively conservatively – or a negative shock – for overly leveraged banks who then default on their initial commitments.

Lenders in the money market do not have the skill to understand interim information immediately and they become aware of the actual repayment at the final date. All that they witness at the interim date is the default of their most leveraged borrowers.<sup>9</sup> If a bank defaults, its lender bears a loss and is able to provide less liquidity under the VaR constraint. This may determine cash-in-the-market pricing and an asset price below its fundamental value, even when there are no fire sales and the market value of collateral equals its intrinsic final value. That may affect banks who undertook conservative investment policies. The key drivers of contagion are opaqueness of collateral, which induce highly heterogeneous valuations, and roll-over frequency, which prevents the entrance of new liquidity providers.

If the heterogeneity of banks' initial evaluation prevents an aggregate shortage of liquidity when assets are fairly priced, policy makers may refrain from intervening. If there is lack of liquidity in the industry, it is necessary to fill such a gap. Yet, solvent banks have no incentive to hoard fresh liquidity. They prefer that it is used to avoid mispricing and bail out the activities of insolvent banks, in their own interest.

### 3. A Model of Indirect Contagion in OTD Banking

Consider a multi-region economy lasting for three dates  $t = 0, 1, 2$ . There is no discounting and the risk-free rate is normalised to zero. The economy is populated by depositors, banks, money market funds, and noninstitutional borrowers. Projects undertaken by the latter are the only source of return in the model.

Each region  $i = 1, \dots, n$  is endowed at the initial date with one unit of liquidity, proportionally owned by a continuum of depositors, and

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<sup>9</sup>The delay in lenders' awareness has the unique purpose of keeping margins unchanged. This allows to focus on wholesale lenders' default and isolate from the margin spiral. The widening of haircuts is neglected in this model, and it would reinforce the effect of asset misvaluation if it was accounted for.

with a piece of relevant information that is shared by the latter and their regional banks. There is no informational asymmetry among players in a region, whereas the fact that each region has its own information determines  $n$  optimal investment strategies.

### 3.1. *Players*

#### a) Depositors

Among depositors there are no early diers: preferences are homogeneous and described by the utility function

$$U(c^t) = \min(c^1, c^2)$$

where  $c^t$  is the amount of liquidity a depositor can use for consumption at date  $t$ . Thus, there is no shock to the demand for liquidity, and depositors strictly prefer smoothing consumption over time.

Depositors have the opportunity to place their endowment in a regional bank, in order to access its lending technology and afford a consumption schedule that yields the highest intertemporal utility. Since they face no preference shock and prefer smoothing consumption over time, a contract that specifies  $c^1 = c^2$  is optimal. Such a contract is akin to limited convertibility, therefore depositors never run their bank at date 1.

#### b) Noninstitutional borrowers

Noninstitutional borrowers face a long-term investment and borrow from banks the cash they need to undertake it.<sup>10</sup> Differently from depositors, borrowers deal with banks from any region. They do not care about the information set of their lender, as long as the loan is granted under the most favourable terms. Loans are granted at the initial date, and their repayment at the final date depends on the outcome of the project.

Borrowers are homogeneous and need exerting no effort to affect a project probability of success, nor can they misrepresent the project

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<sup>10</sup>Noninstitutional borrowers can be consumers who want to buy a home, to account for the category of borrowers the 2007-2009 crisis generated from, as well as agents involved in entrepreneurial ventures.

return. Thus, there are no agency problems between banks and noninstitutional borrowers. The latter are a risky black-box banks can access remunerative projects through.

c) Banks

Each regional banking industry is perfectly competitive. In order to outperform its competitors, a bank offers deposit contracts that maximise its depositors' utility.

Banks lend depositors' liquidity to noninstitutional borrowers. Each bank takes its lending decision on the basis of the repayment it expects at the final date, given its region-specific signal on the realization of exogenous factors that affect loans return. Such decision consists of what share of deposits to invest in loans to noninstitutional borrowers and what to be held as cash, together with a commitment on the periodic payment  $c^t$  to be paid to depositors at dates  $t = 1, 2$ . Every bank chooses its portfolio allocation and commitment with depositors under a solvency constraint. It goes bankrupt if, at any point in time, the liquidity it has available is lower than its commitments.

The liquidity banks in each region can use to finance entrepreneurial projects goes beyond the one unit they collect from depositors: banks can turn to the money market and raise additional liquidity they can invest in noninstitutional loans.

d) Wholesale liquidity providers (ABCP and repo buyers)

Money market funds are awash with liquidity and can only invest it in short-term asset securities such as ABCP and repos. They lack specialized skills necessary to gather any regional signal on entrepreneurial projects backing the securities. Their lending decision is thus based on common knowledge. Funds are subject to a "VaR equal zero" constraint.<sup>11</sup> A fund exits the market if it breaches its VaR constraint.

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<sup>11</sup>See Adrian and Shin (2012) for the derivation of VaR as optimal risk constraint in collateralized loans. For a discussion on the optimality of "VaR equal zero" contract, see Fostel and Geanakoplos (2012).

### 3.2. Investment technologies

#### a) Storage

The storage technology is cash, that is a 1-period investment yielding no return.

#### b) Deposits

The deposit technology is a 2-period fixed commitment that may differ among regions. It entitles depositors in region  $i$  to receive payments  $c_i^t$  at dates  $t = 1, 2$  from a regional bank.

#### c) Entrepreneurial projects

Entrepreneurial projects are homogeneous and illiquid 2-period ventures undertaken by noninstitutional investors. The statistical distribution of their return can be inferred, at the initial date, from information available to all players – e.g. credit rating and historical data.

#### d) Noninstitutional loans

The lending technology is available to banks only. It involves granting liquidity to noninstitutional borrowers at the initial date, in exchange for its repayment with interests two periods later.

Since the return on entrepreneurial projects is random, noninstitutional borrowers' ability to repay the loan is stochastic. It is assumed that the repayment of each unit of lent liquidity follows a Normal distribution  $\tilde{s} \sim N(\bar{s}, \sigma_s^2)$ , conditioned to all public information. All players share such prior belief, whereas banks and depositors in each region  $i$  receive region-specific signals and form a posterior beliefs  $\bar{s}_i$ .

Loans can be securitized by banks and used as collateral, either in repos or to issue ABCP. Without any loss of generality and to keep the model as simple as possible, I assume each unit of liquidity a bank lends is securitized into one unit of MBS.

#### e) Wholesale funding

The wholesale funding technology is a 1-period agreement whereby banks use their pool of loans to borrow liquidity from wholesale liquidity providers. Banks sell their MBSs in exchange for cash either to money market funds or to conduits issuing ABCP. Banks commit to repay their debt one period later and buy back the MBSs.

Banks buy back their MBSs at the price set at time 0 to repay their initial pledgeable value plus interests. The pledgeable value of MBSs – *i.e.* the principal of a wholesale loan granted against them – is equal to its market value at inception minus the margin ABCP issuers and repo lenders charge against market risk. For the sake of simplicity, I assume the interest rate and haircut on wholesale lending are the same independently on whether the loan occurs through ABCP or repos.<sup>12</sup> Parties in a repo agreement are price takers with respect to the remuneration financial markets assign to risk. Therefore, the interest rate  $r$  on repo lending is considered exogenous throughout the model.

### 3.3. Information

Accounting for the aggregation of information in this framework entails technical difficulties because of endogenous haircuts, the shape of consumers' utility, and the nonlinearity of payoffs on collateralized loans. The issue of price formation does not enrich the understanding of the contagion channel. For any market clearing equilibrium price  $p^0$  at the initial date, there exist a set of banks' private signals that causes contagion in the model.<sup>13</sup>

The heterogeneity of regional signals on the future value of ABSs is sufficient to spread insolvency throughout the industry as a consequence of wholesale lenders' default. Heterogeneous signals may be thought of as proprietary pricing models used by a bank and deemed valid by some depositors, who choose then to belong to its same informational region. Since every class of players in the model has homogeneous risk preferences, banks who choose safer (riskier) strategies do so on the basis of relatively bearish (bullish) signals. Thus, they commit to a lower (higher) return to depositors. The reason why some depositors may accept a lower return is that they share the information of a particular bank.<sup>14</sup>

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<sup>12</sup>The main difference between ABCP and repos is that the latter are bankruptcy-remote. Since in ABCP the issuer guarantees for the repayment of loans, neither form of collateral is allocated to other creditors in case the borrowing bank defaults.

<sup>13</sup>Price formation is the focus of a companion paper, "Price Formation of Pledgeable Security" where margins are set by a central clearing house and specific assumptions on preferences and distributions are made to simplify information aggregation.

<sup>14</sup>Depositors who share the signal of a regional bank have no incentive to shop

Formally, each region  $i = 1, \dots, n$  receives an informative signal  $\tilde{f}_i = s + \tilde{\epsilon}_i$  on the final repayment  $s$  of the lending technology. To ensure banks/depositors do not make systematic errors, it is assumed that  $\tilde{\epsilon}_i \sim N(0, \sigma_\epsilon^2)$ ,  $\tilde{s}$  and  $\tilde{\epsilon}_i$  are independent, and errors are independent across banks. Since money market funds cannot observe regional signals, their uninformed expectation  $\bar{s}_p = E(\tilde{s}|p^0)$  generally differ from the informed expectations  $\bar{s}_i = E_i(\tilde{s}|p^0, f_i)$  made by each bank in region  $i = 1, \dots, n$ .

Since banks in any region are perfectly competitive and share the same information and technologies, they are all alike and take the same investment decision. Thus, with no loss of generality and to simplify notation, one representative bank is considered for every region. Banks are then labelled according to their signals, from the most bullish to the most bearish one:  $\bar{s}_1 \geq \bar{s}_2 \geq \dots \geq \bar{s}_n$ .

At the interim date, the true repayment on loans is known to all banks and the latter agree on the value of MBSs.

### 3.4. Time structure

*Initial date:* Bank  $i$  observes its signal  $f_i$  and forms posterior belief  $\bar{s}_i$ . It receives from depositors one unit of liquidity and chooses what portion  $\lambda_i$  of to invest in the storage technology in order to remunerate depositors at  $t = 1$ . The remaining  $(1 - \lambda_i)$  is invested in noninstitutional loans to earn the risky return  $\frac{\tilde{s}}{p^0}$  at  $t = 2$ .

Each bank securitizes its loans to noninstitutional borrowers and use them to borrow liquidity on the money market, with a common interest rate  $r$  and a bank-specific haircut  $h_i$ . Wholesale loans allow banks to leverage and grant (issue) a total amount  $q_i$  of loans (MBS) and to commit to periodic payments  $c_i^t$ .

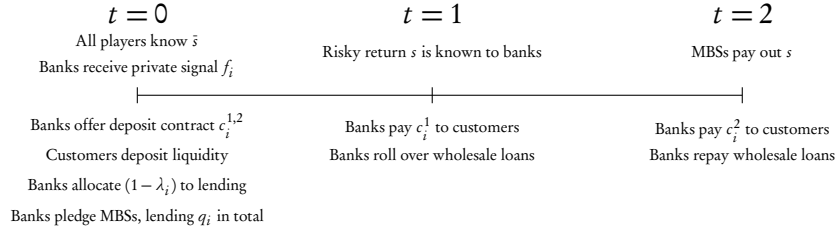
*Interim date:* Banks discover that the repayment of noninstitutional loans will be  $s$ . Each bank  $i$  must pay depositors the sum  $c_i^1$  it specified in the contract at the initial date. The wholesale loan need to be rolled over. Solvent banks may raise an amount of liquidity that equals MBS pledgeable value given the current market price. When a bank

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around for a second bank/region to get a higher return. The combination of such higher return with the risk the second bank undertakes is in fact suboptimal for the depositor from a different region, given the different information sets.

defaults on its wholesale loans MBSs are kept by the ABCP issuers or repo lender, who exit the market if the new MBS price is not sufficient to cover losses.

*Final date:* Entrepreneurial projects pay out. Banks receive the repayment  $s$  from noninstitutional borrowers, repay their debt, and give depositors the sum  $c_i^2$  they committed to at the initial date. For the sake of clarity, the following timeline summarises the time-structure of the model:



### 3.5. MBS pledgeability in the money market

MBS pledgeability is endogenously determined by the need for banks' counterparties to limit the VaR of their loans. Wholesale loans are provided so that expected losses are lower than the VaR limit, with a probability  $(1 - \alpha)$  set by the regulator.

Since banks can invest deposits in either risky loans or safe cash, the usual trade-off between leverage and credit worthiness affects lending terms. The probability that a lender recoups the full value of its credit decreases with the risky investment undertaken by its counterparty. Under the  $\text{VaR}=0$  constraint, bank  $i$  faces a haircut  $h_i$  such that:

$$\lambda_i + q_i p^t (1 - v) \geq q_i p^t (1 - h_i)(1 + r), \quad (1)$$

where  $\lambda_i$  is the portion of deposits bank  $i$  invested in the storage technology,  $q_i$  is the number of MBSs it issued,  $r$  is the wholesale loan rate,  $p^t$  is the security market value at date  $t$ , and  $v$  is its price drop over one period in the  $\alpha\%$  worst scenario, according to the unconditional distribution of  $\tilde{s}$  adopted for risk management purposes. On the left

hand side of the inequality is the sum between the liquidity the bank decided to hold at the interim date and the  $\alpha$ -specific market value of its pledged assets. On the right hand side is the cash flow of the second loan, in case of roll-over at the interim date.

The solution to inequality (1) gives a lower bound for the haircut  $b_i$  that depends on  $r$ ,  $v$ ,  $p^t$ , and bank  $i$  portfolio allocation  $(\lambda_i; 1 - \lambda_i)$ :

$$b_i^{min} = \frac{r + v - \frac{\lambda_i}{q_i p^t}}{1 + r}. \quad (2)$$

Wholesale lenders face a trade-off between applying a higher haircut to hedge risk and earning interests on additional principal amount. Since funds are risk neutral, the lowest admissible value derived in equality (2) is optimal.

### 3.6. Investment and contract design

Each competitive bank chooses its optimal investment in the lending technology, at the initial date, to maximise depositors' intertemporal utility.

Omitting the index  $i$  to simplify notation, the budget constraint of any bank at the initial date is

$$q \leq (1 - \lambda) + q p^0(1 - b)$$

where  $q$  are the units of liquidity a bank that levers through wholesale loans may lend to noninstitutional borrowers,  $(1 - \lambda)$  is the fraction of deposits invested in loans, and  $q p^0(1 - b)$  is the amount of liquidity the bank borrows by pledging the MBSs it originates. The maximum amount of MBSs a bank can originate by allocating  $(1 - \lambda)$  of its deposits to noninstitutional lending is therefore

$$q^{max} = \frac{1 - \lambda}{1 - p^0(1 - b)}, \quad (3)$$

that is also the liquidity a bank is ultimately able to lend to noninstitutional borrowers, given the initial allocation  $\lambda$  and the leverage allowed by the wholesale lending haircut  $b$ .



Ultimate MBS origination  $q$  depends on the wholesale lending technology – given a bank optimal allocation – whereas the margin  $h$  is imposed by risk-constrained liquidity provider. The two values are determined through the simultaneous solution of equalities (2) and (3). Given any initial portfolio allocation  $(\lambda, 1 - \lambda)$ , a bank lends/originates an amount

$$q = \frac{1 + r(1 - \lambda)}{(1 + r) - p^0(1 - v)} \quad (4)$$

and faces the wholesale loan margin

$$h = \frac{p^0(1 - \lambda)(r + v) + \lambda(p^0 - 1)}{p^0(1 + r(1 - \lambda))}. \quad (5)$$

When claims on 2-period noninstitutional loans are offered as collateral in the money market, each bank  $i$  can borrow an amount of liquidity specified by the pledgeability function

$$\begin{aligned} B_i^t(\lambda_i, r, v, p^t) &= (1 - h_i)q_i p^t \\ &= \left( \frac{1 - v + \frac{\lambda_i}{q_i p^t}}{1 + r} \right) q_i p^t. \end{aligned} \quad (6)$$

**Lemma 1.** *As a bank risky investment  $(1 - \lambda_i)$  increases, a lower percentage of its collateral value is pledgeable under the VaR constraint.*

*Proof.* See Appendix A □

In order for banks portfolio allocation to be non-trivial, providers of liquidity must be concerned that the haircut level fulfils their VaR constraint. Otherwise, banks would borrow an infinite amount of liquidity to invest in the lending technology. Liquidity invested in the purchase of MBSs must be expected to yield a return that, in the  $\alpha\%$  worst scenario, is lower than the cost paid for the same liquidity in the money market:

$$(1 + r) > p^0(1 - v). \quad (7)$$

Based upon the regional signal  $f_i$ , bank  $i$  chooses its portfolio allocation to maximize depositors' expected utility.

When a bank solvency constraint does not bind, the optimal portfolio allocation is trivial: every risk neutral bank invests either everything in the risky lending technology – when its posterior belief  $\bar{s}_i$  offsets the costs of funding  $r$  and  $b$  – and it keeps all deposits liquid otherwise. To ensure that the solvency constraint binds, I shall assume throughout the paper that no bank receive either a "too" low or a "too" high signal that induces the choice of a corner solution.

The optimal decision is taken by backward induction and requires knowing the sequence of actions determining the final outcome for any initial allocation  $(\lambda_i, 1 - \lambda_i)$ . Loans and MBS origination depends on  $\lambda_i$  through equality (4). MBSs allow a bank to borrow on the money market an amount of liquidity  $q_i p^0(1 - h_i)$ , that is lent to noninstitutional borrowers together with  $(1 - \lambda_i)$ . Bank  $i$  is left with liquid deposits  $\lambda_i$  it carries on to period 1, when  $q_i p^0(1 - h_i)(1 + r)$  has to be paid to roll over the wholesale loan. Since the decision over  $\lambda_i$  is taken at the initial date, each bank establishes its optimal portfolio allocation upon the expectation that MBSs appreciate at the interim date by

$$\delta_i = \frac{\bar{s}_i}{p^0} - 1. \quad (8)$$

Given its posterior belief  $\bar{s}_i$ , bank  $i$  expects to raise

$$q_i p^0(1 + \delta_i)(1 - h_i)$$

on the money market when its loan is rolled over.

The liquidity bank  $i$  has available to pay its depositors the first periodic instalment  $c_i^1$  amounts to the sum of the deposits  $\lambda_i$  it stored at the initial date and the net cash flow occurring at the interim date:

$$\lambda_i - q_i p^0(1 + r)(1 - h_i) + q_i p^0(1 + \delta_i)(1 - h_i).$$

At the interim date, after depositors receive their first payment, bank  $i$  holds an amount of liquidity

$$\lambda_i + q_i p^0(\delta_i - r)(1 - h_i) - c_i^1. \quad (9)$$

At the final date, entrepreneurial projects yield their return and each MBS pays out  $s$ . The bank must pay  $q_i p^0(1 + \delta_i)(1 - h_i)(1 + r)$  to money market funds, and depositors must be paid the second instalment  $c_i^2$ . Given any belief  $\bar{s}_i$  at the initial date, the residual liquidity bank  $i$  expects to hold at the end of date 2 is therefore

$$\lambda_i - q_i p^0(1 - h_i)(1 + r(2 + \delta_i)) + q_i \bar{s}_i - c_i^1 - c_i^2. \quad (10)$$

Competitive banks offer depositors the highest possible periodic smooth payment  $c_i^{1,2}$ . This induces banks to select their optimal strategy leaving no spare liquidity at the end of the deposit contract. Imposing such condition at the final date allows to identify by backward induction the bank's objective function, that is the expected per period payment  $c_i$ :

$$c_i \equiv \frac{\lambda_i + q_i p^0(\bar{s}_i - 1 - r(2 + \delta_i))(1 - h_i)}{2} \quad (11)$$

The bank's optimization problem at the initial date is therefore:

$$\max_{\lambda_i} : c_i \quad (12)$$

$$s.t. \ c_i \leq \lambda_i + q_i p^0(\delta_i - r)(1 - h_i) \quad (13)$$

$$q_i = \frac{1 + r(1 - \lambda_i)}{(1 + r) - p^0(1 - v)} \quad (4)$$

$$h_i = \frac{p^0(1 - \lambda_i)(r + v) + \lambda_i(p^0 - 1)}{p^0(1 + r(1 - \lambda_i))} \quad (5)$$

Objective function (12) is derived from (10), imposing the final condition that a bank does not expect to hold spare liquidity after depositors are paid. Inequality (13) is the solvency constraint at the interim date, when the periodic payment  $c_i$  has to be lower than the available liquidity that was specified in (9). Equalities (4) and (5) were previously determined as simultaneous solutions to the initial budget constraint and to money market funds VaR limit.

When the bank's solvency constraint (13) does not bind, optimal portfolio allocation is trivial: every risk-neutral bank invests either ev-

everything in the risky lending technology – when its posterior belief  $\bar{s}_i$  offsets the costs of funding  $r$  and  $b$  – and it keeps all deposits liquid otherwise.

In what follows I assume the value of parameters in the model are such that the budget constraint binds for some banks, so that the solution of the optimization problem is the non-trivial interior one. Some banks receive signals that are neither too low nor too high and thus, given their information sets, corner solutions are suboptimal.

**Lemma 2.** *When the solvency constraint binds, the optimal portfolio allocation is:*

$$\lambda_i^* = \frac{p^0 \{(1+r)[p^0(1-v) + \bar{s}_i v] - \bar{s}_i(1-v)\}}{p^0(p^0 r - \bar{s}_i)(1-v) + \bar{s}_i [(1+r)(1+p^0 v) - (p^0 - 1)]}. \quad (14)$$

*The number of MBSs optimally issued by bank  $i$  is*

$$q_i^* = \frac{2+r}{p^0 r v + (2+r) - (2 - p^0 \bar{s}_i^{-1} r) p^0 (1-v)} \quad (15)$$

*Proof.* The result follows directly from the maximization of (12) under the budget constraints (4,5,13)  $\square$

**Corollary 1.** *The higher a bank's posterior belief  $\bar{s}_i$ , the larger its lending and the payment it commits to pay depositors.*

*Proof.* See Appendix A  $\square$

#### 4. Effect of the systemic shock

At the interim date, the risky return  $s$  is known to banks in every region. Since the latter are also involved in the spot market, such information affects the MBS spot price. Label the MBS price change following interim information on loans repayment as

$$\delta_s = \frac{s}{p^0} - 1.$$

The liquidity bank  $i$  has available given new information is not sufficient to fulfill its commitments and causes insolvency when  $\bar{s}_i > s$ . In

such a case,  $\delta_s < \delta_i$  and the available liquidity  $c_i^s$  at the final date is lower than the commitment with depositors.

$$c_i^s = \frac{\lambda_i^* + q_i^* p^0 (\delta_s - r)(1 - b_i) + q_i^* (s - p^0 (1 + \delta_s)(1 - b_i)(1 + r))}{2} < c_i^*. \quad (16)$$

If instead bank  $i$  underestimates loans repayment at the initial date, the intrinsic value of its assets exceeds its commitments.

#### *Scenario 1: considerate banking industry*

The first scenario arises when all banks estimate the realization of  $\tilde{s}$  conservatively. They hold enough cash at the initial date to avoid bankruptcy when interim information is released:  $c_1^s \geq c_1$ .

#### *Scenario 2: inconsiderate banking industry*

The second scenario arises if all banks have access to signals that overestimate the realization of  $\tilde{s}$ :  $c_n^s < c_n$ . If this is the case, new information trail the whole industry to default. Every bank goes bankrupt as a consequence of its wrong investment in the risky technology. There is no contagion channel involved in the crisis, whereas public intervention is necessary to keep banks afloat.

#### *Scenario 3: idiosyncratic effect of the systemic shock*

This is the scenario the rest of the chapter focuses on: one or more banks overestimate the repayment on loans, whereas others make a conservative estimate:

$$c_1^s < c_1, \quad c_n^s \geq c_n.$$

The systemic shock has an idiosyncratic effect. Banks which overvalued loans return is unable to fulfill its commitment with depositors and to roll over its repos at the interim date. The banking industry may have enough aggregate liquidity to cope with the default, depending on the relative sizes of upward and downward misvaluations.

#### *4.1. Contagion*

In scenario 3, the pledgeable value of some banks' collateral at the interim date is in excess of what is necessary to fulfill its commitment. This is true in so far as assets are priced efficiently.

When the MBS value conveyed by banks' interim information is below a threshold level, lenders of defaulting banks breach their VaR limit. This happens if new information on MBSs liquidation value is such that  $s < p^0(1 - v)$ . If this is the case, wholesale liquidity providers "break the buck" and exit the money market.

A defaulting bank cannot roll-over its debt at the interim date. Nevertheless, its lender still offers the dispossessed collateral to try and recoup its loss. Asset supply does not change, thus there is no fire sale.<sup>15</sup> However, the money market is less liquid and may exhibit cash-in-the-market pricing.<sup>16</sup> Overall MBS supply is:

$$Q = \sum_{i=1}^n q_i,$$

where  $q_i$  was specified in (15) and varies according to the realization of regional signals. Only lenders of the  $n - k$  less optimistic regions fulfill their  $\text{VaR}=0$  constraint. Thus, available liquidity  $L$  in the buy side of the market is given by the repayment funds receive from solvent banks at the end of the first wholesale loan:

$$L = p^0 \sum_{i=k+1}^n q_i$$

where  $k$  is the number of insolvent banks. Let  $p^{AG}$  denote the maximum price money market investors are able to pay for an MBS at the interim date in the case of cash-in-the-market pricing á la Allen and Gale (1994):

$$p^{AG} = \frac{L}{Q} = p^0 \frac{\sum_{i=k+1}^n q_i}{\sum_{i=1}^n q_i}. \quad (17)$$

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<sup>15</sup>Since prior to the Subprime crisis a substantial share of bank repos and ABCP were rolled over daily, it makes sense to assume that both the amount of MBSs originated by banks and the liquidity supplied by solvent funds does not vary at the interim date. If defaulting banks had incentive to hold MBSs or owned any other security, their sudden sale would constitute a fire sale like in Cifuentes et al. (2005), among others.

<sup>16</sup>For a comprehensive treatment of cash-in-the-market pricing see Allen and Gale (1994).

Such price is lower than the MBS fundamental value  $s$  when

$$\sum_{i=1}^n q_i > \frac{p^0}{s} \sum_{i=k+1}^n q_i. \quad (18)$$

In line with the too-big-to-fail doctrine, the size of the insolvent institutions portfolio determines the impact their wrong evaluation has on the industry.

**Lemma 3.** *The difference between MBS fair value and its price at the interim date increases with the position held by overleveraged banks.*

*Proof.* See Appendix A □

Over-leveraged banks are those who hold larger positions in the pledgeable asset. Thus, their default is likely to hamper money market ability to provide solvent banks enough liquidity to fulfill their commitments. The consequence of funds breaching their VaR constraint is akin to that of fire sales, even though there is no increase in asset supply.

Although the exit by some money market funds drains funding opportunities, the overall pledgeable value in the industry does not need to fall. Banks who acted conservatively are solvent, but they suffer from the temporary lack of liquidity in the money market.

**Proposition 4.** *When lenders breach their VaR constraint and MBS cash-in-the-market price falls below the initial expectation of a solvent bank, the latter goes bankrupt for lack of liquidity.*

*Proof.* See Appendix A □

With a low roll-over frequency, the model does not exhibit contagion. This happens for two reasons: (1) liquidity providers have time to enter the money market to exploit any mispricing and avoid the cash-in-the-market pricing determined by limited participation; (2) banks funding liquidity is ensured throughout the maturity of their deposit contract.

**Proposition 5.** *Without interim roll-over, the money market does not act as channel for contagion.*

*Proof.* See Appendix A

□

In such a case, overleveraged banks still default because the return on loans is lower than they expected. Default makes banks that took excessive risk bear the cost of its wrong evaluation – so to avoid excessive risk taking and moral hazard – whereas considerate banks are unaffected.

## 5. Policy implications

Public bailouts imply different costs and taxpayers' reactions depending on the type of intervention and the responsibilities of the target bank. The nationalization of banks involve high costs in terms of upfront payment and public debt, possibly inducing the diabolic loop outlined by Brunnermeier et al. (2011) among others. Injections of liquidity incur the risks outlined by Acharya et al. (2012) and Carlin et al. (2007), who find an incentive for solvent banks to hoard liquidity or adopt a vulture-like strategy. The result of the present model is reassuring in this respect.

Conservative banks hold collateral in excess of their liquidity needs at the interim date. That may be used to facilitate the resolution of their defaulting competitors, either directly or by mean of a public fund. On the one hand, the liquidity available to solvent banks and that injected by public intervention may be used to buy distressed collateral from wholesale lenders at a price  $p^{AG}$  below its liquidation value. On the other hand, if the price paid is too low, their portfolio of MBSs used as collateral by solvent institutions loses too much value and they default.

Consider an economy where the most leveraged bank 1 is not able to fulfill its commitments, and assume its exposure to wholesale lenders is sufficient to trigger the contagion pointed out in Proposition 4. In the setting of the present model a nationalization or market operations are needlessly expensive to the government, as private bailouts are feasible either with or without government's intervention – depending on the amount of spare liquidity held by solvent banks.

The government may facilitate a private bailout of bank 1 by according concessional loans. Assume the government picks solvent bank 2 and offer the following scheme: if bank 1 defaults on its loans, the excess supply of MBSs will lower prices and make bank 2 go bankrupt



for lack of liquidity. If bank 2 agrees to buy bank 1's MBSs, the government is ready to grant a loan under concessional rate  $r^G$  and haircut  $h^G$ .<sup>17</sup>

The solvent bank faces a tradeoff: MBS price must be sufficiently high to allow borrowing enough liquidity against its initial holding  $q_2$ , but a high price implies the payment of additional interests. The threat of contagion induces bank 2 to use government's money to buy bank 1's collateral, as long as a bailout price  $p^b$  that prevents contagion exists.

**Proposition 6.** *The bailout price  $p^b$  that is compatible with the prevention of a crisis is:*

$$\frac{(1-h_2)q_2s_2}{(1-h_2)q_2-h^Gq_1} \leq p^b \leq \frac{q_2s_2[(1-h_2)r-h_2]+s(q_1+q_2)}{(1-h_2)q_2(1+r)+(1-h^G)q_1(1+r^G)} \quad (19)$$

*That exists whenever*

$$h^G < \frac{q_2(1-h_2)}{q_1}. \quad (20)$$

*Proof.* See Appendix A □

**Remark 1.** *The haircut applied by the government who injects liquidity through repos has to be lower the higher the insolvent institutions' portfolio of assets  $q_1$ . However, it is always feasible as the upper threshold in (20) is strictly positive.*

A policy maker may even refrain from intervening, if the amount of spare liquidity in the industry is large enough – i.e., if banks adopted on average a prudent pricing model relatively to the realization of the MBS fundamental value  $s$ .

**Proposition 7.** *A private bailout is sufficient to avoid contagion when the liquidation value of the security is above the following threshold:*

$$s > \frac{q_2^2s_2(1-3h_2+2h_2^2(1+r)+2r-4h_2r)+q_1q_2s_2[(1+r)h-r]}{q_1^2-q_2^2+q_2h_2(q_1+q_2)}. \quad (21)$$

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<sup>17</sup>The scheme can be generalized to a coalition of many solvent banks. However, that would imply bargaining among banks and goes beyond the scope of the model.

*Proof.* See Appendix A

□

Proposition 7 shows the relationship between banks' pricing errors and the feasibility of a private bailout. The heterogeneity of pricing models is not bad *per se*, as the mistake by an overly conservative bank increases the probability that the true value of the security is sustainable for the industry. The fact that the fundamental value of solvent institutions' portfolio is higher than they expected when their investment strategy was chosen allows them to prevent a liquidity crisis. This happens when the MBS final realization is sufficiently high relatively to banks' estimates.<sup>18</sup>

## 6. Concluding remarks

The model of contagion developed in the present paper is consistent with four stylized facts of the Subprime crisis: (1) banks-SPVs relied on opaque MBSs to increase their leverage through collateralized borrowing with money market funds; (2) a shock on the cash flow MBSs were expected to pay made overleveraged banks unable to raise the short-term financing they needed to pursue their business strategy; (3) wholesale lenders faced losses; and (4) the market for short-term financing froze, preventing even some banks that did not overestimate MBSs intrinsic value from finding the liquidity they needed to fulfill their commitments.

Far from solving the debate on what allowed the shock in the US subprime mortgage delinquency rate in 2007 to spread worldwide with such a terrific effect, this paper accounts for the possibility that excessive leverage by few institutions precipitates a systemic liquidity crisis. Neglecting the issue of early diers, the model focuses on the misvaluation of opaque financial derivatives as a source of the crisis. Since limited convertibility at the interim date is optimal to depositors who prefer smoothing consumption, there are no bank runs in the model.

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<sup>18</sup>The opaqueness of structured securities allows banks to bias their pricing models towards optimistic scenario and to increase expected returns in a setting with risk-shifting. However, that is outside the scope of the present model.

Moreover, because the model rules out crossed claims among banks, there is no room for domino-contagion.<sup>19</sup>

The main result of the paper is that a market providing liquidity against collateral may act as a channel for contagion. The composite effect of opaque assets and short term collateralized borrowing is able to propagate the negative effect of asset mispricing from an institution to the whole banking sector. Such channel is compatible with stylized facts on the first phase of the crisis. It preceded other channels such as domino effects, fire sales and tightening lending margins that are well known in the extant literature.

Differently from what was found in previous papers on predatory liquidity hoarding, banks have no incentive to adopt vulture-like strategies. The existence of a market for collateralized borrowing introduces a strategic interdependence that limits the advantage solvent banks take from the bankruptcy of their competitors. When banks rely on the money market for their liquidity needs, the additional return they get from cheap collateral may be offset by the amount of liquidity they must give up on the money market.

Policy makers can exploit the individualistic motive of liquid banks to prevent contagion in the first place. When the industry is not biased towards overly positive estimates of the future collateral value, the banking industry shall be left the burden of dealing with the failure by one or more inconsiderate banks. If financial institutions as a whole overestimated the value of collateral and committed to repayments that exceed the liquidity available at the industry level, public intervention is necessary. However, this comes at lower cost if the policy maker recognizes that liquid institutions have incentives to accommodate a bailout.

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<sup>19</sup>Banks are indirectly linked one to each other through the money market. Thus, differently from what happens in domino models, a bank is unable to protect from contagion by means of an appropriate choice of its counterparty.

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## A. Proofs

*Proof of Lemma 1.* The derivative of the pledgeable value per unit of collateral value with respect to the risky investment is

$$\frac{\partial(1-h_i)}{\partial(1-\lambda_i)} = -\frac{1+r-p(1-v)}{p(1+r(1-\lambda_i))^2} < 0.$$

□

*Proof of Corollary 1.* Bank  $i$  optimal investment in the storage technology is lower, the higher is the expected return on the risky investment:

$$\frac{\partial \lambda_i^*}{\partial \bar{s}_i} = -\frac{(p^0)^2(2+r)(1-v)((1+r)-p^0(1-v))}{[(2+r)\bar{s}_i + (p^0)^2 r(1-v) + p\bar{s}_i((2+r)v-2)]^2} < 0 \quad (22)$$

The amount of liquidity  $c_i$  bank  $i$  commits to pay its depositors at each date is given by equality (12), once the optimal portfolio allocation  $\lambda_i^*$  is determined. The periodic payment bank  $i$  commits to pay its depositors increases with the optimal risky investment:

$$\frac{\partial c_i^*}{\partial \lambda_i} = \frac{r((p^0)^2(1-v) - s_i(1+p^0v))}{2p^0((1+r)-p^0(1-v))} < 0. \quad (23)$$

Since the optimal risky investment increases with its expected return, banks with higher expectation  $\bar{s}_i$  commit to higher periodic payments to their depositors. □

*Proof of Lemma 3.* From (17), the level of underpricing at the interim date is

$$\frac{s-p^{AG}}{s} = 1 - \frac{p^0 \sum_{i=k+1}^n q_i}{\sum_{i=1}^n q_i} - 1 \quad (24)$$

Hence, the larger the MBS position  $\sum_{i=1}^k q_i$  of the  $k$  relatively inconsiderate banks, the higher is the difference between the fundamental value of the asset and its cash-in-the-market price. □

*Proof of Proposition 4.* From equation (13), bank  $i' \in [k+1; n]$  commits to a periodic payment

$$c_{i'} = \lambda_{i'} + q_{i'} p^0 (\delta_{i'} - r)(1-h_{i'})$$

whereas, defining by

$$\delta_{AG} = \frac{p^{AG}}{p^0} - 1 \quad (25)$$

the price change of the pledgeable asset in case of cash-in-the-market pricing, the liquidity it has available at the interim date is

$$L_{i'}^1 = \lambda_{i'} + q_{i'} p^0 (\delta_{AG} - r)(1 - h_{i'})$$

The solvent bank becomes unable to fulfil its commitments – that is,  $L_{i'}^1 < c_{i'}^1$  – whenever  $\delta_{AG} < \delta_{i'}$ . Equalities (8) and (25) show that the amount of liquidity  $c_{i'}^1$  considerate bank  $i'$  has available at  $t = 1$  is lower than needed to fulfil depositors' and lenders' claims whenever  $p^1 < \bar{s}_{i'}$   $\square$

*Proof of Proposition 5.* If banks do not roll-over their loan until the end of the deposit contract, optimal portfolio allocation at the initial date is the solution to (12)–(13), but with  $\delta_i = 0$ . In such case with no roll-over, labelled *NR*, the initial optimal allocation is

$$\lambda_i^{NR} = \frac{p^0(1 - v) - \bar{s}_i}{2p^0(1 - v) - (2 + r + r\bar{s}_i)},$$

and MBS origination is

$$q_i^{NR} = \frac{2 + r}{2p^0(1 - v) - (2 + r + r\bar{s}_i)}.$$

With fair pricing, the impact of new information at the interim date is for bank  $i$  an excess liquidity equal to

$$G_i^{NR} = q_i^*(1 - h_i)(s - p^0), \quad (26)$$

that is positive whenever the bank had a conservative estimate on loans repayment. Overleveraged banks may endanger their wholesale lenders. However, differently from the case of cash-in-the-market pricing, this has no negative externality on conservative banks funding.  $\square$

*Proof of Proposition 6.* Bank 2 commitment with depositors at the interim date is

$$c_2^1 = \lambda_2 + q_2 \bar{s}_2(1 - h_2) - q_2 p^0(1 - h_2)(1 + r), \quad (27)$$

whereas the liquidity it has available after buying assets of the insolvent banks at a bailout price  $p^b$  is

$$\lambda_2 + q_2 p^b (1 - h_2) - q_2 p^0 (1 - h_2) (1 + r) - q_1 p^b + q_1 p^b (1 - h^G). \quad (28)$$

The bank avoids contagion if (27)  $\leq$  (28). This happens when it pays a sufficiently high price

$$p^b \geq \frac{(1 - h_2) q_2 s_2}{(1 - h_2) q_2 - h^G q_1}. \quad (29)$$

Nonetheless, an increase in the value of assets pledged at the interim date translates into higher interests to be paid at the final date. The commitment of solvent bank 2 with depositors at the final date is

$$q_2 \bar{s}_2 - q_2 \bar{s}_2 (1 - h_2) (1 + r), \quad (30)$$

whereas it has available liquidity that, after repurchasing assets pledged with the policy maker, amounts to

$$(q_1 + q_2) s - q_2 p^b (1 - h_2) (1 + r) - q_1 p^b (1 - h^G) (1 + r^G). \quad (31)$$

Similarly to the interim date, the bank is solvent if (30)  $\leq$  (31). This yields an additional necessary condition on  $p^b$ :

$$p^b \leq \frac{q_2 s_2 [(1 - h_2) r - h_2] + s (q_1 + q_2)}{(1 - h_2) q_2 (1 + r) + (1 - h^G) q_1 (1 + r^G)} \quad (32)$$

The two conditions (29) and (32) are simultaneously satisfied, when  $r^G \leq r$ , if and only if

$$h^G < \frac{q_2 (1 - h_2)}{q_1}.$$

□

*Proof of Proposition 7.* The proof goes along the same line as that of Proposition 6. The only difference is the amount of liquidity bailing out banks have at the interim and the final date. When the policy maker does not intervene, bank 1 available liquidity at the interim date

is

$$p^b(1-h_2)q_2 - (1-h_2)pq_2(1+r) - p^b q_1. \quad (33)$$

At the final date, the bank holds an amount of liquidity

$$(q_1 + q_2)s - p^b(1-h_2)q_2(1+r). \quad (34)$$

The conditions on  $p^b$  that allow a bailout with no public intervention are therefore

$$p^b \geq \frac{(1-h_2)q_2s_2}{q_1 - (1-h_2)q_2}, \quad (35)$$

$$p^b \leq \frac{-h_2q_2(r+1)s_2 + s(q_1 + q_2) + q_2rs_2}{(1-h_2)q_2(r+1)}. \quad (36)$$

The set defined by such conditions is empty unless

$$s > \frac{q_2^2s_2(1-3h_2+2h_2^2(1+r))+2r-4h_2r+q_1q_2s_2[(1+r)h_2-r]}{q_1^2-q_2^2+q_2h_2(q_1+q_2)}. \quad (37)$$

□



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