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RELATIONSHIP FINANCE, MARKET FINANCE AND ENDOGENOUS BUSINESS CYCLES

Luca Deidda
Bassam Fattouh

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CRENoS – Cagliari
Via San Giorgio 12, I-09100 Cagliari, Italia
tel. +39-070-6756406; fax +39-070- 6756402

CRENoS - Sassari
Via Torre Tonda 34, I-07100 Sassari, Italia
tel. +39-079-2017301; fax +39-079-2017312

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Via Is Mirrionis, 1
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Relationship Finance, Market Finance and Endogenous Business Cycles *

Luca Deidda
University of Sassari, CRENoS, and CeFiMS
Bassam Fattouh
Department of Financial and Management Studies and CeFiMS, SOAS

Abstract
This paper develops an overlapping generation model with asymmetric information in the credit market such that the interplay between relationship finance supplied by investors who monitor investment decisions ex-ante and market finance supplied by investors who relay on public information can be the source of endogenous business fluctuations. Monitoring helps reducing the inefficiency caused by moral hazard. However, the incentives of entrepreneurs to demand relationship finance to induce monitoring –which is also non-contractible – are weaker the lower is the return to investment. If the return to investment is low enough, entrepreneurs demand too little relationship finance. This leads to an inefficiently low level of monitoring and of entrepreneurial effort. Under decreasing marginal returns to capital, the model generates a reversion mechanism that can induce macroeconomic instability. The economy can experience endogenous business cycles characterized by a pro-cyclical behavior of the relative importance of relationship finance. This is consistent with the pro-cyclical behavior of the indicator of relative importance of relationship finance, which we construct based on quarterly and annual data from the US Flow of Funds Accounts for the non-financial corporate business sector.

Keywords: Moral hazard, Endogenous business cycles, relationship finance, market finance, Monitoring
Jel Classification: E32, E44, D82

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1 Introduction

The role of financial market imperfections in short run economic fluctuations has received a wide attention in the literature. Financial frictions propagate exogenous shocks according to the *financial accelerator* principle (Bernanke and Gertler, 1989) and generate cycles in the presence of exogenous idiosyncratic shocks (Kyotaki and Moore, 1997). Also, and more relevant to this paper, financial market imperfections could be the source of macroeconomic instability and endogenous business cycles (Suarez and Sussman, 1997, Matsuyama, 2004, 2007a,b).

This paper develops an overlapping generation model characterized by credit market imperfections. It shows that the interplay between relationship finance supplied by financial investors who privately monitor investment decisions ex ante and market finance supplied by investors who rely on public information can be the source of endogenous business fluctuations.

Entrepreneurs invest to produce capital goods to be used by competitive firms. The probability of success of such investments depends positively upon entrepreneurial effort. Entrepreneurial effort is costly and non-contractible, so that the financing of entrepeneurs' investments is subject to a potential moral hazard (MH) problem. If sufficiently leveraged, entrepreneurs might have the incentive to reduce effort, which would result in excessive risk taking.[1] When MH bites, exerted effort depends positively on the intensity with which financial investors monitor entrepreneurs, which in turns depends positively on the amount of relationship finance demanded by the entrepreneurs, since monitoring is itself non-contractible.[2] Because of monitoring costs, relationship finance is more costly than market finance. Yet, entrepreneurs may still want to demand relationship finance in order to induce financial investors to monitor them – which results in a credible commitment to exert effort – as this would enable to raise cheaper market

[1] The model is abstract enough to neglect any distinction between debt, equity or any other type of financial claim. Matsuyama (2007) also adopts such modelling strategy and argues about the scope for such simplification when assessing the general effects of credit market imperfections referring to Tirole (2006, p.119). Accordingly, in the model, the concept of leverage refers to the ratio of external to internal financing.

[2] This feature makes our model of financial markets close in spirit to that by Holmstrom and Tirole (1997).

finance. This, equivalently to what happens in Holmstrom and Tirole (1997).

Given that the rate of return earned by an entrepreneur equals the marginal product of capital minus financing costs and disutility from effort per unit of investment, such incentive to demand relationship finance becomes weaker as the return to capital gets lower.[3] Hence, under decreasing returns to capital accumulation, the model generates a reversion mechanism that results in potential macroeconomic instability, which could take the form of endogenous business fluctuations. As the economy accumulates capital during an expansionary phase, the return to capital declines so that entrepreneurs might eventually demand less relationship finance, and (therefore) exert lower effort. Since non-contractibility of monitoring services raises the cost of monitoring – and of the relationship finance that goes with it – compared to the case of full information, the demand for relationship finance – and the entrepreneurial effort associated with it – might drop to inefficiently low levels. This would induce a contraction in the net level of output produced by the economy. As the capital stock decreases over time along a recession path, the return to capital it increases to a sufficiently high level to induce entrepreneurs to increase their demand for relationship finance, and (therefore) also the level of effort they exert. At this stage, a new expansionary phase would take place.

Based on US aggregate data, we construct indicators of relationship finance and market finance, and provide evidence on the relative importance of these two sources of funds along the business cycle. Our findings suggest that business cycle fluctuations tend to be associated with changes in the relative importance of market versus relationship finance. On average, expansions witness an increase in the relative importance of relationship finance, while the opposite happens during recessions. Such empirical findings are consistent with the theory we develop according to which the intensity of monitoring, which depends positively on the amount of relationship finance, is procyclical.

Our theoretical results complement those by Suarez and Sussman (1997), and Matsuyama (2004, 2007a). In Sussman and Suarez (1997) insiders have a preference for excessive risk taking in the presence of external financing, which governance mechanisms

[3] When the marginal product of capital is relatively low, so is the expected rate of return that entrepreneurs appropriate, which means that the opportunity cost of exerting no effort is low.

can only partially offset. Economic booms lead to low prices, which reduce liquidity and increase external finance triggering excessive risk taking and high failure rates. In turn, economic busts result in high prices, high liquidity, less external finance and less risk taking triggering a new economic expansion.[4] Our theory also predicts excessive risk taking and high failure rates at the end of expansions and during recessions, and less risk taking and lower failure rates at the end of recessions and during expansions.

In Matsuyama (2004), endogenous fluctuactions arise because of cyclical inefficient changes in the composition of investment. While insiders always choose investments that maximize the return to financial investors, they have the incentive to switch to investments that result in lower wages at the peak of the economic cycle. The reverse holds at the bottom of the cycle. Such behaviour generates a reversion mechanism capable of inducing endogenous business cycle fluctuations. The endogenous fluctuations are the by-product of an unresolved conflict of interest between financial investors and workers. Matsuyama (2007a) develops a model where the composition of the credit is affected by the borrower's net worth, causing an endogenous switch between investment projects with different productivity levels, which produces macroeconomic instability.[5]

In our model, the downturn in the business cycle is also associated with comparatively lower wages. Hence, from a corporate governance perspective, workers are among the outside stakeholders who are being hurt. The conflict of interest between entrepreneurs who control financing decisions and outside stakeholders such as workers is countercyclical. Such conflict of interest could be severe at the peak of the cycle and unimportant at the bottom of the cycle enough to cause a reversion mechanism capable of generating endogenous cycles. The paper is organized as follows: section two describes empirical evidence. Section three presents the model. Section four considers the benchmark case of full contractibility. Section five analyzes the partial equilibrium of financial markets

[4]Relately, Favara (2006) presents a model where entrepreneurs' are subject to moral hazard and financial investors engage in ex post monitoring. Entrepreneurs' incentives toward firms' value maximization and financial investors' incentives to monitor combine in such a way that recessions discourage the adoption of unproductive investment while booms encourage it, so that endogenous fluctuations might arise.

[5]Matsuyama (2007b) provides an exensive and systematic presentation of the aggregate consequences of credit frictions, by means of a unified framework.

under asymmetric information. Section six solves for the dynamic general equilibrium under asymmetric information. Section seven concludes the paper.

2 Relationship vs market finance and business cycle in the US economy

In this section we present the main empirical observations that motivate our theoretical model. Abstracting from the issue of the nature of financial claims, we construct a measure of market finance and a measure of relationship finance. We then construct an indicator of the relative importance of one versus the other - which reflects the importance of outside financiers that could play a monitoring role and analyze how such indicator behaves over the business cycle.

We use seasonally adjusted data from the Flow of Funds Account (FOF) of the United States of the Federal Reserve Statistical Release for the nonfarm nonfinancial corporate business (Table F.102). We construct an indicator of market finance (MF_t) given by the sum of net equity issues (line 38), commercial paper (line 40), municipal securities (line 41), corporate bonds (line 42) and asset-backed-securitised (ABS) issuers (line 50)[6]; and an indicator of relationship finance (RF_t) given by the sum of bank loans (line 43), other loans and advances excluding ABS (line 44-line 50), mortgages (line 51) and trade payable (line 44).[7] We use annual data over the period 1946 to 2007 and quarterly data over the period 1952 Q1 and 2007 Q4.

One feature of market finance is that in some years/quarters it can take a negative value. The retirement of equity in mergers and acquisitions and stock repurchase can explain to a large extent the large negative equity flows observed in the data. As a result, the use of the ratio between RF_t and the sum of RF_t and MF_t to measure the relative importance of RF_t would pose practical problems. To address this problem, Baker and Wurgler (2000) set the equity share to zero in years where net equity issues are negative.[8]. We circumvent this practical issue in a different way, by constructing the

[6] Not including ABS issuers in the construction of the measure does not affect any of the results.
[7] The results are not sensitive to the inclusion of trade payable in the constructed index.
[8] The underlying assumption is that years/quarters in which net equity flows are negative are those

following indicator:[9]

$$FS_t = \frac{e^{RF_t}}{e^{RF_t} + e^{MF_t}}, \tag{1}$$

where RF_t and MF_t are market and relationship finance, respectively.

A simple but informative way to explore the behavior of MF_t, RF_t and FS_t over the business cycle is to examine their cross correlations with the cyclical components of the real GDP. Seasonally-adjusted data for the real GDP were obtained from the National Economic Account of the Bureau of Economic Analysis.

Table 1 reports conditional and unconditional correlations for both annual and quarterly data. As it can be seen, while the level of relationship finance behaves procyclically, the level of market finance behaves countercyclically. Moreover, the relative importance of relationship finance is also procyclical. These results apply both to detrended and not detrended financial series. Moreover, the results also hold, for correlations conditional on the economy being in an expansion/contraction.

Table 2 also shows that the relative importance of relationship finance is statistically higher during expansions than contractions. The null hypothesis that the difference between the mean of FS_t during expansions and that during contractions is zero can be rejected at the 1% level, both using the *T-test* and its non-parametric version, the Wilcoxon-Mann-Whitney statistics.

years in which equity is not the preferred choice for new funding

[9]Note that the ratio takes values between zero and one.

Table 1: Cyclical behaviour of financial structure: unconditional correlation analysis

	Quarterly (1952 : 1 − 2007 : 4		Annual (1946 − 2007)	
	Detrended	Not detrended	Detrended	Not detrended
Unconditional correlations				
$Corr(RGDP_t, RF_t)$	0.5002 (0.0000)	0.4251 (0.0000)	0.5392 (0.0000)	0.4225 (0.0006)
$Corr(RGDP_t, MF_t)$	−0.1930 (0.0037)	−0.1184 (0.0771)	−0.2946 (0.0201)	−0.1058 (0.4133)
$Corr(RGDP_t, FS_t)$	0.4782 (0.0000)	0.4861 (0.0000)	0.5018 (0.0000)	0.4314 (0.0005)
Conditional correlations				
$Corr(RGDP_t, RF_t\|E)$ *112*	0.3952 (0.0000)	0.2928 (0.0017)	0.5506 (0.0011)	0.4531 (0.0092)
$Corr(RGDP_t, MF_t\|E)$ *112*	−0.0951 (0.3185)	0.1603 (0.0914)	−0.0988 (0.5905)	0.1974 (0.2788)
$Corr(RGDP_t, FS_t\|E)$ *112*	0.2690 (0.0041)	0.2288 (0.0152)	0.3250 (0.0695)	0.2814 (0.1187)
$Corr(RGDP_t, RF_t\|C)$ *112*	0.3552 (0.0001)	0.2666 (0.0045)	0.2501 (0.1826)	0.2203 (0.2420)
$Corr(RGDP_t, MF_t\|C)$ *112*	−0.1051 (0.2701)	−0.1923 (0.0422)	0.0736 (0.6989)	−0.1829 0.3333
$Corr(RGDP_t, FS_t\|C)$ *112*	0.2963 (0.0015)	0.3208 (0.0006)	0.3929 (0.0317)	0.2868 (0.1244)

MF refers to Market Finance and is given the sum of net equity issues, commercial paper, municipal securities, corporate bonds and asset-backed-securitised (ABS) issuers. RF refers to Relationship Finance and is given by the sum of bank loans, other loans and advances (excluding ABS), mortgages and trade payable. FS reflects the relative importance of Relationship finance and is given by equation (1). RGDP is GDP in billions of chained 2000 US dollars. All data are seasonally adjusted. All variables in the de-trended column are de-trended using the Hodrick-Prescott (HP) filter. The smoothing parameter is set to 1600 for quarterly data and 6.25 for annual data. In the not-detrended column, only RGDP is deternded using the HP Filter. Figures in parentheses are the significance levels of the correlation coefficients. Numbers in italic are the number of observations.

Table 2: Cyclical behaviour of financial structure: conditional means

	Quarterly (1952 : 1 − 2007 : 4)	Annual (1946 − 2007)
Mean value of FS_t		
Contraction (C)	0.4874892	0.5606653
Expansion (E)	0.8690211	0.8537078
T-test	0.0000	0.006
Wilcoxon-Mann-Whitney Stat.	0.0000	0.0025

The t-test statistics tests the null hypothesis that the mean in both equations are the same. The Wilcoxon-Mann-Whitney statistics which is the non-parametric version of the t- test also points. It tests the null hypothesis that the two populations have the same central location in favour of the alternative that the two population distributions differ. The figures in parentheses are the significance levels of the tests. The number in italics are the number of observations.

Such empirical evidence complements the findings of the extensive literature on the behavior of financial structure indicators such as debt, equity and leverage over the business cycle. Choe, Masulis, and Nanda (1993) find that the volume and the frequency of equity issuance are higher during NBER expansions than during NBER recessions. Covas and den Haan (2006) find that equity issuance is pro-cyclical for all but the largest firms. Korajczyk and Levy (2003) find that unconstrained firms are able to time their equity issues to periods when macroeconomic conditions are improving. As to debt issuance, the picture is rather mixed. Choe, Masulis, and Nanda find that gross debt issuance is countercyclical. In contrast, Covas and den Haan (2006) find that long term debt is pro-cyclical for all but the largest firms. Baker and Wurgler (2000) and Covas and den Haan (2006) find a strong positive correlation between debt and equity issuance.[10] Jerman and Quadrini (2006, 2009) find that while corporate debt is countercyclical, equity payouts are procyclical, implying that there is substitution over the cycle between debt and equity as a source of financing. From this perspective, our findings point out that there could possibly be substitution between relationship and market finance as well, over the business cycle.

[10]Covas and den Haan (2006) examine the cross correlation between GDP and default rates defined as the number of defaults during year divided by the number of outstanding issuers and show strong evidence of counter-cyclicality in default rates.

3 The model

The economy is populated by a continuum of size one of firms and by overlapping generations of two period-living agents.[11] There is only one final good, which is either consumed or invested. Each generation consists of a continuum of individuals, each endowed with one unit of indivisible labor when young, which is supplied inelastically to firms in exchange of a competitive salary, w_t. Within each generation, a time-invariant fraction λ of individuals are *entrepreneurs*, while the remaining $1-\lambda$ are *financial investors*. All entrepreneurs are identical, and so are financial investors.

While all agents can operate as financial investors, only entrepreneurs can operate the technology required to produce physical capital. Production of capital uses physical investment, $I_t \geq 0$, and entrepreneurial effort, $x \geq 0$. An investment of size I_t yields an amount of capital $K_{t+1} = I_t$ at time $t+1$ with probability $\pi(x)$ and zero otherwise, where $\pi(x)$ is a function of effort, x, such that $\pi(x) \in [0,1]$ for any value of x and $\pi(x') > \pi(x)$, for any $x' > x$. Effort can take values, 2, 1 or 0. Capital is rented out in exchange for a rental price q_t. Investment is funded by internal and external sources of finance.

Both entrepreneurs and financial investors derive utility from consumption in their second period of life. Individual utility function of entrepreneurs of generation t is $c_{t+1} - b(x)$, where c_{t+1} is consumption in period $t+1$, and $b(x)$ measures disutility from effort undertaken in period t, both measured in units of the final good, where $b(x) \geq 0$ for any value of x, with $b(0) = 0$ and $b(x') > b(x)$ for $x' > x$.

Firms, which are price takers, produce the final good according to a standard production function of the form $Y_t = G(K_t, L_t)$, where K_t is capital and L_t is labor,[12] $G(.,.)$ is a strictly concave linearly homogeneous function, with the intensive form, $g(k_t) \equiv G(K_t/L_t, 1)$, where $k_t \equiv K_t/L_t$, satisfying the Inada conditions and $g(0) = 0$. We further impose $lim_{k_t \to 0} \pi(0) g''(k_t) k_t > 1$ to ensure the existence of a non-trivial steady state.

[11] Equivalently to Bernanke and Gertler (1989) generations should be thought of as representative of entry and exit of firms from capital markets.

[12] We do not consider (exogenous) technological progress as we are only interested in business cycle fluctuations.

The economy functions as follows. At time t, firms rent capital from old entrepreneurs and hire labor from young individuals to produce final goods. Young entrepreneurs receive w_t in exchange for their labor and invest in order to produce capital. Alternatively, they can finance other entrepreneurs. Young financial investors of generation t, who also receive w_t for their labor, can only save by financing entrepreneurs. At time $t+1$, old entrepreneurs of generation t whose investments have been successful rent capital to firms. These firms produce and pay q_{t+1} per unit of capital to entrepreneurs of generation t and w_{t+1} to workers of generation $t+1$. (Old) entrepreneurs and financial investors of generation t consume their returns and die, while young invididuals of generation $t+1$ engage in investment as explained before. The time-line of the model is summarized in figure 1.

We first solve the model in the benchmark case of perfect information, and then under the hypothesis that effort is non-observable and non-contractible. In the latter case, we assume that financial investors have access to the following monitoring technology: by monitoring entrepreneurs with intensity 1 financial investors prevent entrepreneurs from choosing effort equal to 0; by monitoring entrepreneurs with intensity 2 financial investors force entrepreneurs to exert a level of effort equal to 2. Total monitoring costs per entrepreneur are increasing in the amount of external finance, f_t, as well as in the monitoring intensity, $i \in \{0, 1, 2\}$, where $i = 0$ means no monitoring, and decreasing in the amount of internal finance, e_t, according to the following expression, $M(f_t, e_t, i) = m(i)f_t/e_t$, with $m(2) = m(1) + \Delta m$ and $\Delta m > 0$. Monitoring is non-observable and non-verifiable and therefore non contractible.

4 The benchmark case: Perfect information

In this section we characterize the macroeconomic equilibrium in the benchmark case of perfect information. In particular, entrepreneurs' effort choices are fully observable and verifiable, and therefore contractible.

4.1 Agents' behavior

The expected utility of a young entrepreneur of generation t who undertakes and investment of size $f_t + e_t$, where f_t and e_t are external and internal funds used to finance it, and chooses a level of effort x is

$$u^E_{t+1} = u(e_t, f_t, x) = \pi(x)[q_{t+1}(e_t + f_t) - R_{t+1}(x)f_t] - b(x)(f_t + e_t), \tag{2}$$

where, at each time t, $R_{t+1}(x)$ is the gross rate of return – contingent on the level of effort, x – to be paid (in case of success) to financial investors at time $t+1$.[13] Agents take $R_{t+1}(x)$ as well as the other prices, as given. Accordingly, the entrepreneur's demand for external funds, conditional on effort, x, is

$$f^d_t = \begin{cases} \infty & \text{if } \pi(x)\,[q_{t+1} - R_{t+1}(x)] - b(x) > 0 \\ [0, \infty) & \text{if } \pi(x)\,[q_{t+1} - R_{t+1}(x)] - b(x) = 0. \\ 0 & \text{if } \pi(x)\,[q_{t+1} - R_{t+1}(x)] - b(x) < 0 \end{cases} \tag{3}$$

An entrepreneur exerting effort x, self-finances investment if and only if

$$e_t = \begin{cases} w_t & \text{if } \max_x(\pi(x)R_t(x)) < \pi(x)q_{t+1} + b(x) \\ [0, w_t) & \text{if } \max_x(\pi(x)R_t(x)) = \pi(x)q_{t+1} + b(x). \\ 0 & \text{if } \max_x(\pi(x)R_t(x)) > \pi(x)q_{t+1} + b(x) \end{cases} \tag{4}$$

Entrepreneurs choose x, e_t and f_t in order to maximize u^E_{t+1}.

Financial investors supply funds if and only if the expected return exceeds zero. Therefore, the supply of funds by a financial investor is

$$f^s_t = \begin{cases} w_t & \text{if } \pi(x)R_{t+1}(x) > 0 \text{ for any } x \in \{0,1,2\} \\ [0, w_t] & \text{if } \pi(x)R_{t+1}(x) \leq 0 \text{ for all } x \in \{0,1,2\}, \text{ with strict equality for some } x. \\ 0 & \text{if } \pi(x)R_{t+1}(x) < 0 \text{ for all } x \in \{0,1,2\} \end{cases} \tag{5}$$

Firms hire labor and rent capital in order to maximize their profits given by

$$G(K_t, L_t) - w_t L_t - q_t K_t, \tag{6}$$

[13] Note that, under limited liability, financial investors receive nothing in the event of failure, because in this case the gross return generated by the entrepreneur equals zero.

taking the wage rate, w_t, and the rental price of capital, q_t, as given. Inverse demand functions for capital and labor are defined by the first order conditions of firms' maximization problem:

$$q_t = g'(k_t^d) \tag{7}$$

$$w_t = [g(k_t^d) - g'(k_t^d)k_t^d], \tag{8}$$

where $g(k_t^d) = G(k_t^d, 1)$, and $k_t^d = K_t^d / L_t^d$, is the capital-labor ratio demanded by each firm, L_t^d and K_t^d being the demand for labor and capital, respectively, for a given pair of prices of capital and labor, $\{w_t, q_t\}$.

Supply of capital and labour are inelastic and positive for $q_t \geq 0$ and $w_t \geq 0$, respectively.

4.2 Macroeconomic equilibrium

Define $\theta_t(x) \geq 0$ the fraction of aggregate investment of quality x at time t, which is associated with level $x \in \{0, 1, 2\}$ of entrepreneurial effort. Correspondingly, the average quality of aggregate investment is defined as

$$\chi_t = \sum_{x=0}^{2} \theta_t(x)x, \tag{9}$$

where

$$\sum_{x=0}^{2} \theta_t(x) = 1. \tag{10}$$

Given that investments of quality x are successful with probability $\pi(x)$,

$$\pi_t = \Pi(\chi_t) = \sum_{x=0}^{2} \theta_t(x)\pi(x), \tag{11}$$

measures the average probability of success of investments undertaken in period t. π_t is an increasing function of χ_t such that: i. For any given values $\chi_t^{'} > \chi_t^{''}$, $\Pi(\chi_t^{'}) > \Pi(\chi_t^{''})$ follows; ii. $\Pi(\chi_r) = \pi(x)$, for $\chi_t = x$, with $x \in \{0, 1, 2\}$.

Label z_t the fraction of aggregate funds channeled to investment at time t. Then,

Definition 1 (Intertemporal equilibrium)**.** *An intertemporal macroeconomic equilibrium is a sequence* $\left\{k_{t+1}, \chi_t, z_t, q_t, w_t, \{R_{t+1}(x), \theta_t(x)\}_{x=0}^{2}\right\}_{t=t_0}^{\infty}$*, where* t_0 *is the starting date of the economy, such that, given the initial level of capital at time* t_0*,* K_{t_0}*,* *i. Financial investors and entrepreneurs act optimally based upon their rational expectations;*

ii. Markets clear;

for all $t \geq t_0$.

Consider an equilibrium at time t characterized by production and possibly financial exchange, such that a fraction $z_t > 0$ of aggregate funds is channeled toward investment, χ_t is the average quality of investment, and existing capital and labor are fully employed in production.

Let K_{t+1} the amount of capital available at $t+1$. In equilibrium, the capital-labour ratio at time $t+1$, k_{t+1}, should be the same across firms. Then, given that the aggregate amount of labor in the economy equals one, if factors of production are fully employed, $k_{t+1} = K_{t+1}$, must hold.

Imposing the market clearing condition, $k^d_{t+1} = k_{t+1}$, and given equations (7) and (8) we write the equilibrium prices of capital and labor at time $t+1$ as

$$q_{t+1} = = g'(k_{t+1}) \equiv q(k_{t+1}); \tag{12}$$

$$w_{t+1} = = g(k_{t+1}) - g'(k_{t+1})k_{t+1} \equiv w(k_{t+1}). \tag{13}$$

Given demand and supply of financial funds, as given by equations (4, 5) and imposing market clearing, for any level of effort x such that $\theta_t(x) > 0$, the equilibrium cost of external funds for entrepreneurs exerting x is

$$R_{t+1}(x) = q(k_{t+1}) - \frac{b(x)}{\pi(x)} \equiv R(k_{t+1}, x). \tag{14}$$

Accordingly, entrepreneurs' expected return per unit of self-financing when exerting such level of effort is

$$\pi(x)q(k_{t+1}) - b(x). \tag{15}$$

Aggregate funds at time t amount to w_t. If a fraction z_t is invested in the production of capital, aggregate capital available at time $t+1$ is $K_{t+1} = \pi_t z_t w_t$, where π_t measures the fraction of successful investments. Given $K_{t+1} = k_{t+1}$, we can write, $k_{t+1} = \pi_t z_t w_t$. Then, using equations (13) and (11) to substitute in for w_t and π_t respectively,

$$k_{t+1} = \Pi(\chi_t) z_t [g(k_t) - g'(k_t)k_t] \equiv k(k_t, \chi_t, z_t), \tag{16}$$

which gives the equilibrium level of capital-labor ratio at time t+1 as a function of the capital-labour ratio at time t, k_t, the average quality of investment χ_t, and the fraction of aggregate funds channeled toward investment, z_t.

Given k_t, z_t and χ_t, the aggregate product of the economy would be

$$y_{t+1} = g(k_{t+1}). \tag{17}$$

and aggregate value of entrepreneurs' disutility from effort measured in units of final good would be

$$b_{t+1} \equiv \sum_{i=0}^{2} \theta_t(x) b(x) z_t [g(k_t) - g'(k_t) k_t]. \tag{18}$$

so that, $y_{t+1} - b_{t+1}$ measures the level of aggregate output net of entrepreneurs' disutility from effort.

With no loss of generality we impose

Assumption 1. *Let* $\Delta_2 \equiv \pi(2) - \pi(1)$ *and* $\Delta_1 \equiv \pi(1) - \pi(0)$. *Then,*

$$\frac{b_2 - b_1}{\Delta_2} > \frac{b_1}{\Delta_1} \tag{19}$$

$$0 < \frac{b(1)}{\pi(1)} < \frac{b(2)}{\pi(2)} \tag{20}$$

The above assumption is to ensure that in equilibrium: (i) If x' and x'' are two levels of effort exerted with positive probability, then any level of effort x''' such that $x' < x''' < x''$ will also be exerted with positive probability; (ii) If only part of the funds are channeled toward investment, then the average quality of investment quality should equal zero (see Lemma 1 in the Appendix).

The main result for the perfect information case is as follows.

Proposition 1 (Equilibrium uniqueness and characterization)**.** *Given the initial level of capital* $K_{t_0} > 0$*, the intertemporal macroeconomic equilibrium is unique and characterized as follows:*

a. *The fraction of funds chaneled toward investment,* z_t*, satisfies* $z_t = 1$ *for all t; capital and labor are fully employed at all t; the average quality of aggregate investment,* $\{\chi_t\}_{t=t_0}^{\infty}$*, and the capital-labour ratio,* $\{k_t\}_{t=t_0}^{\infty}$ *converge monotonically to the steady state values*

$$\widehat{\chi} = \sum_{x=0}^{2} \widehat{\theta}(x) x; \tag{21}$$

$$\widehat{k} = \kappa(\widehat{\chi}, \widehat{z}) \ : \widehat{k} = k(\widehat{k}, \widehat{\chi}, \widehat{z}). \tag{22}$$

where $\widehat{\chi} \in [0,2]$ *and* $\widehat{\theta}(x)$ *is the steady state fraction of aggregate investment of quality* x*, with* $\sum_{x=0}^{2} \widehat{\theta}(x) = 1$*;*

b. *The steady state value of* $\widehat{\chi}$ *is uniquely determined as follows:*

   i. $\widehat{\chi} = 0$*, if and only if* $g'(\kappa(0,1)) \leq \frac{b(1)}{\Delta_1}$*;*

   ii. $\widehat{\chi} = 1$ *if and only if* $g'(\kappa(1,1)) \in \left[\frac{b_1}{\Delta_1}, \frac{b(2)-b(1)}{\Delta_2}\right]$*;*

   iii. $\widehat{\chi} = 2$ *if and only if* $g'(\kappa(2,1)) \geq \frac{b(2)-b(1)}{\Delta_2}$*;*

   iv. $\widehat{\chi} \in (0,1)$ *if and only if* $g'(\kappa(0,1)) > \frac{b(1)}{\Delta_1}$ *and* $g'(\kappa(1,1)) < \frac{b(1)}{\Delta_1}$*;*

   v. $\widehat{\chi} \in (1,2)$ *if and only if* $g'(\kappa(1,1)) > \frac{b(2)-b(1)}{\Delta_2}$ *and* $g'(\kappa(2,1)) < \frac{b(2)-b(1)}{\Delta_2}$*.*

Proof. See appendix.

The above proposition states that along the transition process toward the steady state, the average quality of investment either stays constant or monotonically converges to a (steady state) constant; and the level of capital-labor ratio – and therefore the level of income per capita – also monotonically converge to a (steady state) constant. Before turning to the case of imperfect information it is worth mentioning the following result:

Remark 1. *Under complete information, the quality of investment,* $\{\chi_t\}_{t_0}^{\infty}$*, associated with the competitive equilibrium is Pareto-optimal.*

Proof. See appendix.

We now turn to the analysis of the real effects induced by the non-contractibility of entrepreneurs' choice of effort in the presence of non-contractible monitoring services.

5 Asymmetric information and non-contractibility

Asymmetric information introduces an element of interaction among agents to the extent that less informed parties take the incentives of other agents into account when assessing the profitability of alternative investments. Financial investors can either operate as *bankers* who provide *relationship finance*, possibly subject to monitoring, or as *market investors* who provide *market finance* relying on public information. Bankers choose monitoring intensity taking into account entrepreneurs' incentives to exert effort. Market investors assess entrepreneurs' incentives to exert effort taking into account bankers' incentives to monitor. Entrepreneurs, will now choose not just: (i) Whether to invest in capital production or not; (ii) The level of effort; (iii) The amount to invest, but also; (iv) The composition of the external funds they demand in terms of relationship and market finance. Note that, differently from the full information case, payments to financial investors cannot be made contingent on effort or monitoring since both are non-contractible.

Before turning to the characterization of the macroeconomic equilibrium, we analyze the partial equilibrium of the financial markets, other things given.

5.1 Financial markets

The time-sequence of entrepreneurs' and financial investors' financial decisions is as follows:

Stage 1a. Each financial investor decides whether to be a banker or a market investor, and whether to supply finance or not;

Stage 1b. Each entrepreneur decides: (i) Whether to invest or provide finance to other entrepreneurs; (ii) whether to demand external funds, f_t^d, (iii) whether to self-finance investment, e_t, and; (iv) The fraction of relationship finance to demand per unit of external funds, l_t^d;

Stage 2. Markets for market and relationship finance clear;

Stage 3. Entrepreneurs privately choose effort, x, and bankers privately choose monitoring intensity, i.

We require that, in equilibrium, agents' decisions are optimal at each stage, given the information available and given that they correctly anticipate the returns on market and relationship finance. Accordingly, the following definition of equilibrium applies

Definition 2 (Partial equilibrium of financial markets)**.** *For a given price of capital at time $t+1$, q_{t+1} , a partial equilibrium of the financial markets at any time t consists of two return-functions, $R^B_{t+1}(l, f, e, q_{t+1})$ and $R^M_{t+1}(l, f, e, q_{t+1})$ and a set of strategies played by agents such that, other things given:*

i. Agents' strategies should be optimal at each stage, given the return functions, and other agents' strategies;

ii. $R^B_{t+1}(l, f, e, q_{t+1})$ *and* $R^M_{t+1}(l, f, e, q_{t+1})$ *are consistent with agents' optimal behavior and clear the markets for relationship and market finance.*

The action, $\{l_t, f_t, e_t\}$, played by an entrepreneur, might be informative about the monitoring intensity, i, the entrepreneur is subject to and (therefore) the effort, x, he exerts, which ultimately determines the probability of success of their investments. Accordingly, equilibrium returns are defined by the return-functions $R^M_{t+1}(l, f, e, q_{t+1})$ and $R^B_{t+1}(l, f, e, q_{t+1})$ that map entrepreneurs' actions $\{l_t, f_t, e_t\}$ into values of returns for relationship and market funds, given the rental price of capital, q_{t+1}.[14] According to the above definition, we require the return functions $R^B_{t+1}(l, f, e, q_{t+1})$, and $R^M_{t+1}(l, f, eq_{t+1})$ to be consistent with the subgame perfect equilibrium (SPE) strategies played by the agents'.

In order to characterize the partial equilibrium of the financial markets, we first describe agents' financial behaviour for given return-functions $R^B_{t+1}(l, f, e, q_{t+1})$ and $R^M_{t+1}(l, f, e, q_{t+1})$.

5.1.1 Entrepreneurs' behavior

The expected utility of an entrepreneur who invests $f_t + e_t$ and puts effort x, is

$$u^E_{t+1} = \pi(x)[q_{t+1}(e_t + f_t) - R_{t+1}(l_t, f_t, e_t, q_{t+1})f_t] + b(x)(e_t + f_t), \tag{23}$$

where,

$$R_{t+1}(l_t, f_t, e_t, q_{t+1}) \equiv R^B_{t+1}(l_t, f_t, e_t, q_{t+1})l_t + (1 - l_t)R^M_{t+1}(l_t, f_t, e_t, q_{t+1}), \tag{24}$$

is the weighted average cost of external funds faced by the entrepreneur.

If engaging in entrepreneurial activity, entrepreneurs choose l_t, f_t, e_t and x in order to maximize u^E_{t+1} taking $R^B_{t+1}(l, f, e, q_{t+1})$ and $R^M_{t+1}(l, f, e, q_{t+1})$ as given.[15] Accordingly, they undertake entrepreneurial activity if and only if $u^E_{t+1} \geq 0$ for some feasible choice $\{l_t, e_t, f_t, x\}$.

[14] Under full information, there is no need to introduce return-functions explicitly as entrepreneurs' choices, $\{l_t, f_t, e_t\}$, do not carry additional information about the effort being exerted by the entrepreneur.

[15] Note that, other things equal, maximizing u^E_{t+1} requires choosing l_t so to minimize $R_{t+1}(l_t, f_t, e_t, q_{t+1})$.

5.1.2 Financial investors' behavior

Define $\alpha(x|l, f, e, i)$ the probability that an entrepreneur playing $\{l, f, e\}$ exerts effort x when he is monitored with intensity i. Then, the expected rate of return for a banker financing an entrepreneur who plays $\{l_t, f_t, e_t\}$, and therefore demands $l_t f_t$ units of relationship finance, is

$$\max_{i\in\{0,1,2\}} \sum_{x=0}^{2} \alpha(x|l_t, f_t, e_t, i)\pi(x) R^B_{t+1}(l_t, f_t, e_t, q_{t+1}) - \frac{m(i)}{l_t e_t}. \tag{25}$$

In equilibrium, bankers supply finance if and only if the above expression is positive for some $\{l_t, f_t, e_t\}$ played with positive probability.

Define $\tau(i|l, f, e)$ the probability that an entrepreneur playing $\{l, f, e\}$ is monitored with intensity i. Then, the expected rate of return for a market investor who is financing an entrepreneur who plays $\{l_t, f_t, e_t\}$ is

$$\sum_{i=0}^{2} \tau(i|l, f, e)\alpha(x|l_t, f_t, e_t, i)\pi(x) R^M_{t+1}(l_t, f_t, e_t, q_{t+1}) \tag{26}$$

In equilibrium, market investors supply finance if and only if the above expression is positive for some $\{l_t, f_t, e_t\}$ played with positive probability. Finally, financial investors decide whether to be bankers or market investors depending on the expected returns from such activities given the actions played by entrepreneurs.

We now turn to the analysis of entrepreneurs' incentives to exert effort and bankers' incentives to monitor.

5.1.3 Potential moral hazard

Consider an entrepreneur who plays an action $\{l_t = 0, f_t > 0, e_t = 0\}$ and is not being monitored, i.e. $\tau(0|l_t = 0, f_t > 0, e_t = 0) = 1$. Let R_M be the equilibrium value of the cost of market finance associated to such action, and q the value of the rental price of capital. It is immediate to verify that as long as

$$\Delta_1(R_M - q) + b(1) > 0, \tag{27}$$

the entrepreneur prefers to exert effort equal to zero, $x = 0$, as opposed to $x = 1$. Similarly, if

$$\Delta_2(R_M - q) + b(2) - b(1) > 0, \tag{28}$$

the entrepreneur prefers $x = 1$ to $x = 2$.[16] This occurs independently of whether such choice is efficient or not, that is independently of whether it maximizes the expected value of the NPV generated by the investment, net of the disutility from effort.

Consider now an unmonitored entrepreneur playing $\{l_t = 0, f_t > 0, e_t > 0\}$. Let R_M be the associated value of the return to market finance. Assuming inequalities (27) and (28) hold – given her expected utility as given by equation (23) – the entrepreneur prefers $x = 2$ ($x = 0$) as opposed to $x = 0$, ($x = 2$), so long as $f_t < (>)\phi_2 e_t$. Similarly, if $f_t < (>)\phi_1 e_t$ the entrepreneur prefers $x = 1$ ($x = 0$) to $x = 0$ ($x = 1$) where:

$$\phi_2 = \frac{\Delta_{\max} q_{t+1} - b(2)}{\Delta_{\max}(R^M - q_{t+1}) + b(2)}; \tag{29}$$

$$\phi_1 = \frac{\Delta_1 q_{t+1} - b(1)}{\Delta_1(R^M - q_{t+1}) + b(1)}, \tag{30}$$

with $\Delta_{\max} \equiv \pi_2 - \pi_0$.[17] Finally, the entrepreneur prefers $x = 2$ (one) against $x = 1$ if $f_t < (>)\phi_{2,1} e_t$, where[18]

$$\phi_{2,1} = \frac{\Delta_2 q_{t+1} - [b(2) - b(1)]}{\Delta_2(R^M - q_{t+1}) + [b(2) - b(1)]}. \tag{31}$$

Given assumption 1, $\phi_{2,1} < \phi_2 < \phi_1$ holds. Therefore, assuming $\phi_{2,1} > 0$, unmonitored entrepreneurs exert $x = 2$ if $f_t < e_t\phi_{2,1}$; $x = 1$ if $f_t \in [e_t\phi_{2,1}, e_t\phi_1]$, and $x = 0$ if $f_t > \phi_1 e_t$. This occurs independently of whether such effort levels are efficient or not. Hence, the moral hazard problem: if sufficiently leveraged, entrepreneurs might exert inefficiently low effort at the expenses of third stakeholders.

Since monitoring is non-contractible, bankers should also be provided with incentives if they are to monitor with intensity $i \neq 0$. Let R_B the return to relationship finance

[16] Also, note that – given assumption 1– whenever entrepreneurs prefers $x = 0$ to $x = 1$ they also prefer $x = 0$ to $x = 2$ and $x = 1$ to $x = 2$.

[17] This, as long as the denominators of the two expressions are positive. Otherwise the inequality signs are reversed, and entrepreneurs always exert the level of effort, x, which yields the maximum net present value.

[18] Again, provided that the denominator of the expression for $\phi_{2,1}$ is positive.

associated with some entrepreneurial financial action $\{l_t, f_t, e_t\}$. Given equation (25), a banker has incentive to monitor with intensity $i \neq 1, 2$ as opposed to $i = 0$ if and only if the following inequality holds

$$l_t \geq \frac{m(i)}{e_t R_B \sum_{x=0}^{2} \left[\alpha(x|l_t, f_t, e_t, i)\pi(x) - \alpha(x|l_t, f_t, e_t, 0)\right]\pi(x)}. \tag{32}$$

Similarly, a banker has incentive to monitor with intensity 2 as opposed to intensity 1 if and only if:

$$l_t \geq \frac{\Delta m}{e_t R_B \sum_{x=0}^{2} \left(\alpha(x|l_t, f_t, e_t, 2) - \alpha(x|l_t, f_t, e_t, 1)\right)\pi(x)}. \tag{33}$$

Define $\Delta_2 = \pi_2 - \pi_1$ and $\Delta_1 = \pi_1 - \pi_0$. We impose the following:

Assumption 2.

$$\frac{\Delta m}{\Delta_2} > \frac{m(1)}{\Delta_1}$$

The above assumption ensures that, other things equal, the monitoring intensity chosen by a banker is increasing in the fraction of relationship finance, l_t.

5.1.4 Partial equilibrium analysis

We now characterize the equilibrium in the financial markets for a given value of the rental price of capital, q_{t+1}. We restrict attention to symmetric equilibria in which entrepreneurs do not randomize with respect to their choice regarding the demand of external finance f_t^d – and focus on situations in which moral hazard is pervasive, which means that the equilibrium strategy of entrepreneurs entails $x = 0$ if unmonitored.[19] Moreover, we assume that the cost-parameter $m(1)$ and Δm are sufficiently small for the monitoring technology to be feasible in the sense, in equilibrium, the fraction of relationship finance l necessary to induce bankers to monitor with intensity $i = 2$ is always strictly less than one.[20]

[19] The necessary and sufficient conditions for moral hazard to be pervasive in the sense just described, are provided in the proof of lemma 1, section A.3 of the appendix.

[20] See proof of in the proof of lemma 1, section A.3 of the appendix.

Let

$$q_0 \equiv (1-\lambda)\frac{m(1)}{w_t\Delta_1}+\frac{b(1)}{\Delta_1} \tag{34}$$

$$q_1 \equiv (1-\lambda)\left(\frac{m(2)}{w_t\Delta_2}-\frac{m(1)}{w_t\Delta_1}\frac{m(1)}{\Delta m}\right)+\frac{b(2)-b(1)}{\Delta_2} \tag{35}$$

$$q_2 \equiv (1-\lambda)\left(\frac{\Delta m}{w_t\Delta_2^2}\Delta_{max}-\frac{m(1)}{w_t\Delta_2}\right)+\frac{b(2)-b(1)}{\Delta_2}, \tag{36}$$

be three critical values of the price of capital at time $t+1$, q_{t+1}, where, given assumptions 1 and 2, $0<q_0<q_1<q_2$ hold for any w_t.

Lemma 1 (Partial equilibrium of financial markets). *Given $q_{t+1}>0$, when moral hazard is pervasive, symmetric equilibria where entrepreneurs do not randomize with respect to the demand of external financial resources, f_t^d, are as follows:*

1. *All financial resources are channeled toward investment in capital accumulation: $z_t=1$. For each entrepreneur, self-financing and external funds amount to $e_t=w_t$ and $f_t=w_t(1-\lambda)/\lambda$ respectively*

2. *Average quality of investment, χ_t:*

   i. *If $q_{t+1}<q_0$, then $\chi_t=0$;*

   ii. *If $q_{t+1}\in(q_0,q_1)$, then $\chi_t=1$;*

   iii. *If $q_{t+1}\in(q_2,\infty]$, then, $\chi_t=2$;*

   iv. *If $q_{t+1}\in[q_1,q_2]$, there exist multiple equilibria and $\chi_t\in[1,2]$. If $q_{t+1}=q_0$, there are multiple equilibria, and $\chi_t\in[0,1]$.*

Proof. See appendix.

A level of effort, x, is efficient if and only if it maximizes the expected present value of investment, net of monitoring costs and disutility from effort. Accordingly, a symmetric partial equilibrium of the financial markets in which $e_t=w_t$ and $f_t=w_t(1-\lambda)/\lambda$, would be efficient, if associated with effort $x=2$, if and only if:

$$\begin{aligned}\left[\pi((2)q_{t+1}-b(2)\right]\frac{w_t}{\lambda}-m(2)\frac{(1-\lambda)}{\lambda} &\geq \left[\pi((1)q_{t+1}-b(1)\right]\frac{w_t}{\lambda}-m(1)\frac{(1-\lambda)}{\lambda}\\ &\Rightarrow\\ q_{t+1} &\geq q_H\equiv\frac{b(2)-b(1)}{\Delta_2}+\frac{\Delta m}{w_t}(1-\lambda).\end{aligned} \tag{37}$$

Similarly, such equilibrium would be efficient if associated with $x=0$, if and only if

$$q_{t+1}\leq q_L\equiv\frac{b(1)}{\Delta_1}+\frac{m_1}{w_t}(1-\lambda). \tag{38}$$

Finally, the equilibrium would be efficient if associated with $x = 1$ if and only if $q_{t+1} \in [q_L, q_H]$. Given (34), (35) and (36), it is immediate to verify othat $q_L = q_0$, and $q_H < q_1 < q_2$. Accordingly, it follows directly from Lemma 1 that – under pervasive moral hazard – symmetric partial equilibria in which $\chi_t = 0(2)$ are efficient.

However, and more importantly, lemma 1 also implies that symmetric partial equilibria characterized by $\chi_t = 1$ could be inefficient in the sense that the effort exerted by entrepreneurs – and the related quality of investment – is lower than the level that would maximize the expected value net of monitoring costs and disutility from effort.

6 Macroeconomic equilibrium

In this section, we verify whether the inefficient equilibria characterized by means of the partial equilibrium analysis, survive when the general equilibrium concept is applied, and explore the macroeconomic consequences of such inefficiency.

Given the fraction, $\theta_t(x)$, of aggregate investment associated with entrepreneurial effort x at time t, the average quality of aggregate investment, χ_t, and the average probability of success of investment, π_t, are still given by equations (9) and (11), respectively. Similarly, the price of capital, q_t, and that of labour, w_t, are still given by equations (12) and (13).

The following definition of intertemporal equilibrium applies to the case of non-contractibility of effort and monitoring,

Definition 3 (Intertemporal equilibrium under asymmetric information)**.** *A symmetric intertemporal macroeconomic equilibrium under non-contractibility of effort and monitoring is a sequence e* $\left\{k_t, \chi_t, z_t, q_t, w_t, l_t, f_t, e_t, \{\theta_t(x)\}_0^2\right\}_{t=t_0}^{\infty}$ *and two associated sequences of return-functions* $\left\{R_{t+1}^B(l, f, e, q), R_{t+1}^M(l, f, e, q)\right\}_{t=t_0}^{\infty}$ *for relationship and market finance, respectively, such that, given* k_{t_0}*:*

i. All agents act optimally at any given stage, taking into account other agents' optimal behavior as well as the return functions and the prices that they anticipate based upon rational expectations;

ii. Return-functions and prices are consistent with agents' optimal behavior and clear all markets;

at any time $t \geq t_0$*.*

Consider a symmetric equilibrium at time t such that $z_t > 0$ is the fraction of financial resources channeled toward investment, $\chi_t \in [0,2]$ is the average quality of investment resulting from the fractions $\{\theta_t\}_0^2$, $e_t = w_t$ and $f_t = (1-\lambda)w_t/\lambda$ are the levels of self-financing and external funds per entrepreneur, and l_t is the fraction of relationship finance. The capital-labor ratio at time $t+1$, k_{t+1}, and the aggregate production, gross of monitoring costs, y_{t+1}, associated with such equilibrium are still described by equations (16) and (17), respectively. Let $\tau_t(i|l_t, f_t, e_t)$ be the probability that an entrepreneur is monitored with intensity i, so that

$$\sum_{i=0}^{2} \tau_t(i|l_t, f_t, e_t) = 1 \tag{39}$$

Then, aggregate production net of monitoring costs is,

$$\tilde{y}_{t+1} = y_{t+1} - (1-\lambda)\sum_{i=0}^{2} \tau_t(i|l_t, f_t, e_t)m(i). \tag{40}$$

Finally, aggregate production net of monitoring costs and disutility from effort would be given by $\tilde{y}_{t+1} - b_{t+1}$, where b_{t+1} is the aggregate disutility from effort measured in units of the final good as given by equation (18). In order to characterize the possible equilibrium paths of the economy, we first study the temporary macroeconomic equilibrium at time t. Again, our focus is on symmetric equilibria in economies in which the moral hazard problem is pervasive, and monitoring is a feasible technology.

6.1 Temporary equilibrium at time t

Recall that, for a given level of k_t, $k_{t+1} = k(k_t, \chi_t, z_t)$ according to equation (16), while $q_{t+1} = q(k_{t+1})$ according to equation (12). Then, the following result applies:

Lemma 2 (Temporary equilibrium). *Given $K_t > 0$, a symmetric temporary equilibrium at time t in which entrepreneurs do not randomize over f_t^d always exists, and is characterized as follows:*

i. Period t labor and capital are fully employed in exchange for a salary $w_t = g(k_t) - g'(k_t)k_t > 0$, and a rental price $q_t = g(k_t) > 0$, respectively, where $k_t = K_t > 0$;

ii. All financial resources are channeled toward investment, i.e. $z_t = 1$, $e_t = w_t$ and $f_t = w_t(1-\lambda)/\lambda$;

iii. If and only if any of the following condition holds, the average quality of investment χ_t is uniquely determined as follows:

a. $q(k(k_t, 2, 1)) \geq q_2$*, in which case,* χ_t*, satisfies,* $\chi_t = 2$*;*

b. $q(k(k_t, 1, 1)) \in [q_0, q_1]$*, in which case* $\chi_t = 1$*;*

c. $q(k(k_t, 0, 1)) \leq q_0$*, in which case* $\chi_t = 0$*;*

d. $q(k(k_t, 2, 1)) < q_1$ *and* $q(k(k_t, 1, 1)) > q_2$*, in which case* $\chi_t \in (1, 2)$*;*

e. $q(k(k_t, 0, 1)) > q_0$*, and* $q(k(k_t, 1, 1) < q_0$*, in which case* $\chi_t \in [0, 1]$*;*

iv. Multiple equilibria exist, each associated with a different value of χ_t*, if and only if any of the following conditions holds:*

a. $q(k(k_t, 2, 1)), q(k(k_t, 1, 1) \in [q_1, q_2]$*, in which case,* $\chi_t \in [1, 2]$*;*

b. $q(k(k_t, 1, 1)) > q_2$ *and* $q(k(k_t, 2, 1)) \in [q_1, q_2]$*, in which case,* $\chi_t \in (1, 2]$*;*

c. $q(k(k_t, 2, 1)) < q_1$ *and* $q(k(k_t, 1, 1)) \in [q_1, q_2]$*, in which case,* $\chi_t \in [1, 2)$*.*

Proof. See appendix.

The above lemma has three main interesting implications. First, for sufficiently high values of k_t, the economy's temporary macroeconomic equilibrium is characterized by a quality of investment equal to zero, $\chi_t = 0$, while for sufficiently low values of k_t, the equilibrium entails $\chi_t = 2$. This follows directly from lemma 2 given that: (a) The equilibrium price of capital, q_{t+1}, as defined by equation (12), is a strictly decreasing and continuous function of the capital-labor ratio, k_{t+1}, with $q_{t+1}(0) = \infty$; (b) k_{t+1}, as defined by equation (16), is a strictly increasing and continuous function of k_t, z_t and χ_t, with $k(0, \chi_t, z_t) = 0$ for any χ_t, z_t .

Second, the inefficient equilibria envisaged by means of the partial equilibrium analysis conducted in the previous section, are indeed a possibility. In particular, while equilibria characterized by an average quality of investment, χ_t, either equal to 0 or 2, could emerge only if efficient, equilibria characterized by $\chi_t \in (1, 2)$ exist even when they are not efficient. We know from partial equilibrium analysis that $x = 2$ is the efficient level of entrepreneurial effort whenever q_{t+1} exceeds q_H (see equation 37). Therefore, given $q_1 > q_H$, lemma 2 implies that in cases *iv.a-c*, any temporary equilibrium characterized by $\chi_t < 2$ is inefficient. Similarly, the equilibrium prevailing in case *iv.b* is inefficient whenever $q_{t+1}(k(k_t, 2, 1)) \in (q_H, q_1)$. Finally, in cases *iv.d*, equilibria such that χ_t below a critical treshold greater than one, are also inefficient.

An inefficient temporary equilibrium at time t always results in a lower level of capital accumulation and, even more importantly, a lower level of aggregate product net of the monitoring costs an disutility of effort, at time $t + 1$. Given a symmetric equilibrium characterized by $\chi_t = 1$, $e_t = w_t$ and $f_t = w_t(1 - \lambda)/\lambda$ the effect of an increase in monitoring intensity on next period capital, k_{t+1} is given by $\Delta_2 w_t$, where $w_t = g(k_t) - g'(k_t)k_t$ is aggregate investment at time t. Furthermore, the effect on aggregate product net of monitoring costs is positive whenever

$$q_{t+1}(k_{t+1})\Delta_2 - \frac{\Delta m}{w_t}(1 - \lambda) > 0 \tag{41}$$

We know that the above inequality holds whenever q_{t+1} exceeds q_H. Hence, inefficient equilibria are characterized by a level of output that is lower than the economy could end up in a temporary equilibrium where net aggregate output is less than it would be in the absence of asymmetric information.[21]

Third, Lemma 2 states that the symmetric macroeconomic equilibrium - even when we restrict attention to equilibria where entrepreneurs do not randomize with respect to f_t^d - is generally not unique. Obviously, this translates into the possibility of multiple intertemporal symmetric equilibria and associated equilibrium paths for the economy.

6.2 Intertemporal equilibrium paths: macroeconomic instability and endogenous cycles

The consequences of financial market imperfections on the dynamics of the economy are examined next. In order to reduce the extent of equilibrium indeterminacy, we restrict our attention to *history selected equilibria* as defined by Cooper (1994). In our setup, the key property of such selection criterion can be stated as follows. Let E be the equilibrium (outcome) at time $t - 1$, characterized by an average quality of investment equal to χ_{t-1}^E and by a fraction z_{t-1}^E of resources channeled to investment. Then, the temporary equilibrium of the economy at time t is characterized by $z_t^E = z_{t-1}^E$ and $\chi_t^E = \chi_{t-1}^E$ so long as such equilibrium exists.

[21] In fact, whenever $q_{t+1} > q_H$, inefficient equilibria result in lower income net not just of monitoring costs but also of the disutility of effort.

Define,

$$k_2 \; : \; q(k_2) = q_2 \tag{42}$$

$$k_1 \; : \; q(k_1) = q_1 \tag{43}$$

$$k_0 \; : \; q(k_0) = q_0 \tag{44}$$

where q_0, q_1, and q_2 are given by equations, (34), (35), and (36). We note that, $k_0 > k_2 > k_1$ holds, given that the equilibrium rental price of capital, $q(k_{t+1})$, is strictly decreasing in k_{t+1}.

Given such critical values of k, lemma 2 implies that a temporary equilibrium characterized by an average quality of investment $\chi_t = 2$ fails to exist if the resulting capital-labor ratio, $k(k_t, 2, 1)$, exceeds k_1. In other words, k_1 defines the maximum capital-labor ratio at time $t+1$ that sustains a temporary equilibrium characterized by $\chi_t = 2$ at time t.

Similarly, a temporary equilibrium characterized by an average quality of investment $\chi_t = 1$ does not exist at time t if the resulting capital-labor ratio, $k(k_t, 1, 1)$ falls below k_2 or above k_0. In the first case, the equilibrium would be characterize by $\chi_t = 2$, while in the second case, $\chi_t = 0$ would hold. Finally, a temporary equilibrium characterized by an average quality of investment $\chi_t \in (1, 2)$ at time t, exists if and only if the resulting capital-labor ratio, $k(k_t, \chi_t, 1) \in [k_2, k_1]$.

This means that, at least potentially, as the economy's capital-labor ratio, k_t, increases along the expansionary phase of an hypothetical intertemporal equilibrium path, the average quality of investment, χ_t, might drop. Viceversa, in a contraction, as k_t decreases over time, χ_t, might actually increase.

Recall that $\kappa(\chi, z)$ defines the steady state value of the capital-labor ratio as a function of average quality of investment, χ, and fraction of resources channelled toward investment. Then, restricting attention to economies where moral hazard is pervasive and focusing on symmetric equilibria in which entrepreneurs do not randomize with respect to f_t^d, – given lemma 1 – the following result holds:

Proposition 2 (Intertemporal equilibrium)**.** *Given an initial condition $K_{t_0} > 0$, a symmetric intertemporal macroeconomic equilibrium always exists. In all equilibria, $z_t = 1$ for all t. History selected equilibria are characterized as follows:*

i. If $\kappa(2,1) < k_2$, the economy always converges to a steady state where $\chi_t = \widehat{\chi} = 2$ is the average quality of investment;

ii. If a. $\kappa(2,1) > k_1$, and $\kappa(1,1) \in [k_2, k_0)$, the economy always converge to a steady state where $\widehat{\chi} \in [1,2)$, while if b. $\kappa(2,1) \in [k_2, k_1]$, and $\kappa(1,1) < k_2$, the economy always converge to a steady state characterized by $\widehat{\chi} \in (1,2]$;

iii. If $\kappa(2,1), \kappa(1,1) \in [k_2, k_1]$, economy always converges to a steady state characterized by $\widehat{\chi} \in [1,2]$;

iv. If $\kappa(2,1) > k_1$ and $\kappa(1,1) < k_2$ then two types of equilibria exist: one in which the economy converges to a steady state characterized by $\widehat{\chi} \in (1,2)$; and the other in which the economy experiences endogenous business cycles;

v. If $\kappa(1,1) \in [k_2, k_0]$, then the economy converges to a steady state where $\widehat{\chi} = 1$ and $\widehat{k} = \kappa(1,1)$;

vi. If $\kappa(1,1) > k_0$, then the economy converges to a steady state where $\widehat{\chi} = 0$, and $\widehat{k} = \kappa(0,1)$.

Proof. See appendix.

Restricting attention to history selected intertemporal equilibria makes the equilibrium outcome more predictable, albeit not entirely. Aside from indeterminacy, the dynamics of the economy depends on the initial condition and the structural parameters that determine both the shape of the accumulation equations and the critical values k_0, k_1, and k_2.

In order to understand the possible dynamic paths of our economy, we focus on figure 2, which plots some of the possible equilibrium paths described in Proposition 2.

Suppose that the economy starts with a level of capital-labor ratio k_{t_0}, such that, assuming the equilibrium average quality of investment is $\chi_{t_0} = 2$ and all resources are channeled toward investment, $z_{t_0} = 1$, the implied capital-labor at time $t_0 + 1$, $k_{t_0+1} = k(k_{t_0+1}, 2, 1)$, is less than the value of the capital-labor ratio, $\kappa(2,1)$, associated with a steady state equilibrium where $\widehat{\chi} = 2$, $\widehat{z} = 1$.

Consider first case *i*, in which $\kappa(2,1) < k_2$. Given $k(k_{t_0+1}, 2, 1) < \kappa(2,1)$, the temporary equilibrium at time t_0 will be characterized by an average quality of investment,

$\chi_{t_0} = 2,$. Accordingly, the economy will start accumulating capital along $k(k_t, 2, 1)$ in a monotone fashion. Furthermore, since the steady state $\kappa(2,1)$ is strictly lower than k_2 the economy moves along an equilibrium path characterized by $\chi_t = 2$ until it reaches the steady state, $\kappa(2,1)$.

Consider now case *ii.a*, in which $\kappa(2,1) > k_1$ and $\kappa(1,1) \in [k_2, k_0]$. Again, suppose that k_{t_0} is such that, $k(k_{t_0}, 2, 1) < k_2$ so that a temporary equilibrium such that $\chi_{t_0} = 2$ exists. Assume that the economy finds itself in such equilibrium at time t_0. Since, $k(k_{t_0}, 2, 1) < k_2$ (which also implies $k(k_{t_0}, 2, 1) < \kappa(2,1)$), that the economy expands along the accumulation path, $k(k_t, 2, 1)$. The economy will move along an equilibrium path characterized by $\chi_t = 2$ until it reaches a level of capita-labor ratio, k_t, such that $k(k_t, 2, 1) > k_1$. At this stage, a temporary equilibrium such that $\chi_t = 2$ no longer exists. Hence, the economy switches to a temporary equilibrium where the average quality of investment would be strictly lower than two. There is, however, indeterminacy, as to which equilibrium will be selected. It could be any equilibrium characterized by $\chi_t \in (1,2)$ such that $k(k_t, \chi_t, 1) \leq k_1$, or an equilibrium where $\chi_t = 1$. From there onward, the economy will be characterized by a sequence of temporary equilibria such that average quality of investment would be non-increasing over time until a stable steady state would be reached characterized by an average quality of investment $\widehat{\chi} \in [1,2)$ such that the steady state value of the capital-labor ratio satisfies $\kappa(\widehat{\chi}, 1) \geq$.

The above logic can be applied to understand why also in cases *ii.b*, *iii*, and *v* history selected intertemporal equilibrium paths are characterized by a process of convergence to a stable state.

Consider now case *iv* in which, $\kappa(2,1) > k_1$, and $\kappa(1,1) < k_2$ hold. Suppose that $k(k_{t_0}, 2, 1) < k_1$), so that the temporary equilibrium at time t_0 is characterized by an average quality of investment, $\chi_{t_0} = 2$. Restricting attention to history selected equilibria, the economy will accumulate capital along $k(k_t, 2, 1)$. However, by doing so it will necessarily reach a level of capital-labor ratio, k_t, such that $k(k_t, 2, 1) > k_1$. At this stage, a temporary equilibrium such that $\chi_t = 2$ no longer exists. The economy will thus switch to another equilibrium characterized by a lower average quality of investment. We know that there will be indeterminacy as to which equilibrium, and therefore which average quality of investment, the economy will switch to. Suppose that the new temporary equilibrium involves $\chi_t = 1$. Given $\kappa(1,1) < k_2$, if such equilibrium exists, it must be the case that $k(k_t, 2, 1) > \kappa(1,1)$, so that the economy starts contracting and k_t decreases over time along $k(k_t, 1, 1)$. However, since $k_2 > \kappa(1,1)$, the economy eventually reaches a level of k_t such that $\chi_t = 1$ is no longer an equilibrium. At this stage, the economy switches to a new equilibrium. Again, there will be indeterminacy with respect to which equilibrium the economy switches to. Anyway, for the seek of the argument, suppose it switches to an equilibrium where $\chi_t = 2$. Then an expansion takes place, which comes to an end as the economy hits again a level of k_t such that $\chi_t = 2$ is not an equilibrium. The above dynamics rensembles that of an endogenous cycle.

Anyway, such cyclical fluctuations correspond to just one of the many possible equilibrium paths. For instance, other than experiencing cycles, the economy could simply converge to a stable steady state characterized by a an average quality of investment $\widehat{\chi} \in (1,2)$ such that the associated steady-state value of the capital-labor ratio, $\kappa(\widehat{\chi}, 1)$ satisfies $\kappa(\widehat{\chi}, 1) \in [k_2, k_1]$.

To put it differently, Lemma 2 states some necessary conditions for endogenous business fluctuations. The following result establishes a set of sufficient conditions for the economy to experience endogenous business cycles when entrepreneurs play pure strategies.

Define, $\underline{k}_1$ such that $k(\underline{k}_1, 2, 1) = k_1$. Then,

Corollary 1 (Endogenous business cycles.). *If the following conditions are satisfied:*

$$k(k_2, 2, 1) \geq k_1 \tag{45}$$

$$k(\underline{k}_1, 1, 1) \geq k_2 \tag{46}$$

$$\kappa(2, 1) > k_1 \tag{47}$$

$$\kappa(1, 1) < k_2 \tag{48}$$

then there exist a unique intertemporal symmetric equilibrium in pure strategies, which is characterized by endogenous fluctuations in the level of capital-labor ratio, k_{t+1}, and output net of monitoring costs, $\tilde{y}_{t+1}$ such that:

i. During slow-down phases, both capital-per capita k_{t+1} and aggregate output net of monitoring costs $\tilde{y}_{t+1}$ keep dropping, while $\chi_t = 1$;

ii. During expansions, both k_{t+1} and $\tilde{y}_{t+1}$ keep rising, while $\chi_t = 2$.

Proof. See appendix.

6.3 Empirical implications: Financial structure cycles

As we have seen, under asymmetric information, entrepreneurs' financing decisions induce macroeconomic instability, which could take the form of endogenous business cycles. Endogenous cycles emerge –under the sufficient conditions set by corollary 1– as the consequence of the following reversion mechanism. Consider figure and suppose the economy starts at $k_{t_0} < k_2$ so that the unique symmetric equilibrium involves $\chi_t = 2$. As the economy accumulates capital during expansions, at time t_1 it finally reaches a level of capital k_{t_1} such that $k(k_{t_1}, 2, 1) > k_1$ so that $\chi_t = 2$ no longer constitute an equilibrium (see dotted-arrow line on figure) , that is entrepreneurs have the incentive to toward a financial structure that favours too much market financing, as opposed to relationship finance. If we restrict attention to pure strategy equilibria –according to corollary 1– this means that the average quality of investment drops to $\chi_{t_1} = 1$. The decline in χ_t results in lower capital accumulation and the economy enters a recession phase, where both k_t and aggregate output $\tilde{y}_t$ fall. Opposite to that, as the economy contracts, it will finally reach a level of capital-labor ratio, k_{t_2}, such that $\chi_t = 1$ no longer constitutes an equilibrium as $k(k_{t_2}, 1, 1) < k_2$ (see dotted-arrow line on figure). At this stage, entrepreneurs toward a financial structure that favours relationship finance. This induces higher monitoring intensity and a higher quality of investment ($\chi_t = 2$ if we restrict attention to pure strategy equilibria). As a consequence, k_t jumps up, and the economy enters an expansion phase in which both $\tilde{y}_t$ and k_t rise over time.

The above reversion mechanism entails the possibility of endogenous business cycles as part of the intertemporal equilibrium path of the economy. Corollary 1 states a set of sufficient conditions for the existence of a unique intertemporal equilibrium in pure strategies that is characterized by such endogenous business cycle fluctuations. We now study the systematic changes in the financial structure associated with the phases of expansions and of contraction that characterize the intertemporal equilibrium path associated with such equilibrium.

A phase of expansion (contraction) corresponds to a subsequence of temporary equilibria such that $\tilde{y}_t$ expands (contracts). According to corollary 1 in the temporary equilibria associated with the expansionary phases (E) entrepreneurs are monitored with intensity $i = 2$; so that the average quality of investment, $\chi_t^E = 2$. Viceversa, the temporary equilibria associated with contractions (C) are characterized by a monitoring intensity equal to $i = 1$; and hence an average quality of investment $\chi_t^C = 1$.

Accordingly, given the characterization of type 1 and type 2 symmetric equilibria provided in the appendix (proof of lemma 1, the amount of relationship finance during expansions is given by

$$l_t^E = \frac{1}{w_t}\left(\frac{\Delta m}{\Delta_2} - \frac{m(2)}{\pi(2)}\right)\frac{1}{R_{t+1}^{M,E}}, \tag{49}$$

where

$$R_{t+1}^{M,E} = R^E(q_{t+1}) - \frac{m(2)}{w_t\pi(2)}, \tag{50}$$

is the equilibrium return on market finance when monitoring intensity is $i = 2$, where we know from proof of lemma 1 that the equilibrium WACC, $R^E(q_{t+1})$, is a decreasing function of q_{t+1}. Similarly, during contractions:

$$l_t^C = \frac{1}{w_t}\pi(0)\frac{m(1)}{\pi(1)\Delta_1}\frac{1}{R_{t+1}^{M,C}}, \tag{51}$$

where

$$R_{t+1}^{M,C} = R^E(q_{t+1}) - \frac{m(1)}{w_t\pi(1)}, \tag{52}$$

where $R^{M,C}$ is the equilibrium return of market finance when monitoring intensity is $i = 1$. Given the above expressions - knowing that $q(k_{t+1})$ varies inversely with k_{t+1}, it is immediate to verify that:

$$\frac{dl_t^j}{dR^{M,j}} < 0 \tag{53}$$

$$\frac{dR^{M,j}}{dk_{t+1}} < 0, \tag{54}$$

for $j = C, E$. The equilibrium return to market finance declines (increases) as k_{t+1} increases; while the fraction of relationship finance declines as the return to market finance increases. These observations directly imply that the fraction of relationship finance varies inversely with respect to k_{t+1}. Furthermore, we know that according to the intertemporal path associated with the equilibrium in pure strategies studied by corollary 1, during an expansion both k_{t+1} and aggregate output net of monitoring costs $\tilde{y}_{t+1}$ increase over time, while the opposite is true during a contraction. Hence, we can conclude that along that the intertemporal path, the fraction of relationship finance behaves procyclically. In other words, the correlation between l_t and $\tilde{y}_t$ is positive.

Importantly, this prediction is consistent with the empirical evidence about the procyclical behavior of the proxy for relative importance of relationship finance for the US economy, reviewed in section 2 of the paper.

As for the level of l_t the following result holds:

Remark 2. *If*

$$\left(\frac{\Delta m}{\Delta_2} - \frac{m(2)}{\pi(2)}\right) \geq \pi(0)\frac{m(1)}{\pi(1)\Delta_1} \tag{55}$$

then relationship finance always drops as the economy enters a contraction phase and raises as the economy enters an expansion phase.

Proof. See appendix.

Hence, whenever condition (55) is satisfied, the level of relationship finance is always strictly higher during expansions than it is during contraction. Therefore, the model also accounts for the fact reported in section 2 that the average value of relationship finance conditional on the economy being in a expansion is, for US listed non-financial firms, significantly larger than that conditional on the economy being in a contraction.

7 Conclusions

We develop a simple macroeconomic model of financial imperfections, in which the interplay between relationship finance supplied by financiers who actively monitor investment decisions and market finance supplied by financiers who rely on public information can generate endogenous business fluctuations. In our model, expansions are associated with an increase in the relative importance of relationship finance. This ensures high monitoring intensity and therefore high entrepreneurial effort, which results in a high probability of success of investments. However, as the economy accumulates capital, the marginal return to capital could eventually drop to a sufficiently low level that entrepreneurs find it profitable to reduce their demand for relationship finance. This causes an inefficient lowering of monitoring intensity and entrepreneurial effort, causing a decline in the probability of success of investment. This induces a contraction, which would last until the return to capital is high enough to induce entrepreneurs to increase their demand for relationship finance.

Based on quarterly and annual data from the US flow of funds accounts, we construct a new measure of financial structure for the US non-financial corporate business, that reflects the importance of relationship finance relative to market finance. Consistent with our model, we find that this indicator is strongly procyclical.

It is important to stress that, although the discussion in the paper has been mainly framed in terms of endogenous business cycles, the theoretical results are more general and indicate that the interplay between relationship and market finance is a source of macroeconomic instability. Such findings provide fruitful avenues for future research.

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A Appendix

A.1 Proposition 1

In order to study the intertemporal equilibrium we need to analyze the temporary equilibrium at time t, first.

A.1.1 Temporary equilibrium

Definition 4. *Given the pre-determined value of capital at time t, K_t, a symmetric temporary equilibrium at time t is a set $\left\{k_{t+1}, \chi_t, z_t, q_t, w_t, \{R_{t+1}(x), \theta_t(x)\}_{x=0}^{2}\right\}$, such that,*

i. Financial investors and entrepreneurs act optimally based upon their rational expectations;

ii. Markets clear.

Given the above definition,

Lemma 3 (Temporary equilibrium)**.** *Given $K_t > 0$, the symmetric temporary equilibrium at time t always exists, is unique, and characterized as follows:*

i. Labor and capital are fully employed in exchange for a salary $w_t = g(k_t) - g'(k_t)k_t > 0$, and a rental price $q_t = g(k_t) > 0$, respectively, where $k_t = K_t > 0$;

ii. All financial resources are channeled toward investment, i.e. $z_t = 1$;

iii. The average quality of investment, χ_t, is uniquely determined and satisfies:

(a) $\chi_t = 0$, if $g'(k(k_t, 0, 1)) \leq \frac{b(1)}{\Delta_1}$;

(b) $\chi_t = 1$ if $g'(k(k_t, 1, 1)) \in \left[\frac{b(1)-b(0)}{\Delta_1}, \frac{b(2)-b(1)}{\Delta_2}\right]$;

(c) $\chi_t = 2$, if $g'(k(k_t, 2, 1)) \geq \frac{b(2)-b(1)}{\Delta_2}$;

(d) $\chi_t \in (0, 1)$, if $g'(k(k_t, 0, 1)) > \frac{b(1)}{\Delta_1}$ and $g'(k(k_t, 1, 1)) < \frac{b(1)}{\Delta_1}$;

(e) $\chi_t \in (1, 2)$, if $g'(k(k_t, 1, 1)) > \frac{b(2)-b(1)}{\Delta_2}$ and $g'(k(k_t, 2, 1)) < \frac{b(2)-b(1)}{\Delta_2}$.

Proof. We first (i) prove that, given $K_t > 0$: *a.* In any equilibrium characterized by production, firms hire all labor and capital available in the economy; *b.* There exist no equilibrium with no production. Then, *(ii)*, we prove that in any equilibrium, $z_t = 1$ holds, and finally, (iii) characterize – thereby showing existence and uniqueness – the temporary equilibrium at time t.

i. (a) In any equilibrium with production capital and labor are fully employed; (b) Given $K_t > 0$, no production is never an equilibrium.

(a) Consider a candidate equilibrium E at time t in which firms demand labor and capital and produce. $k_t^E > 0$ holds, and $q_t^E = q(k_t^E) > 0$, $w_t^E = w(k_t^E) > 0$, where $w(\cdot)$, $q(\cdot)$ are given by (12) and (13). But then, the optimal behavior of old and young agents implies the aggregate supply of capital be $K_t > 0$ and the aggregate supply of labor be one, so that inputs are fully employed; *(b)* Consider now an alternative

candidate equilibrium, E', in which firms are not producing. A condition necessary to such equilibrium is, for at least one of the two inputs, demand and supply equal zero. However, given $K_t > 0$, there is no combination of prices of capital and labor that satisfies such condition and clears the markets at the same time, so that E' never exists.

ii.Given $K_t > 0$, in equilibrium, all financial resources are always channeled toward investment, $z_t = 1$.

The proof is by contradiction. Consider a candidate equilibrium E with, $k_t^E = K_t > 0$, $0 \leq z_t^E < 1$ and $\chi_t^E \geq 0$, so that $K_{t+1}^E = k(k_t, \chi_t, z_t^E)$ equals zero if and only if if $z_t^E = 0$, and it is strictly positive otherwise. Suppose $z_t^E = 0$, so that $K_{t+1}^E = 0$. Then, market clearing implies $q_{t+1}^E = +\infty$, and $w_{t+1}^E = 0$. Given, $q_{t+1}^E > 0$, entrepreneurs' equilibrium strategy is to finance in capital accumulation at time t. To see why, note that investing in capital accumulation while exerting $x = 0$ yields $\pi(0)q_{t+1}^E > 0$. However, if entrepreneurs finance capital accumlation, $z_t^E = 0$ does not hold; a contradiction. Suppose now $z_t^E \in (0, 1)$. Then, $K_{t+1}^E = k(k_t, \chi_t, z_t^E) > 0$. Then, as proved in part (i) above, equilibrium requires all inputs to be fully employed, so that $K_{t+1}^E = k_{t+1}^E > 0$. Accordingly, $q_{t+1}^E(k_{t+1}^E) > 0$. Hence, entrepreneurs must be earning a strictly positive expected return. To see this, note that they could earn $\pi(0)q_{t+}^E > 0$ by investing and exerting $x = 0$. Financial markets' clearing requires $\pi(x)R_{t+1}^E(x) = \pi(x)q_{t+1}^E - b(x)$, for any level of effort x exerted with positive probability in equilibrium. Correspondingly, for levels of efforts played with positive probability, the equilibrium expected marginal rate of return for the entrepreneurs is $\pi(x)q_{t+1}^E - b(x)$. Since entrepreneurs could earn $\pi(0)q_{t+1}^E > 0$ by exerting $x = 0$, $\pi(x)q_{t+1}^E - b(x) \geq \pi(0)q_{t+1}^E > 0$, and, therefore, $\pi(x)R_{t+1}^E(x) \geq \pi(0)q_{t+1}^E > 0$ must hold for any $x \neq 0$ played with positive probability. Hence, both entrepreneurs and financial investors are making a strictly positive expected return and, therefore, they must be investing all their financial resources. Finally, this implies $z_t^E = 1$, which contradicts the initial assumption that $z_t^E < 1$.

iii.a. Temporary equilibrium with $\chi_t = 0$.

Consider a candidate equilibrium E in which entrepreneurs exert effort $x = 0$ (so that $\chi_t^E = 0$) and $z_t^E = 1$, such that $K_{t+1}^E = k(k_t, 0, 1) > 0$, where

$$k(k_t, 0, 1) = \pi(0)\left[g(k_t) - g'(k_t)k_t\right]. \tag{A.1}$$

Note that, given $K_{t+1}^E > 0$, point i above implies that aggregate supply of labor equals one at time $t + 1$, so that $k_{t+1}^E = K_{t+1}^E$. Since entrepreneurs are exerting effort $x = 0$, financial markets' clearing implies $\pi(x)R_{t+1}^E(x) \geq \pi(x)q_{t+1}^E - b(x)$, with strict equality for $x = 0$. Entrepreneurs' expected return per unit of capital is $\pi(0)q_{t+1}^E$, which equals the expected return $\pi(0)R_{t+1}^E(0)$ received by financial investors, where $q(k_{t+1}^E) = g'(k(k_t, 0, 1)) \in (0, \infty)$. Entrepreneurs could always exert $x \neq 0$, which would yield at most $\pi(x)q_{t+1}^E - b(x)$[22] Accordingly, given assumption 1, it is immediate to verify that it is optimal for the entrepreneurs to choose $x = 0$ if and only if

$$q(k_{t+1}^E) \leq \frac{b(1)}{\Delta_1}. \tag{A.2}$$

Henceforth, given that $k_{t+1} = k(k_t, \chi_t, 1)$ is strictly increasing in χ_t, while $q(k_{t+1}) =$

[22] This would be the expected return for an entrepreneur who exerts $x \neq 0$ and relies only on internal financing.

$g'(k_{t+1})$ is strictly decreasing in k_{t+1},

$$g'(k(k_t, 0, 1)) \leq \frac{b(1)}{\Delta_1}, \tag{A.3}$$

constitutes a necessary and sufficient condition for E to exist. Furthermore, the following observation shows that E is unique whenever (A.3) holds. Consider an alternative equilibrium E' where some entrepreneurs are choosing a levels of effort x' greater than 0, so that $\chi_t^{E'} > 0$. Financial markets' clearing implies $\pi(x)R_{t+1}^E(x) \geq \pi(x')q_{t+1}^{E'} - b(x)$ $\forall x$, with strict equality for any x' played with positive probability: $\pi(x')q_{t+1}^{E'} - b(x')$ is the expected return earned both by entrepreneurs exerting x' and financial investors financing them. Furthermore, $\pi(x')q_{t+1}^{E'} - b(x') \geq \pi(0)q_{t+1}^{E'} > 0$ must also be true for some entrepreneurs to optimally choose $x' \neq 0$ as entrepreneurs could earn $\pi(0)q_{t+1}^{E'} > 0$ by exerting x and not relying on external resources. Hence, (as proved in part i above), all agents must be financing capital production so that $z_t^{E'} = 1$. This implies $K_{t+1}^{E'} = k(k_t, \chi_t, 1) > 0$, with $K_{t+1}^{E'} = k_{t+1}^{E'}$ since as proved din part i above, all labor is employed in production given that $K_{t+1}^{E'} > 0$. Therefore, given the properties of $k(k_t, \chi_t, 1)$, $k_{t+1}^{E'} = k(k_t, \chi_t^{E'}, 1) > k_{t+1}^E = k(k_t, 0, 1)$. Finally, given that $q(k_{t+1})$ is decreasing in k_{t+1}, this implies

$$q(k_{t+1}^{E'}) < \frac{b(1)}{\Delta_1}, \tag{A.4}$$

so long (A.3) is satisfied. But then, no agent would choose $x' \neq 0$, which contradicts our starting hypothesis that E', with $\chi_t^{E'} \neq 0$, constitutes an equilibrium.

iii.b. Temporary equilibrium with $\chi_t = 1$.

Given $K_t > 0$, consider a candidate equilibrium E in which entrepreneurs exert effort $x = 1$ (so that $\chi_t^E = 1$) and $z_t^E = 1$, such that $k_{t+1}^E = k(k_t, 1, 1)$ is the implied capital-labor ratio at time $t + 1$.

The financial markets' clearing condition requires $\pi(x)R_{t+1}^E(x) \geq \pi(x)q_{t+1}^E - b(x)$, for all x, with strict equality for $x = 1$ since entrepreneurs are exerting $x = 1$ with positive probability in E. Accordingly, the expected return of entrepreneurs playing $x = 1$ would be $\pi(1)q(k_{t+1}^E) - b(1)$, while entrepreneurs playing $x \neq 1$ would get at most $\pi(x)q(k_{t+1}^E) - b(x)$ (which is the expected return earned by an entrepreneur exerting x and not relying on external funds). Accordingly, given assumption 1, entrepreneurs choose $x = 1$ only if

$$q(k_{t+1}^E) : \frac{b(1)}{\Delta_1} \leq q(k_{t+1}^E) \leq \frac{b(2) - b(1)}{\Delta_2}. \tag{A.5}$$

Note also that when the above condition is satisfied the necessary and sufficient condition for $z_t^E = 1$ (see part i of the proof) is always verified. Then, given the properties of $q(k_{t+1}) = g'(k_{t+1})$ and $k(k_t, \chi_t, 1)$, the necessary and sufficient condition for the existence of E is

$$k(k_t, 1, 1)) :\in \frac{b(1)}{\Delta_1} \leq g'(k(k_t, 1, 1)) \leq \frac{b(2) - b(1)}{\Delta_2}. \tag{A.6}$$

Whenever the above condition is satisfied, uniqueness follows from the following observations. First, consider an alternative equilibrium E' where entrepreneurs exert all effort levels, 0, 1, and 2, with positive probability. Such equilibria require

$$g'(k_{t+1}^{E'}) = \frac{b(1)}{\Delta_1}; \tag{A.7}$$

$$g'(k_{t+1}^{E'}) = \frac{b(2)-b(1)}{\Delta_2}, \tag{A.8}$$

be satisfied simultaneously. This, however, violates assumption 1. Consider now an alternative equilibrium E' where entrepreneurs choose levels of efforts 0 and 2 with positive probability. Such equilibrium require

$$g'(k_{t+1}^{E'}) = \frac{b(2)}{\Delta_{\max}}. \tag{A.9}$$

However, assumption 1 entails

$$\frac{b(2)-b(0)}{\Delta_{\max}} > \frac{b(1)}{\Delta_1}, \tag{A.10}$$

so that, under (A.9), entrepreneurs would prefer to exert a level of effort equal to one, rather than zero, which contradicts the hyptothesis that entrepreneurs find it optimal not choose $x = 1$. Finally, consider an alternative equilibrium E' where entrepreneurs exert levels of effort, 1 and 2, so that $\chi_t^{E'} > 1$. Given $z_t^{E'} = 1$, the capital-labor ratio at time $t+1$ implied by this equilibrium, $k_{t+1}^{E'} = k(k_t, \chi_t^{E'}, 1)$ would always exceed $k(k_t, 1, 1)$, since $k(k_t, \chi_t, 1)$ is an increasing function of χ_t. Therefore, so long as (A.6) holds, it would not be optimal to choose $x = 2$, which contradicts the starting hypothesis that E' - where entrepreneurs choose $x = 2$ with positive probability- constitutes an equilibrium. Hence, the only equilibrium is the one in which $\chi_t = 1$.

iii.c. Temporary equilibrium with $\chi_t = 2$.

Consider a candidate equilibrium E in which entrepreneurs exert $x = 2$ (so that $\chi_t^E = 2$) and $z_t^E = 1$, such that $k_{t+1}^E = k(k_t, 2, 1)$ is the implied capital-labor ratio at time $t+1$. Applying the same logic used in parts *ii.a-b* above, we conclude that the necessary and sufficient condition for E to be an equilibrium is[23]

$$q(k(k_t, 2, 1)) \geq \frac{b(2)-b(1)}{\Delta_2}. \tag{A.11}$$

Whenever the above condition holds as a strict inequality, uniqueness follows from the following observation. Consider an alternative equilibrium E' where some entrepreneurs exert levels of effort different from 2. Such equilibrium would imply a value of $k_{t+1}^{E'}$ lower than $k(k_t, 2, 1)$. But then, under (A.11), the only optimal choice of effort would $x = 2$, which contradicts the hypothesis that E' - in which entrepreneurs choose $x \neq 2$ with positive probability - constitutes an equilibrium.

iii.d. Temporary equilibrium with $\chi_t \in (0, 1)$.

Given the properties of $g'(.)$ and $k(., ., .)$ it is possible that the following inequalities hold simultaneously:

$$g'(k(k_t, 0, 1)) > \frac{b(1)}{\Delta_1} \tag{A.12}$$

$$g'(k(k_t, 1, 1)) < \frac{b(1)}{\Delta_1}. \tag{A.13}$$

[23]Note that when condition () holds, the necessary and sufficient condition for $z^E = 1$, see part i of the proof, is always satisfied.

In this case, neither $\chi_t = 0$ nor $\chi_t = 1$ or $\chi_t = 2$ is an equilibrium. Consider a candidate equilibrium E with $z_t^E = 1$, $\chi_t^E \in (0,1)$, such that $k_{t+1}^E = k(k_t, \chi_t^E, 1)$ is the implied capital-labour ratio at time $t+1$. As usual, market clearing implies $\pi(x)R_{t+1}^E(x) \geq \pi(x)q(k_{t+1}^E - b(x)$, with strict equality for all x played with positive probability. As $k(k_t, \chi_t, 1)$ is a continuous increasing function of χ_t, there exist a unique value of χ_t, call it χ_t^E,such that

$$g'(k(k_t, \chi_t^E, 1)) = \frac{b(1) - b(0)}{\Delta_1}, \tag{A.14}$$

where, given conditions (A.12) and (A.13), $\chi_t^E \in (0,1)$ holds. Note that, when the above inequality holds, the necessary and sufficient condition for $z_t^E = 1$ (see part i of the proof) is satisfied. Given (A.14), entrepreneurs are indifferent between choosing levels of effort, 0 and 1, and strictly prefer such levels of effort to $x = 2$. Accordingly, $\theta_t^E(2) = 0$, so that $\theta_t^E(1) + \theta_t^E(0) = 1$. Moreover, given $\chi_t^E \in (0,1)$, $\theta_t^E(x) > 0$ for $x = 0, 1$ follows. The unique equilibrium value $\theta_t^E(1)$ solves

$$\chi_t^E = \pi(1)\theta_t^E(1) + \pi(0)(1 - \theta_t^E(1)), \tag{A.15}$$

where χ_t^E is determined by condition (A.14). This proves existence and uniqueness of the equilibrium where efforts takes values 0 and 1 with positive probability, under condition (A.14). As for overall uniqueness, we are left with checking whether there exist other combinations of entrepreneurial efforts that could constitute an equilibrium. We already know from previous discussion that equilibria where either, a. All possible levels of effort are played with positive probability, or; b. Entrepreneurs choose levels of effort 0 and 2 with positive probability, are not possible. Given that, under (A.12 ,A.13) equilibria where all entrepreneurs choose all the same level of effort are not possible either, the only other possibility left is equilibria where levels of effort 1 and 2 are chosen. However, given the properties of $k(.,.,.)$ and $q(.)$ it is immediate to verify that under (A.13), such equilibria do not exist.

ii.e. Temporary equilibrium with $\chi_t \in (1,2)$.

Given the properties of $g'(.)$ and $k(.,.,.)$ it is possible for the following inequalities to hold simultaneously:

$$g'(k(k_t, 1, 1)) > \frac{b(2) - b(1)}{\Delta_2} \tag{A.16}$$

$$g'(k(k_t, 2, 1)) < \frac{b(2) - b(1)}{\Delta_2}. \tag{A.17}$$

In this case, neither $\chi_t = 0$ nor $\chi_t = 1$ or $\chi_t = 2$ is an equilibrium. Consider a candidate equilibrium with $z_t^E = 1$, $\chi_t^E \in (1,2)$, such that $k_{t+1}^E = k(k_t, \chi_t^E, 1)$ is the implied capital-labor ratio at time $t+1$. As usual, market clearing implies $\pi(x)R_{t+1}^E(x) = \pi(x)q(k_{t+1}^E - b(x)$ for all x. As $k(.,.,.)$ is a continuous increasing function of χ_t, there exist a unique value of χ_t, call it χ_t^E, such that

$$g'(k(k_t, \chi_t^E, 1)) = \frac{b(2) - b(1)}{\Delta_2}, \tag{A.18}$$

where, given conditions (A.16) and (A.17), $\chi_t^E \in (1,2)$ holds. Note that when the above equality holds, the necessary and sufficient condition for $z_t^E = 1$ (see part i of the proof) is satisfied. Given (A.18), entrepreneurs will be indifferent between choosing levels of effort, 1 and 2, and strictly prefer such levels of effort to $x = 0$. Accordingly, $\theta_t^E(0) = 0$, so that $\theta_t^E(1) + \theta_t^E(2) = 1$. Moreover, since $\chi_t^E \in (1, 2)$, $\theta_t^E(x) > 0$ for $x = 1, 2$ holds. The unique value $\theta_t^E(1)$ solves

$$\chi_t^E = \pi(1)\theta_t^E(1) + \pi(2)(1 - \theta_t^E(2)), \tag{A.19}$$

where χ_t^E is determined by condition (A.18). This proves existence and uniqueness of the equilibrium where efforts takes values 1 and 2 with positive probability. As for overall uniqueness, we are left with checking whether there exist other combinations of entrepreneurial effort that could give rise to an equilibrium. We already know from previous discussion that equilibria where either, a. All possible levels of effort are played with positive probability, or b. Entrepreneurs choose levels of effort 0 and 2 with positive probability, are never an equilibrium. Given (A.16) and (A.17) , equilibria where all entrepreneurs choose the same effort are not possible either. Finally, consider equilibria where only levels of effort 0 and 1 are played with positive probability. Such equilibria entail $z_t = 1$. Given the properties of $k(.,.,.)$ and $g'(.)$ it is then immediate to verify that, under condition (A.16), such equilibria do not exist. □

A.1.2 Intertemporal equilibrium

First, (i) we characterize and prove the existence of the unique non trivial steady state equilibrium. Then, (ii) we prove the existence and uniqueness of the intertemporal equilibrium, and characterize the associated equilibrium path in terms of the variables k_{t+1} and χ_t.

i. Characterization and uniqueness of the non-trivial steady state.

Definition 5 (Steady state)**.** *A steady state (or stationary) competitive equilibrium is a time-invariant set of values* $\{\widehat{k}, \widehat{\chi}, \widehat{z}, \widehat{q}, \widehat{w}, \{\widehat{R}(x), \widehat{\theta}(x)\}_{x=0}^2\}$*, where*

$$\widehat{k} = \kappa(\widehat{\chi}, \widehat{z}) \ : \ \widehat{k} = k(\widehat{k}, \widehat{\chi}, \widehat{z}) \tag{A.20}$$

$$\widehat{\chi} = \sum_{x=0}^{2} \widehat{\theta}(x)x \tag{A.21}$$

$\widehat{q} = q(\widehat{k})$*,* $\widehat{w} = w(\widehat{k})$*,* $\widehat{R}(x) = R(\widehat{k}, x)$*, for* $x = 0, 1, 2$*, where* $w(\cdot)$*,* $q(\cdot)$ *and* $R(.,.)$ *are defined by equations (12) and (13), and (14), such that for any* $t \in \{t_0, \infty\}$*,* $q_t = \widehat{q}$*,* $w_t = \widehat{q}$*,* $R_{t+1}(x) = \widehat{R}(,x)$ *for* $x = 0, 1, 2$*,* $z_t = \widehat{z}$*,* $\chi_t = \widehat{\chi}$*,* $k_t = \widehat{k}$*, markets clear and agents act optimally based upon their rational expectations.*

We know from lemma 3 that when $K_t > 0$, the temporary equilibrium at time t is unique and characterized by $z_t = 1$, so that

$$K_{t+1} = k(k_t, \chi_t, 1) = \Pi(\chi_t)\left[g(k_t) - g'(k_t)k_t\right], \tag{A.22}$$

which is strictly positive for any possible value of χ_t. Given $K_{t+1} > 0$, the temporary equilibrium at time $t+1$ implies $k_{t+1} = K_{t+1}$. Hence, $k_{t+1} = k(k_t, \chi_t, 1)$. Given the properties of $g(.)$, for $\chi_t = \widehat{\chi} \in [0, 2]$ such accumulation equation admits a unique non trivial fixed point $\widehat{k} = \kappa(\widehat{\chi}, 1) > 0$, which is a strictly increasing function of $\widehat{\chi}$. Let

$K_{t+1} = k_{t+1} = \kappa(\widehat{\chi}, 1) > 0$. Then, according to lemma 3, the temporary equilibrium at time t is unique and characterized by $z_t = \widehat{z} = 1$, while $\widehat{\chi}$ – given $\widehat{k} = \kappa(\widehat{\chi}, 1) > 0$, where $\widehat{k}$ is strictly increasing in $\widehat{\chi}$ – is determined as follows:

- $\widehat{\chi} = 0$, if and only if
$$g'(\kappa(0,1)) \leq \frac{b(1)}{\Delta_1}; \tag{A.23}$$
- $\widehat{\chi} = 1$ if and only if
$$g'(\kappa(1,1)) \in \left[\frac{b(1)}{\Delta_1}, \frac{b(2)-b(1)}{\Delta_2}\right]; \tag{A.24}$$
- $\widehat{\chi} = 2$ if and only if
$$g'(\kappa(2,1)) \geq \frac{b(2)-b(1)}{\Delta_2}; \tag{A.25}$$
- $\widehat{\chi} \in (0,1)$ if and only if
$$g'(\kappa(0,1)) > \frac{b(1)}{\Delta_1} \tag{A.26}$$
$$g'(\kappa(1,1)) < \frac{b(1)}{\Delta_1}; \tag{A.27}$$
  in which case $\widehat{\chi}$ is uniquely determined by the condition
$$g'(\kappa(\widehat{\chi},1)) = \frac{b(1)}{\Delta_1}; \tag{A.28}$$
- $\widehat{\chi} \in (1,2)$ if and only if
$$g'(\kappa(1,1)) > \frac{b(2)-b(1)}{\Delta_2} \tag{A.29}$$
$$g'(\kappa(2,1)) < \frac{b(2)-b(1)}{\Delta_2} \tag{A.30}$$
  in which case $\widehat{\chi}$ is uniquely determined by the condition
$$g'(\kappa(\widehat{\chi},1)) = \frac{b(2)-b(1)}{\Delta_2}. \tag{A.31}$$

ii. Characterization and uniqueness of the equilibrium path.

Lemma 3 says that given $K_t > 0$, the temporary equilibrium always exist, is unique, and characterized by $\chi_t \in [0,2]$, $z_t = 1$, and $K_{t+1} = k(k_t, \chi_t, 1) > 0$, where χ_t is negatively associated with k_{t+1}. It then follows by induction that, given $K_{t_0} > 0$, the intertemporal equilibrium –defined as a sequence of temporary equilibria – exists and it is unique. Furthermore, we can show that the equilibrium sequence $\{k_t, \chi_t\}_{t_0}^{\infty}$ converges monotonically to the steady state values $\{\widehat{k}, \widehat{\chi}\}$ associated with definition 5 above. Consider a temporary equilibrium at time t_0 such that $k_{t_0} \in \left(0, \widehat{k}\right)$ and $\chi_{t_0} \in [0,2]$ are the capital-labor ratio and the quality of investment, so that $k_{t_1} = k(k_{t_0}, \chi_{t_0}, 1) > 0$ defines the implied capital-labor ratio at time $t_1 = t_0 + 1$. Given $k_{t_0} < \widehat{k}$, $k_{t_1} > k_{t_0}$. Then, $\chi_{t_0} \geq \widehat{\chi}$ and $k_{t_1} < \widehat{k}$. Suppose $\chi_{t_0} \geq \widehat{\chi}$. Then, $k_{t_1} < \widehat{k}$ must be true since χ_t is negatively associated with k_{t+1}, and $k_{t_0} < \widehat{k}$. Suppose now $\chi_{t_0} < \widehat{\chi}$. Then, $k_{t_1} < \widehat{k}$ follows, since $k(.,.,1)$ is non decreasing in k_t and in χ_t, which entails a contradiction given that χ_t is negatively related to k_{t+1}.

Given $k_{t_1} > k_{t_0}$, and $k_{t_1} < \widehat{k}$, $k_{t_2} > k_{t_1}$, and $\chi_{t_1} \leq \chi_{t_0}$ follow. Suppose $\chi_{t_1} \leq \chi_{t_0}$. Then, given $k_{t_0} < k_{t_1}$, $k_{t_2} > k_{t_1}$ follows from the fact that k_{t+1} is negatively associated with χ_t and k_t. Suppose now $\chi_{t_1} > \chi_{t_0}$, then - given $k_{t_1} > k_{t_0}$, $k_{t_2} > k_{t_1}$ follows from the fact that $k(.,.,1)$ is an increasing function of k_t and χ_t, which would give a contradiction since χ_t is decreasing in k_{t+1}. Furthermore, the same argument used above leads us to the conclusion that $k_{t_1} < k_{t_2}$ implies $k_{t_2} < \widehat{k}$ and $\chi_{t_1} \geq \widehat{\chi}$. Iterating these arguments yield $k_{t_n} \geq k_{t_n - 1}$, $k_{t_n} \leq \widehat{k}$, $\chi_{t_n} \leq \chi_{t_n - 1}$ and $\chi_{t_n} \leq \widehat{\chi}$. Hence, both $\{k_{t+1}\}_{t_0}^{\infty}$ and $\{\chi_{t+1}\}_{t_0}^{\infty}$ are monotonic sequences.

Continuity of $k(k_t, \chi_t, 1)$ with respect to χ_t and k_t then implies that k_t will finally reach a value k' close enough to $\widehat{k}$ such that, $\chi_t = \widehat{\chi}$ for any $k_t \in [k', \widehat{k}]$. Hence, by continuity, we are able to state that the sequences $\{k_{t+1}\}_{t_0}^{\infty}$ and $\{\chi_{t+1}\}_{t_0}^{\infty}$ converge to their steady state values $\widehat{k}$ and $\widehat{\chi}$. Convergence is monotonic since $\{k_{t+1}\}_{t_0}^{\infty}$ and $\{\chi_{t+1}\}_{t_0}^{\infty}$ are monotonic sequences.

A.2 Remark 1

According to previous results, in any temporary equilibrium at time t, entrepreneurs choose at most two values of effort, call them i and j, with $i > j$, with positive probability. Accordingly,

$$\theta_t(j) = 1 - \theta_t(i), \tag{A.32}$$

so that:

$$\chi_t = \theta_t(i)i + [1 - \theta_t(i)]\, j \tag{A.33}$$

$$\pi_t = \theta_t(i)\pi(i) + [1 - \theta_t(i)]\, \pi(j). \tag{A.34}$$

From (A.33), we can express $\theta_t(i)$ as a function of χ_t and obtain:

$$\theta_t(i) = \frac{\chi_t - j}{i - j}. \tag{A.35}$$

Substituting back into (A.34) and rearranging gives:

$$pi_t = \Pi(\chi_t) = \pi(i)\frac{\chi_t - j}{i - j} + \pi(j)\frac{i - \chi_t}{i - j}. \tag{A.36}$$

Similarly, by substituting in equation (18), we can rewrite aggregate private benefits as

$$b_{t+1} = \left(b(i)\frac{\chi_t - j}{i - j} + b(j)\frac{i - \chi_t}{i - j} \right) z_t[g(k_t) - g'(k_t)k_t] \tag{A.37}$$

Next period capital and aggregate product net of disutility of effort can the be written as:

$$k_{t+1} = \left(\pi(i)\frac{\chi_t - j}{i - j} + \pi(j)\frac{i - \chi_t}{i - j} \right) z_t\, [g(k_t) - g'(k_t)k_t] \tag{A.38}$$

$$y_{t+1} - b_{t+1} = g(k_{t+1}) - \left(b(i)\frac{\chi_t - j}{i - j} + b(j)\frac{i - \chi_t}{i - j} \right) z_t[g(k_t) - g'(k_t)k_t] \tag{A.39}$$

The Pareto-optimal average quality of investment is defined as the value of χ_t that maximizes y_{t+1}. An increase in the average quality Taking the derivative of y_{t+1} with respect to χ_t, where $\chi_t \in [0, 2]$, yields:

$$\frac{\partial(y_{t+1} - b_{t+1})}{\partial \chi_t} = \{g'(k_{t+1})\Delta_i - [b(i) - b(j)]\} z_t w_t z_t [g(k_t) - g'(k_t)k_t] \tag{A.40}$$

where $\Delta_i = \pi(i) - \pi(j)$. It is then immediate to verifty that the first order condition for χ_t to be Pareto-optimal matches the condition for χ_t to be part of an equilibrium (see proof of proposition 1).[24]

A.3 Proof of Lemma 1

We characterize symmetric partial equilibria of the financial markets under pervasive moral hazard –with no randomization over f_t^d, and provide the related necessary and sufficient conditions for existence of the various types of equilibrium. For each type of equilibrium, we derive the necessary and sufficient condition for moral hazard to be pervasive for $\lambda > 0$ sufficiently close to zero. Then, we discuss uniqueness and multipliticy of equilibria.

i. "Type 0" equilibrium.

Consider an equilibrium E such that $\forall \{l_t, f_t, e_t\} \in \mathcal{E}^E$, where $\mathcal{E}^E$ is the set of actions played by entrepreneurs by with positive probability in E:

a. Entrepreneurs exert $x = 0$ effort if not monitored (MH is pervasive):, $\alpha(0|l_t, f_t, e_t, 0) = 1$;

b. Financial investors monitor with intensity $i = 0$: , $\tau(0, l_t, f_t, e_t) = 1$;

c. $z_t^E = 1$, $f_t^d = f_t^s = f_t = (1 - \lambda)w_t/\lambda \equiv f_t^E$, $e_t = w_t \equiv e_t^E$; and $R^{B,E}(l_t, f_t, e_t, q_{t+1})$, $R^{M,E}(l_t, f_t, e_t, q_{t+1})$, are the return-functions.

1. Characterization. (a) and (b) imply that for all $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$, the probability of success of entrepreneurs' investments is $\pi(0)$. Then, market clearing requires that $\forall \{l, f_t^E, e_t^E\} \in \mathcal{E}^E$ the weighted average cost of external financial capital (WACC)

$$R^E(l, f_t^E, e_t^E, q_{t+1}) = R^{B,E}(l, f_t^E, e_t^E, q_{t+1})l + (1 - l)R^{M,E}(l, f_t^E, e_t^E, q_{t+1}), \tag{A.41}$$

satisfies:

$$\pi(0)q_{t+1}\left[e_t^E + f_t^E\right] - R^E(l, f_t^E, e_t^E, q_{t+1})f_t^E = \max_i \left(\pi(i)q_{t+1} - b(i)\right) w_t, \tag{A.42}$$

where the RHS is the entrepreneur's outside option to rely only on internal finance (autarky). Note that the value of autarky is strictly greater than zero, since the expected return earned by an autarkic entrepreneur exerting $x = 0$ is $\pi(0)q_{t+1}$, which is strictly positive, given $q_{t+1} > 0$. Note that, according to (A.42), given f_t^E and e_t^E, the WACC, takes a unique value R^E independently of l. Note that R^E is increasing in q_{t+1}, that is $R^E(q_{t+1})$ with $dR^E/dq_{t+1} > 0$.

[24] We note that the second order condition is always satisfied as the second order derivative of $g(k_{t+1})$ is strictly negative, given the properties of $g(.)$.

Given (a) and (b) market clearing also requires that, for any type $j = B, M$ of external finance exchanged with positive probability in equilibrium, the associated return-function, $R^{j,E}(l, f, e, q_{t+1})$, takes a unique value $R^{j,E}$:

$$R^{j,E}(l', f_t^E, e_t^E, q_{t+1}) = R^{j,E}(l'', f_t^E, e_t^E, q_{t+1}) = R^{j,E} \quad \forall \left\{l', f_t^E, e_t^E\right\}, \left\{l'', f_t^E, e_t^E\right\} \in \mathcal{E}^E. \tag{A.43}$$

The expected return earned by bankers and market investors are $\pi(0)R^{B,E}$, and $\pi(0)R^{M,E}$, respectively. Accordingly, if both type of finance are exchanged with positive probability, $R^{B,E} = R^{M,E}$. Financial investors supply only relationship (market) finance if $R^{B;E}(>)(<)R^{M,E}$. In any case, given expression (A.41), $R^{j,E} = R^E$, where $j = B, M$ is the type of finance exchange with positive probability in equilibrium.

Finally, since no monitoring occurs in equilibrium, bankers' should be given no incentives to monitor. Therefore, given equations (32) and (33) and the equilibrium values e_t^E and f_t^E, any l such that $\{l, f_t^E e_t^E\} \in \mathcal{E}^E$ must satisfy

$$l < \frac{m(1)}{R^{B,E}\Delta_1 w_t}. \tag{A.44}$$

Finally, given that MH is pervasive and no monitoring occurs in equilibrium, entrepreneurs choose $x = 0$, so that $\chi_t^E = 0$.

2. Existence. A necessary condition for E to be an equilibrium is that the LHS of (A.42) weakly exceeds the RHS for $R^E = 0$. Such condition is always satisfied for λ sufficiently close to zero. Given $R^E \geq 0$, it is optimal for financial investors to supply financial resources, which – under symmetry – sustains $f_t^E = (1-\lambda)/\lambda)w_t$. Moreover, (A.42) implies $R^E \leq q_{t+1} - b(i)/\pi(i)$, since $\max_i (\pi(i)q_{t+1} - b(i)) \geq \pi(0)q_{t+1}$, so that it is optimal for entrepreneurs finance their own investment, which sustains $e_t^E = w_t$. Finally, this supports $z_t^E = 1$.

MH is pervasive when $f_t^E > \phi_1 w_t$, where ϕ_1 is given by (30) (see subsection 5.1.3). Substituting in for f_t^E and ϕ_1, the inequality reduces to:

$$\frac{1-\lambda}{\lambda} w_t > \frac{\Delta_1 q_{t+1} - b(1)}{\Delta_1(R^E - q_{t+1}) + b(1)}, \tag{A.45}$$

It follows from (A.42) that R^E is decreasing in λ and reaches its upper bound, q_{t+1}, as λ goes to zero. Therefore, the condition for MH pervasiveness is always satisfied for λ sufficiently close to 0, so long as

$$q_{t+1} > \frac{b(1)}{\Delta_1}, \tag{A.46}$$

holds.

For E to be an equilibrium, market-clearing implies that for all $\{l, f, e\} \notin \mathcal{E}^E$ such that $\alpha(x|l, f, e, 0) = 1$ and $\tau(0|l, f, e) = 1$, so that $\pi(x)$ is the probability of success of entrepreneurial investment, the weighted average cost of capital, $R^E(l, f, e)$, should be such that:

$$\pi(x)q_{t+1}(e+f) - \pi(x)R^E(l, f, e) - b(x)\left[f + e\right] \leq \max_x \left(\pi(x)q_{t+1} - b(x)\right), \tag{A.47}$$

so that deviating to any $\{l, f, e\}$ that satisfies the above is never profitable. However, it is always possible to construct equilibrium return functions such that the above condition is satisfied.

Finally, for E to be an equilibrium, given f_t^E and e_t^E, for any $\{l, f_t^E, e_t^E\} \notin \mathcal{E}^E$, $R^E(l, f_t^E, e_t^E) \geq R^E$. With no loss of generality, consider a deviation, $\{l', f_t^E, e_t^E\} \notin \mathcal{E}^E$

$$l' = \frac{m(1)}{e_t^E \Delta_1 R^{B,E}}, \tag{A.48}$$

where we recall that $R^{B,E} = R^E$ and note that $l' < 1$ for sufficiently small values of $m(1)$ (feasibility of monitoring technology). Given the above value for l', monitoring with intensity, $i = 1$, constitutes part of SPE strategies of bankers. Accordingly, the probability of success of investments undertaken by the entrepreneur rises to $\pi(1)$. Accordingly – given their SPE strategy – financial investors supply finance if

$$R^{M,E}(l', f_t^E, e_t^E, q_{t+1}) \geq \frac{\pi(0)}{\pi(1)} R^E. \tag{A.49}$$

The correspondent WACC is

$$R' = \frac{m(1)}{e_t^E \Delta_1} + (1 - \frac{m(1)}{e_t^E \Delta_1 R^E}) \frac{\pi(0)}{\pi(1)} R^E = \frac{m(1)}{e_t^E} + \frac{\pi(0)}{\pi(1)} R^E. \tag{A.50}$$

Deviating makes an entrepreneur strictly better off if and only if:

$$\pi(1) q_{t+1} \left(e_t^E + f_t^E\right) - \pi(1) R' f_t^E - b(1)(e_t^E + f_t^E) > \pi(0) q_{t+1} \left(e_t^E + f_t^E\right) - \pi(0) R^E f_t^E, \tag{A.51}$$

while he would be indifferent if $q_{t+1} = q_0$. Substituting for R', given the value of R^E as determined by (A.42), and for the equilibrium values e_t^E and $f - t^E$, condition (A.50) reduces to

$$q_{t+1} > (1 - \lambda) \frac{m(1)}{w_t \Delta_1} + \frac{b(1)}{\Delta_1} \equiv q_0. \tag{A.52}$$

Consider now a deviation, $\{l', f_t^E, e_t^E\} \notin \mathcal{E}^E$ such that $\alpha(0|l', f_t^E, e_t^E, 0) = 1$ and

$$l' = \frac{\Delta m}{e_t^E \Delta_2 R^{B,E}} \tag{A.53}$$

Note that $l' < 1$ for Δm sufficiently small (feasibility of the monitoring technology). Given such deviation, bankers' monitoring intensity $i = 2$ forms part of a SPE strategy, so that the probability of success of investments rises to $\pi(2)$. According to their SPE strategy, financial investors are willing to supply finance if,

$$R^{M,E}(l', f_t^E, e_t^E, q_{t+1}) \geq \frac{\pi(0)}{\pi(2)} R^E. \tag{A.54}$$

Therefore, the WACC associated with the deviation is

$$R' = \frac{\Delta m}{e_t^E \Delta_2} + (1 - \frac{\Delta m}{e_t^E \Delta_2 R^E}) \frac{\pi(0)}{\pi(2)} R^E = \Delta m \frac{\Delta_{Max}}{e_t^E \Delta_2} + \frac{\pi(0)}{\pi(2)} R^E \tag{A.55}$$

That given, the deviation would be strictly profitable if and only if:

$$\pi(2) q_{t+1} \left(e_t^E + f_t^E\right) - \pi(2) R' f_t^E - b(2)(e_t^E + f_t^E) > \pi(0) q_{t+1} \left(e_t^E + f_t^E\right) - \pi(0) R^E f_t^E. \tag{A.56}$$

Substituting for the value of R, given the value of R^E and $e_t^E = w_t$, the above inequality reduces to:

$$q_{t+1} > (1-\lambda)\frac{\Delta m}{w_t \Delta_2}\frac{\Delta_{Max}}{\Delta_2} + \frac{b(2)}{\Delta_2}. \tag{A.57}$$

The RHS of the above inequality is strictly greater than q_0. hence, we conclude that - under pervasive moral hazard (condition (A.46)) -, a "type 0" equilibrium exists if and only if $q_{t+1} \leq q_0$.

Consider now the special case in which $q_{t+1} = q_0$. Entrepreneurs are indifferent between playing $\{l_t^E, f_t^E, e_t^E\}$ such that they are not monitored (type 0 equilibrium) and $\{l', f_t^E, e_t^E\}$, with l' given by (A.48). The latter strategy implies $R^{M,E}(l', f_t^E, e_t^E, q_{t+1}) = \pi(0)R^E/\pi(1)$ so that market investors always earn the same expected return. Given (A.48) bankers' return from supplying l' also equals $\pi(0)R^E/\pi(1)$ so that they are also indifferent. Finally, entrepreneurs are making zero profits. Hence, there exist alternative equilibria such that: a. Entrepreneurs play $\{l_t^E, f_t^E, e_t^E\}$ with probability $1-p$ and $\{l', f_t^E, e_t^E\}$ with probability p, where $p \in [0,1]$; b. the values of the returns are R^E, $R^{B,E} = R^E$, and $R^{M,E} = R^E$, and $R^{M,E}(l', f_t^E, e_t^E, q_{t+1}) = R^E\pi(0)/pi(1)$, where R^E is determined by (A.42) ; c. The average quality of such alternative equilibria is $\chi_t \in [0,1]$, and it is strictly increasing in p.

ii. " Type 1 " Equilibrium.

Let E be a symmetric equilibrium such that, $\forall \{l_t, f_t, e_t\} \in \mathcal{E}^E$, where $\mathcal{E}^E$ is the set of actions played by entrepreneurs by with positive probability in E:

a. MH is pervasive: $\alpha(0|l_t, f_t, e_t, 0) = 1$;

b. Financial investors monitor with intensity $i = 1$: $\tau(1, l_t, f_t, e_t) = 1$;

c. $z_t^E = 1$, $f_t^d = f_t^s = f_t = (1-\lambda)w_t\lambda \equiv f_t^E$, $e_t = w_t \equiv e_t^E$; and $R^{B,E}(l_t, f_t, e_t, q_{t+1})$, $R^{M,E}(l_t, f_t, e_t, q_{t+1})$, are the return-functions.

1. Characterization. We know from discussion in section 5.1.3 that, whenever entrepreneurs exert $x = 0$ if unmonitored, they would exert $x = 1$ if monitored with intensity, $i = 1$. Then, given (a) and (b), $\alpha(1|l, f_t^E, e_t^E) = 1$ for all $\left\{l, f_t^E, e_t^E\right\} \in \mathcal{E}^E$. Accordingly, the probability of success of financed investments equals $\pi(1)$. Hence, for any $\left\{l, f_t^E, e_t^E\right\} \in \mathcal{E}^E$, market clearing requires that the weighted average cost of capital, R^E, satisfies:

$$\pi(1)q_{t+1}\left[e_t^E + f_t^E\right] - R^E(l, f_t^E, e_t^E, q_{t+1})f - b(1)\left(e_t^E + f_t^E\right) = \max_i \left(\pi(i)q_{t+1} - b(i)\right)w_t, \tag{A.58}$$

where the RHS is the entrepreneur's outside option to rely only on internal finance, which is strictly positive. According to A.58, given f_t^E, e_t^E, and q_{t+1}, $R^E = R^E(q_{t+1})$ takes the same value for all $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$. Note that R^E increases with q_{t+1}.

The expected return for banker funding an entrepreneur who is playing $\left\{l, f_t^E, e_t^E\right\} \in \mathcal{E}^E$ is:

$$\pi(1)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) - \frac{m(1)}{e_t^E l}, \tag{A.59}$$

while the expected return for market investors would be

$$\pi(1)R^{M,E}(l, f_t^E, e_t^E, q_{t+1}). \tag{A.60}$$

In order for demand of relationship finance to be positive:

$$\pi(1)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) - \frac{m(1)}{el} \geq \pi(1)R^{M,E}(l, f_t^E, e_t^E, q_{t+1}) \quad \forall\{l, f_t^E, e_t^E\} \in \mathcal{E}^E, \tag{A.61}$$

which implies $R^{M,E}(l, f_t^E, e_t^E, q_{t+1}) < R^{B,E}(l, f_t^E, e_t^E, q_{t+1})$, $\forall(l, f_t^E, e_t^E) \in \mathcal{E}^E$. Note that (A.61) is satisfied as strict equality if both bank and market finance are supplied with positive probability (that is if $l \in (0,1)$) for some $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$.

In order for bankers to be monitoring with intensity $i = 1$, they must be given incentives to do so. Hence, the following incentive-compatibility constraint must be satisfied for $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$:

$$\pi(1)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) - \frac{m(1)}{el} \geq \pi(0)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) \Rightarrow lR^{B,E}(l, f_t^E, e_t^E, q_{t+1}) \geq \frac{m(1)}{e\Delta_1} \tag{A.62}$$

Since, $R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) > R^{M,E}(l, f_t^E, e_t^E, q_{t+1})$ entrepreneurs optimal play is to minimize the use of bank-finance, so that they all choose the level of l that satisfies the above constraint as strict equality:

$$l : l = \frac{m(1)}{e\Delta_1 R^{B,E}(l, f_t^E, e_t^E, q_{t+1})} \tag{A.63}$$

Note that, for $e = w_t$, $l < 1$ holds for $m(1)$ sufficiently small (feasibility of the monitoring technology). Accordingly, the equilibrium values of the return on relationship and market finance, $R^{B,E}$ and, $R^{M,E}$, respectively, are uniquely determined as a function of R^E, by combining equations, (A.61), (A.63), given the expression for the WACC, $R^E = R^{B,E}l + (1-l)R^{M,E}$, and $e_t^E = w_t$

$$R^{M,E} = R^E - \frac{m(1)}{w_t \pi(1)} \tag{A.64}$$

$$R^{B,E} = \frac{\pi(1)}{\pi(0)} R^{M,E} \tag{A.65}$$

while the unique value of R^E is being determined by condition (A.58), given $e_t^E = w_t$ and $f_t^E = w_t(1-\lambda)/\lambda$.

2. Existence. The same arguments used for *Type 0 equilibrium* hold so that, under symmetry – for λ sufficiently close to zero – $z_t^E = 1$, with $e_t = w_t$ and $f_t^E = w_t(1-\lambda)/\lambda$ are supported as equilibrium outcomes.

It follows from (A.58) that the upper bound of R^E is given by $q_{t+1} - b(1)/\pi(1)$ and moreover, R^E converges monotonically to this upper bound as λ goes to zero. Therefore, given the equilibrium value of $R^{M,E}$ (see equation A.64) the condition for moral hazard pervasiveness,

$$\frac{1-\lambda}{\lambda} w_t > \frac{\Delta_1 q_{t+1} - b(1)}{\Delta_1(R^{M,E} - q_{t+1}) + b(1)}, \tag{A.66}$$

inequality is always satisfied for λ sufficiently close to 0 so long as the following inequality holds:

$$q_{t+1} > \frac{b(1)}{\Delta_1} \tag{A.67}$$

$$\frac{b(1)}{\Delta_1} > \frac{m(1)}{w_t \pi(0)}. \tag{A.68}$$

For E to be an equilibrium, market-clearing and sub-game perfection implies that for all $\{l, f, e\} \notin \mathcal{E}^E$ such that $\alpha(x|l, f, e, 1) = 1$ and $\tau(0|l, f, e) = 1$, the weighted average cost of capital should satisfy condition (A.47). However, one can always construct (equilibrium) return functions that satisfy this property, so that deviating to any $\{l, f, e\}$ that satisfies the above is never profitable.

Finally, for E to be an equilibrium, given f_t^E and e_t^E, for any $\{l, f_t^E, e_t^E\} \notin \mathcal{E}^E$, $R^E(l, f_t^E, e_t^E) \geq R^E$. With no loss of generality, consider a deviation, $\{l', f_t^E, e_t^E\} \notin \mathcal{E}^E$ such that $\alpha(0|l', f_t^E, e_t^E, 0) = 1$ and:

$$l' \geq \frac{\Delta m}{R^{B,E} e \Delta_2}, \tag{A.69}$$

or

$$l' < \frac{m(1)}{R^{B,E} e_t^E \Delta_1}. \tag{A.70}$$

where in the case of (A.69), $l' < 1$ holds for sufficiently small Δm (feasibility of monitoring) . Suppose that an entrepreneur deviates to l' that satisfies (A.69) above as a strict equality.[25] Given their SPE strategy, bankers' best reply would be monitor with intensity, $i = 2$, which induces a probability of success $\pi(2)$. Accordingly, market investors' SPE strategy implies

$$R^{M,E}(l', f_t^E, f_t^E, q_{t+1}) = \frac{\pi(1)}{\pi(2)} R^{M,E}. \tag{A.71}$$

The weighted average cost of capital associated with the deviation would then be:

$$R' = \frac{\Delta m}{e_t^E \Delta_2} + (1 - l') \frac{\pi(1)}{\pi(2)} R^{M,E}. \tag{A.72}$$

Substituting for l' evaluated at $e = e_t^E$, and the equilibrium value of $R^{M,E}$, and imposting condition A.61 (as strict equality), yields,

$$R' = \frac{\Delta_{\max} \Delta m}{e_t^E \pi(2) \Delta_2} + \frac{\pi(1)}{\pi(2)} R^E - \frac{m(1)}{e_t^E \pi(2)}. \tag{A.73}$$

Deviating would be strictly profitable for the entrepreneur if

$$\pi(2)\left[q_{t+1}(f_t^E + e_t^E) - R' f_t^E\right] - b(2)\left(e_t^E + f_t^E\right) > \pi(1)\left[q_{t+1}(f_t^E + e_t^E) - R^E f_t^E\right] - b(1)\left(e_t^E + f_t^E\right) \tag{A.74}$$

Substituting for R', given the equilibrium value of R^E as given by expression (A.58), and for the values of f_t^E and e_t^E, the above condition reduces to:

$$q_{t+1} > q_2 \equiv (1 - \lambda) \frac{b(2) - b(1)}{\Delta_2} + \left[\frac{\Delta_{\max}}{w_t \Delta_2^2} \Delta m - \frac{m(1)}{w_t \Delta_2}\right]. \tag{A.75}$$

Take now the alternative deviation as given by (A.70). Bankers' best reply would be to monitor with intensity $i = 0$, so that, given market investors' SPE strategy, the cost of market finance will increase to $R^{M,E}(l', f_t^E, e_t^E, q_{t+1})\pi(1)/\pi(0)$, which would also be the weighted average cost of capital faced by the entrepreneur. Taking the same steps

[25] deviating to higher values of l is dominated.

as before to compute the return from deviation and comparing it to the equilibrium expected payoff of the entrepreneur yields that such deviation would be profitable if and only if:

$$q_{t+1} < q_0 \tag{A.76}$$

We note that $q_0 < q_2$. Hence, the equilibrium E exists so long as $q_{t+1} \in [q_0, q_2]$.

iii. Type 2 equilibrium.

Let E be a symmetric equilibrium such that, $\forall \{l_t, f_t, e_t\} \in \mathcal{E}^E$, where $\mathcal{E}^E$ is the set of actions played by entrepreneurs by with positive probability in E:

a. MH is pervasive: $\alpha(0|l_t, f_t, e_t, 0) = 1$;

b. Financial investors monitor entrepreneurs with intensity $i = 2$: $\tau(2, l_t, f_t, e_t) = 1$;

c. $z_t^E = 1$, $f_t^d = f_t^s = f_t = (1-\lambda)w_t\lambda \equiv f_t^E$, $e_t = w_t \equiv e_t^E$; and $R^{B,E}(l_t, f_t, e_t, q_{t+1})$, and $R^{M,E}(l_t, f_t, e_t, q_{t+1})$, are the return-functions.

1. Characterization. Given $i = 2$, entrepreneurs have no choice but to exert $x = 2$,. Then, given (a) and (b), $\alpha(2|l, f_t^E, e_t^E, 2) = 2$ for all $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$. Accordingly, the probability of success of financed investments equals $\pi(2)$. Hence, for any $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$, market clearing requires the weighted average cost of capital to satisfy:

$$\pi(2)q_{t+1}\left[e_t^E + f_t^E\right] - R^E(l, f_t^E, e_t^E)f_t^E - b(2)\left(e_t^E + f_t^E\right) = \max_i \left(\pi(i)q_{t+1} - b(i)\right)w_t, \tag{A.77}$$

where the RHS is the entrepreneur's outside option to rely only on internal finance, which is strictly positive. According to (A.77), given f_t^E and e_t^E and q_{t+1}, the weighted average cost of capital takes a unique value $R^E = R^E(q_{t+1})$, which is increasing in q_{t+1}, for all $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$.

Bankers' expected return for financing an entrepreneur playing $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$ is:

$$\pi(2)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) - \frac{m(2)}{l}, \tag{A.78}$$

while for market investors it would be:

$$\pi(2)R^{M,E}(l, f_t^E, e_t^E, q_{t+1}), \tag{A.79}$$

In order for financial investors to operate as bankers,

$$\pi(2)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) - \frac{m(2)}{l} \geq \pi(2)R^{M,E}(l, f_t^E, e_t^E, q_{t+1}), \tag{A.80}$$

which entails $R^{M,E}(l, f_t^E, e_t^E, q_{t+1}) > R^{B,E}(l, f_t^E, e_t^E, q_{t+1})$. For supply of market and bank finance to be both positive in equilibrium the above condition must hold as strict equality.

In order for bankers to be monitoring with intensity $i = 2$, they must be given incentives to do so. Hence, the following incentive-compatibility constraint must be satisfied for each $\{l, f_t, e_t\} \in \mathcal{E}^E$:

$$\pi(2)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) - \frac{m(2)}{l} \geq \pi(1)R^{B,E}(l, f_t^E, e_t^E, q_{t+1}) \Rightarrow lR^{B,E}(l, f_t^E, e_t^E, q_{t+1}) \geq \frac{\Delta m}{\Delta_2}. \tag{A.81}$$

Since, $R^{B,E}(l,f,e) > R^{M,E}(l,f,e)$ entrepreneurs optimal play is to minimize the use of bank-finance:

$$l : R^{B,E}(l, f_t^E, e_t^E) = \frac{\Delta m}{e_t^E \Delta_2 l}. \tag{A.82}$$

Note that $l < 1$ for Δm sufficiently small (feasibility of monitoring). Accordingly, given $e_t^E = w_t$, $f_t^E = (1-\lambda)w_t/w_t$, the equilibrium values of the return on relationship finance, $R^{B,E}$ and market finance (when supplied with positive probability), $R^{M,E}$ are uniquely determined as a function of R^E, by combining equations, (A.61), (A.63), and $R^E = lR^{B,E} + (1-l)R^{M,E}$:

$$R^{M,E} = R^E - \frac{m(2)}{w_t \pi(2)} \tag{A.83}$$

$$R^{B,E} : \pi(2)\frac{\Delta m}{\Delta_2} - m(2) = \pi(2)\frac{\Delta m}{\Delta_2}\frac{R^{M,E}}{R^{B,E}}, \tag{A.84}$$

while R^E is being determined by condition (A.77).

2. Existence. The same arguments used for *Type 0 equilibrium* hold so that, under symmetry – for λ sufficiently close to zero – $z_t^E = 1$, with $e_t = w_t$ and $f_t^E = w_t(1-\lambda)/\lambda$ are supported as equilibrium outcomes.

It follows from (A.77) that the upper bound of R^E is given by $q_{t+1} - b(2)/\pi(2)$ and moreover, R^E converges monotonically to this upper bound as λ goes to zero. Therefore, the condition for moral hazard pervasiveness,

$$\frac{1-\lambda}{\lambda} w_t > \frac{\Delta_1 q_{t+1} - b(1)}{\Delta_1(R^M - q_{t+1}) + b(1)}, \tag{A.85}$$

inequality is satisfied for λ sufficiently close to 1 if and only if the following inequalities hold:

$$q_{t+1} > \frac{b(1)}{\Delta_1} \tag{A.86}$$

$$\frac{b(1)}{\Delta_1} > \frac{b(2)}{\pi(2)} + \frac{m(2)}{w_t \pi(2)}. \tag{A.87}$$

For E to be an equilibrium, market-clearing and sub-game perfection implies that for all $\{l,f,e\} \notin \mathcal{E}^E$ such that $\alpha(x|l,f,e,1) = 1$ and $\tau(0|l,f,e) = 1$, the weighted average cost of capital should satisfy condition (A.47). However, one can always construct (equilibrium) return functions that satisfy this property, so that deviating to any $\{l,f,e\}$ that satisfies the above is never profitable.

Finally, entrepreneurs must have no incentive to deviate to $\{l', f_t^E, e_t^E\} \notin \mathcal{E}^E$, where

$$l' = \frac{m(1)}{R^{B,E} e_t^E \Delta_1}, \tag{A.88}$$

or

$$l' = 0 \tag{A.89}$$

Suppose that an entrepreneur deviates to $l' > 0$ as given by (A.88). Note that $l' < 1$ for $m(1)$ sufficiently small (feasibility of monitoring). Given their SPE strategy, bankers would then monitor with intensity, $i = 1$, which induces a probability of success $\pi(1)$. Accordingly, the return on market finance such market investors are willing to supply finance is $R^{M,E}\pi(2)/\pi(1)$. By comparing the resulting expected payoff of the entrepreneur (computed given the equilibrium values of $R^{M,E}$, $R^{B,E}$, R^{E}, e_t^E and f_t^E) with the equilibrium one, it can be shown that such deviation would be profitable if and only if

$$q_{t+1} < q_1 \equiv (1-\lambda)\frac{b(2)-b(1)}{\Delta_2} + \left(\frac{m(2)}{w_t\Delta_2} - \frac{m(1)}{\Delta_1}\frac{m(1)}{\Delta m}\right) \tag{A.90}$$

Suppose now that an entrepreneur deviates to $l' = 0$. Applying the same logic as the one used above, it follows that such deviation is strictly profitable if and only if

$$q_{t+1} < (1-\lambda)\frac{m(2)}{\Delta_{\max}} + \frac{b(2)}{\Delta_{\max}} \tag{A.91}$$

It is immediate to verify that - under assumptions 1 and 2 the RHS is lower than q_1. Hence, E exists so long as $q_{t+1} \geq q_1$.

iv. Equilibrium with $\chi_t \in (1,2)$.

Let E be a symmetric equilibrium in mixed strategies such that:

a. $\mathcal{E}^E = \mathcal{E}_1^E \cup \mathcal{E}_2^E$ is the set of actions played with positive probability by entrepreneurs, with $\{l, f, e\} \in \mathcal{E}_1^E$ played with probability $p > 0$ and $\{l, f_t, e_t\} \in \mathcal{E}_2^E$ played with probability $1-p$.

b. MH is pervasive: $\alpha(0|l_t, f_t, e_t, 0) = 1$ for all $\{l_t, f_t, e_t\} \in \mathcal{E}^E$.

c. $\tau(1, l_t, f_t, e_t) = 1$, $\forall\{l_t, f_t, e_t\} \in \mathcal{E}_1^E$ and $\tau(2, l_t, f_t, e_t) = 1$, $\forall\{l_t, f_t, e_t\} \in \mathcal{E}_2^E$;

d. $z_t^E = 1$, $e_t^E = w_t$, $f_t^E = (1-\lambda)w_t/\lambda$; and $R^{B,E}(l_t, f_t, e_t, q_{t+1})$, and $R^{M,E}(l_t, f_t, e_t, q_{t+1})$, are the return-functions.

1. Characterization.

Let $R_i^E(l, f, e, q_{t+1})$, with $i = 1, 2$ the equilibrium weighted average cost of capital associated with $\{l, f_t^E, e_t^E\} \in \mathcal{E}_1^E$ and $\{l, f_t^E, e_t^E\} \in \mathcal{E}_2^E$, respectively. The equilibrium values of the weighted average cost of capital associated with $\{l, f_t^E, e_t^E\} \in \mathcal{E}_j^E$, $j = 1, 2$, R_1^E and R_2^E, are uniquely determined by the conditions (A.58) and (A.77), which are the equilibrium values of the WACC associated with type 1 and type 2 equilibria, respectively, for $\{l, f_t^E, e_t^E\} \in \mathcal{E}_1^E$ and $\{l, f_t^E, e_t^E\} \in \mathcal{E}_2^E$. Since $\alpha(0|l, f_t^E, e_t^E, 0) = 1$ for all $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$,

$$R^B(l, f_t^E, e_t^E, q_{t+1}) = R^B(l', f_t^E, e_t^E, q_{t+1}) = R^{B,E} \quad \forall \{l_t, f_t^E, e_t^E\}, \{l', f_t^E, e_t^E\} \in \mathcal{E}^E \tag{A.92}$$

As for the equilibrium return function for market finance, $\forall \{l, f_t^E, e_t^E\}, \{l', f_t^E, e\} \in \mathcal{E}_1^E$,:

$$R_1^M(l, f_t^E, e_t^E, q_{t+1}) = R_1^M(l', f_t^E, e_t^E, q_{t+1}) = R_1^{M,E} \tag{A.93}$$

and $\forall \{l, f_t^E, e_t^E\}, \{l', f_t^E, e\} \in \mathcal{E}_2^E$

$$R_2^M(l, f_t^E, e_t^E, q_{t+1}) = R_2^M(l', f_t^E, e_t^E, q_{t+1}) = R_2^{M,E} \tag{A.94}$$

where $\pi(2)R_2^{M,E} = \pi(1)R_1^{M,E}$.

In order for financial investors to operate as bankers, their expected return faced by bankers should be no less than the expected return of market investors. Let $\{l_1, f_t^E, e_t^E\} \in \mathcal{E}_1^E$, and $\{l_2, f_t^E, e_t^E\} \in \mathcal{E}_2^E$ be two actions played with probability p and $1-p$ respectively. Then:

$$p\left[\pi(1)R^{B,E} - \frac{m(1)}{e_t^E l_1}\right] + (1-p)\left[\pi(2)R^{B,E} - \frac{m(2)}{e_t^E l_2}\right] \geq \pi(2)R_2^{E,M} \tag{A.95}$$

must hold for financial investors to operate as bankers, where we recall $\pi(2)R_2^{E,M} = \pi(1)R_1^{E,M}$ holds. The above condition implies $R^{B,E} > R_j^M$ for all $\{l, f_t^E, e_t^E\} \in \mathcal{E}^E$. Note that the above condition hold as strict equality if both relationship and market finance are supplied in equilibrium. Therefore, entrepreneurs' optimal choice is to demand the minimum fractions of relationship finance necessary in order to be monitored at level one or two, that is:

$$l_1 = \frac{m(1)}{\Delta_1 R^{B,E} e_t^E} \tag{A.96}$$

$$l_2 = \frac{\Delta m}{\Delta_2 R^{B,E} e_t^E} \tag{A.97}$$

Note that, for $e = e_t^E = w_t$ both l_1 and l_2 are less than one for sufficiently small values of $m(1)$ and Δm, respectively. Let $R^{B,E}$ and market finance $R^{M,E}(j)$ with $j = 1, 2$ equilibrium values of the return to relationship and market finance respectively. Combining condition (A.95) –taken as strict equality– and the expression for the weighted average cost of capital $R_j^E = R^{B,E}l + (1-l)R_j^{M,E}$, with $j = 1, 2$, and imposing $e_t^E = w_t$, we obtain

$$\frac{m(1)}{w_t\Delta_1} + R_1^{E,M} - \frac{m(1)}{w_t\Delta_1}\frac{R_1^{E,M}}{R^{E,B}} = R_1^E \tag{A.98}$$

$\forall\{l, f_t^E, e_t^E\} \in \mathcal{E}_1^E$, and

$$\frac{\Delta m}{w_t\Delta_2} + R_2^{E,M} - \frac{\Delta m}{w_t\Delta_2}\frac{R_2^{E,M}}{R^{E,B}} = R_2^E. \tag{A.99}$$

while $\forall\{l, f_t^E, e_t^E\} \in \mathcal{E}_2^E$. The values of $R_j^{M,E}$, $R^{B,E}$ and R_j^E, with $j = 1, 2$ are uniquely determined as a function of p, by combining (A.98), (A.99), (A.96), (A.97), (A.77), (A.58), and condition (A.95), all taken as strict equalities.[26]

In order for E to be an equilibrium, entrepreneurs should be indifferent between playing an action that induces monitoring with intensity $i = 2$ or $i = 1$, that is:

$$\pi(2\left[q_{t+1}(f_t^E + e_t^E) - R_2^E f_t^E\right] - b(2)(f_t^E + e_t^E) = \pi(1)\left[q_{t+1}(f_t^E + e_t^E) - R_1^E f_t^E\right] - b(1)(f_t^E + e_t^E) \tag{A.100}$$

Substituting in for the equilibrium values of R_2^E, R_1^E, e_t^E, f_t^E and using all other equilibrium conditions, yields the equilibrium value of p associated with E, given q_{t+1},

$$p : q_{t+1} = \frac{b(2) - b(1)}{\Delta_2} + \frac{1-\lambda}{\Delta_2 w_t}\{\pi(2)\frac{\Delta m}{\Delta_2} - \pi(1)\frac{m(1)}{\Delta_1} \tag{A.101}$$

$$+\left(\frac{m(1)}{\Delta_1} - \frac{\Delta m}{\Delta_2}\right)\left[p\pi(0) + (1-p)\left(\pi(2) - \frac{m(2)\Delta_2}{\Delta m}\right)\right]\}. \tag{A.102}$$

[26]These equations yield a system in seven unknowns, $R_1^{M,E}, R_2^{M,E}, R_1^E, R_2^E, R^{B,E}, l_1, l_2$.

2. Existence.

We note that –given the above condition– $p = 0$ if $q_{t+1} = q_1$, while $p = 1$ if $q_{t+1} = q_2$. Furthermore, we know from part ii and iii that entrepreneurs playing $\{l_t, f_t, e_t\} \in \mathcal{E}_1^E$ have no profitable deviation if $q_{t+1} \leq q_1$, while agents playing $\{l_t, f_t, e_t\} \in \mathcal{E}_2^E$ have no profitable deviation if $q_{t+1} \geq q_2$. Hence, $q_{t+1} \in [q_2, q_1]$, is the necessary and sufficient condition for the equilibrium E to exist. □

vi. Uniqueness and multiplicity.

Consider a candidate symmetric equilibrium with no randomization over f_t^E such that $e_t^E \neq w_t$ and $f_t^E \neq w_t(1-\lambda)/\lambda$. Under the existence conditions of any of the equilibrium types described above, $e_t^E \neq w_t$ is not optimal, and $f_t^E \neq w_t(1-\lambda)/\lambda$ violates market clearing. Hence, in any symmetric equilibrium $e_t^E = w_t$ and $f_t^E = w_t(1-\lambda)/\lambda$. It then follows from the above analysis that if $q_{t+1} < q_0$, the type 0 equilibrium is the unique symmetric equilibrium. Similarly, the type 1 equilibrium is the unique symmetric equilibrium if $q_{t+1} \in (q_0, q_1)$, while the type 2 equilibrium is the unique symmetric equilibrium if $q_{t+1} > q_2$. Multiplicity arises if $q_{t+1} = q_0$, in which case $\chi_t \in [0,1]$, and $q_{t+1} \in (q_1, q_2)$. In this latter case, the equilibrium is unique given that there is a unique value of p that satisfies (A.101) condition, and the arguments made with respect to equilibria of type 1 and 2 hold.

A.4 Proof of lemma 2

i. Labor and capital are fully employed, and; ii. The fraction of financial resources, z_t channeled toward investment equals one. With respect to part *i*, the proof part *i* of 3 directly applies. Regarding part *ii*, we observe that, according to Lemma 1, given $q_{t+1} > 0$, $z_t^E = 1$ holds in any equilibrium, E. In turns, $z_t^E = 1$ implies $K_{t+1}^E = k_{t+1}^E > 0$ – according to part *i* above, so that $q(k_{t+1}^E) > 0$, which sustains $z_t^E = 1$. Then, symmetry and market clearing imply $f_t = (1-\lambda)/\lambda$ (see also proof of lemma 1), which completes the proof.

iii. Unique equilibrium and the associated value of χ_t. The argument is equivalent to that developed in the proof of lemma (3), parts *iii.a-e*. Consider a candidate equilibrium, E, with $\chi_t^E \in [0,2]$ and $z_t^E = 1$, so that $K_{t+1}^E = k(k_t, 1, \chi_t^E)$, where

$$k(k_t, \chi_t^E, 1) = \Pi(\chi_t^E)\left[g(k_t) - g'(k_t)k_t\right]. \tag{A.103}$$

Since $\Pi(\chi_t^E)$ is strictly positive for any possible value of χ_t^E, $k_{t+1}^E > 0$ holds and part *i* above applies so that $k_{t+1}^E = K_{t+1}^E$. Given Lemma 1, the partial equilibrium of financial markets exists and it is characterized by: i. $\chi_t = 0$ if $q_{t+1} < q_0$; ii. $\chi_t = 1$ if $q_{t+1} \in (q_0, q_2)$; iii. $\chi_t = 2$ if $q_{t+1} > q_1$; and either $\chi_t = 1$ or $\chi_t = 2$ or $\chi_t \in [1,2]$ if $q_{t+1} \in [q_1, q_2]$. Then, given that: (a) $k(k_t, \chi_t^E, 1)$ is strictly positive and increasing in $\chi_t^E \in [0,2]$, and (b) $q_{t+1} = q(k_{t+1}^E)$ is strictly decreasing in k_{t+1}^E, it follows directly from Lemma 1 that the temporary symmetric macroeconomic equilibrium exists and it is characterized by a unique value of χ_t as follows: a. $\chi_t^E = 0$, if $q(k(k_t, 0, 1) < q_0$; b. $\chi_t^E = 1$ if $q(k(k_t, 1, 1) \in [q_0, q_1)$; c. $\chi_t^E = 2$ if $q(k(k_t, 2, 1) > q_1$; d. $\chi_t^E \in [1,2]$ if $q(k(k_t, 1, 1) > q_2$ and $q(k(k_t, 2, 1) < q_1$ (so that type 1 and type 2 equilibria do not exist); e. $\chi_t^E \in [0,1]$ if $q(k(k_t, 0, 1) > q_0$ and $q(k(k_t, 1, 1) < q_0$ (so that type 0 and type 1 equilibria do not exist).

iv. Multiple equilibria and associated values of χ_t. The proof follows directly from the following observations:*(1)* According to Lemma 1, part *iv*, there exist multiple equilibria in the following instances: *a.* $q_{t+1} \in [q_2, q_1]$, in which case $\chi_t \in [1, 2]$; *b.* $q_{t+1} = q_0$, in which case either $\chi_t \in [0, 1]$; *(2)* $k^E_{r+1} = k(k_t, \chi^E_t, 1)$ is strictly positive for any χ^E_t, and strictly increasing in χ^E_t, and; *(3)* $q_{t+1} = q(k^E_{t+1})$ is strictly decreasing in k^E_{t+1}. $\square$

A.5 proof of Proposition 2

According to Lemma 2, given $K_t > 0$, a temporary symmetric equilibrium exists (although it might be not unique), and it is characterized by $\chi_t \in [0, 2]$, and $z_t = 1$. Therefore, $K_{t+1} = k_{t+1} = k(k_t, \chi_t, 1) > 0$ holds, so that – by induction – we conclude that, given $K_{t_0} > 0$ an intertemporal equilibrium always exists; and all intertemporaly equilibria are chara cterized by $z_t = 1$.

i. Stable steady state with $\widehat{\chi} = 2$. Let $\kappa(2, 1) < k_1$be the steady state value, $\widehat{k}$, of the capital-labor ratio associated with an intertemporal equilibrium path such that the steady-state quality of investment satisfies $\widehat{\chi} = 2$ for all t. Note that – given Lemma 2 and the definition of k_1, if $\kappa(2, 1) < k_1$, $\widehat{k} = \kappa(2, 1)$, $\widehat{\chi} = 2$, is the unique non-trivial steady state equilibrium. Consider a temporary equilibrium at time t_0 where $k_{t_0} > 0$ and χ_{t_0} is the average quality of investment, so that $k_{t_1} = k(k_{t_0}, \chi_{t_0}, 1)$ is the capital-labor ratio at time $t_1 = t_0 + 1$. Suppose $k_{t_0} < \kappa(2, 1)$. We know from Lemma 2 that the equilibrium average quality of investment, χ_t, is (weakly) decreasing in k_t. Hence, $k_{t_0} < \kappa(2, 1)$ implies $\chi_{t_0} = 2$. Then, $k_{t_1} > k_{t_0}$. Then, the proof of Lemma 3 applies. Accordingly, for any $n > 0$, $k_{t_n} < \kappa(2, 1)$, with $k_{t_{n-1}} < k_{t_n}$ and $\chi_{t_n} = 2$. Hence, the sequences $\{k_{t+n}\}$ and $\{\chi_{t+n}\}$ are monotonic and therefore converge to their limit equal to $\kappa(2, 1)$ and $\widehat{\chi} = 2$, respectively (with are elements of the sequences).

The same logic can be applied to show convergence to the unique steady state associated with intertemporal symmetric equilibrium paths, in cases *v-vi*.

ii-vi. Multiple equilibria: Convergence to steady state.

Let χ^M be the highest quality of investment associated with a steady state equilibrium. Given Lemma 2 and the definition of k_1, $\kappa(2, 1) > k_2$ implies $\chi^M < 2$. Let χ^m be the lowest quality of investment associated with a steady state equilibrium. Then, given Lemma 2 and the definition of k_1 and k_0 $\kappa(1, 1) \in [k_1, k_0]$, $\chi^m = 1$ follows. Define $\mathcal{S} = \{\widehat{k} = \kappa(\widehat{\chi}, 1), \widehat{\chi} : \kappa(\chi^M, 1) \geq \widehat{k} \geq k_1; \chi^M \geq \widehat{\chi} \geq 1\}$ the set of steady state equilibria.

Consider now a temporary equilibrium at time t_0 such that $k_{t_0} < k_2$ and $\chi_{t_0} \leq \chi^M$, so that $k_{t_1} > k_{t_0}$. Then the proof of lemma 3 applies, so that $\{k_{t+n}\}$ and $\{\chi_{t+n}\}$ are monotonic sequences converging to $\kappa(\chi_{t_0}), \chi_{t_0}$.

Consider now a temporary equilibrium at time t_0 such that $k_{t_0} < k_2$and $\chi_{t_0} > \chi^M$. Then, the economy would eventually reach a value of k_{t+n} such that $k_{t+n+1} > k_2$. At this stage, the average quality of investment would drop to $\chi_{t+n} = 1$. Then, the economy would monotonically converge to $\widehat{k} = \kappa(1, 1), \widehat{\chi} = 1$.

The following logic can be applied to show convergence to any steady state equilibrium $\widehat{k} = \kappa(\widehat{\chi}, 1), \widehat{\chi}$ (cases *ii-vi* whenever such equilibrium exists.

iv. Endogenous business cycles.

Define $\underline{k}_1$ such that $k(\underline{k}_1, 2, 1) = k_1$ and restrict attention to pure strategy equilibria. Suppose that $\kappa(2, 1) > k_1$; $\kappa(1, 1) < k_2$; $k(k_2, 2, 1) \leq k_1$, and $k(\underline{k}_1, 1, 1) \geq k_2$. Let $k_{t_0} < k_2$ be the initial condition so that –given $k(k_1, 2, 1) < k_1$ –, $k_{t_1} = k(k_{t_0}, 2, 1) < k_2$ and the temporary equilibrium at time t_0 is characterized by $\chi_{t_0} = 2$. The economy will move along an equilibrium path in which $k_{t+n+1} > k_{t+n}$ and $\chi_{t+n} = 2$ until it reaches a level of k_{t+n} such that $k_{t+n+1} > k_1$. At this stage the only equilibrium under pure strategies involves $\chi_{t+n} = 1$. Since $k_{t+n+1} \geq k_1$, $k_{t+n} \geq \underline{k}_1$ must hold. Then, $k(\underline{k}_1 1, 1) > k_2$ implies that $k_{t+n+1} = k(k_{t+n}, 1, 1) > k_2$ so that a pure strategy equilibrium with $\chi_t = 1$ exists and $k_{t+n+1} > \kappa(1,1)$, so that the economy shrinks until it reaches a value of k_{t+n} such that $k_{t+n+1} < k_2$. At this stage the only pure strategy equilibrium would be characterized by $\chi_{t+n} = 2$ and the economy expands again. □

A.6 Proof of Corollary 1

In part *iv. Endogenous business cycles* of the proof of proposition 2, we prove that $\kappa(2,1) > k_1$; $\kappa(1,1) < k_2$; $k(k_2, 2, 1) \leq k_1$, and $k(\underline{k}_1, 1, 1) \geq k_2$ constitute sufficient conditions for the economy to experience endogenous fluctuations in k_t. Given the expression of the aggregate output net of monitoring costs, (40), it is immediate to verify that during expansion phases, in which $\chi_t = 2$, $\tilde{y}_t = y(k_{t+1}) - (1-\lambda)m(2)$, while during recessions $\tilde{y}_t = y(k_{t+1}) - (1-\lambda)m(1)$. In both cases the aggregate output net of monitoring costs, $\tilde{y}_t$, is a monotone strictly increasing function in k_t. Hence, as k_t accumulates (decumualtes) along expansion (recession) phases, aggregate output increases (falls). □

A.7 Proof of Remark 2

We know that the relative fraction of relationship finance during expansions (E) and contractions (C) is as follows:

$$l_t^E = \frac{1}{w_t}\left(\frac{\Delta m}{\Delta_2} - \frac{m(2)}{w_t \pi(2)}\right)\frac{1}{R_{t+1}^{M,E}} \tag{A.104}$$

$$l_t^C = \frac{1}{w_t}\pi(0)\frac{m(1)}{\pi(1)\Delta_1}\frac{1}{R_{t+1}^{M,C}} \tag{A.105}$$

where

$$R_{t+1}^{M,E} = R^E(q(k_{t+1})) - \frac{m(2)}{w_t \pi(2)} \tag{A.106}$$

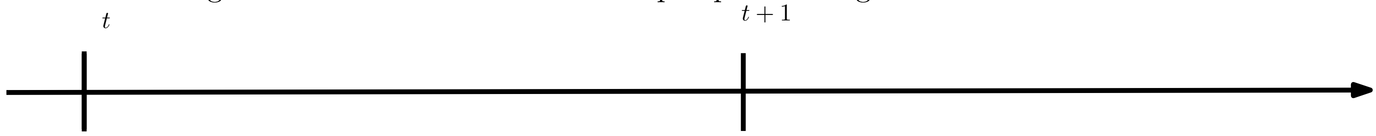
$$R_{t+1}^{M,C} = R^E(q(k_{t+1})) - \frac{m(1)}{w_t \pi(1)} \tag{A.107}$$

Let k_t the capital labor ratio associated with the temporary equilibrium at time t. Suppose the economy enters a contraction phase $k_{t+1} < k_t$. Given assumption (2), $m(2)/\pi(2) > m(1)/\pi(1)$.Given that $R^{M,j}$, with $j = E, M$ is decreasing in k_{t+1}, this implies $R_t^{M,E} < R_{t+1}^{M,C}$. Also, if the economy has been expanding until time t, $w_{t-1} < w_t$. Accordingly, given the expressions for l_t^E and l_t^C,

$$\left(\frac{\Delta m}{\Delta_2} - \frac{m(2)}{\pi(2)}\right) \geq \pi(0)\frac{m(1)}{\pi(1)\Delta_1} \Rightarrow l_{t-1}^E > l_t^C \tag{A.108}$$

Applying the same reasoning yields the he opposite conclusion when the economy enters an expansion. □

Figure 1: Model timeline from the perspective of generation t



At t:

1. Generation t is born
2. Young agents receive w_t in exchange for labor
3. Young entrepreneurs and financial investors exchange financial resources and investment takes place

At $t+1$:

1. Old entrepreneurs whose investments succeed receive q_{t+1} in exchange for capital
2. Old financial investors receive their return to financing, $R_{t+1}(x)$, which is contingent on the effort x exerted by the entrepreneurs they finance, and translates in an expected return $\pi(x)R_{t+1}(x)$
3. Firms produce combining labor supplied by members of generation t+1 and capital supplied by old entrepreneurs
4. Old agents consume and die

Figure 2: History selected equilibrum paths

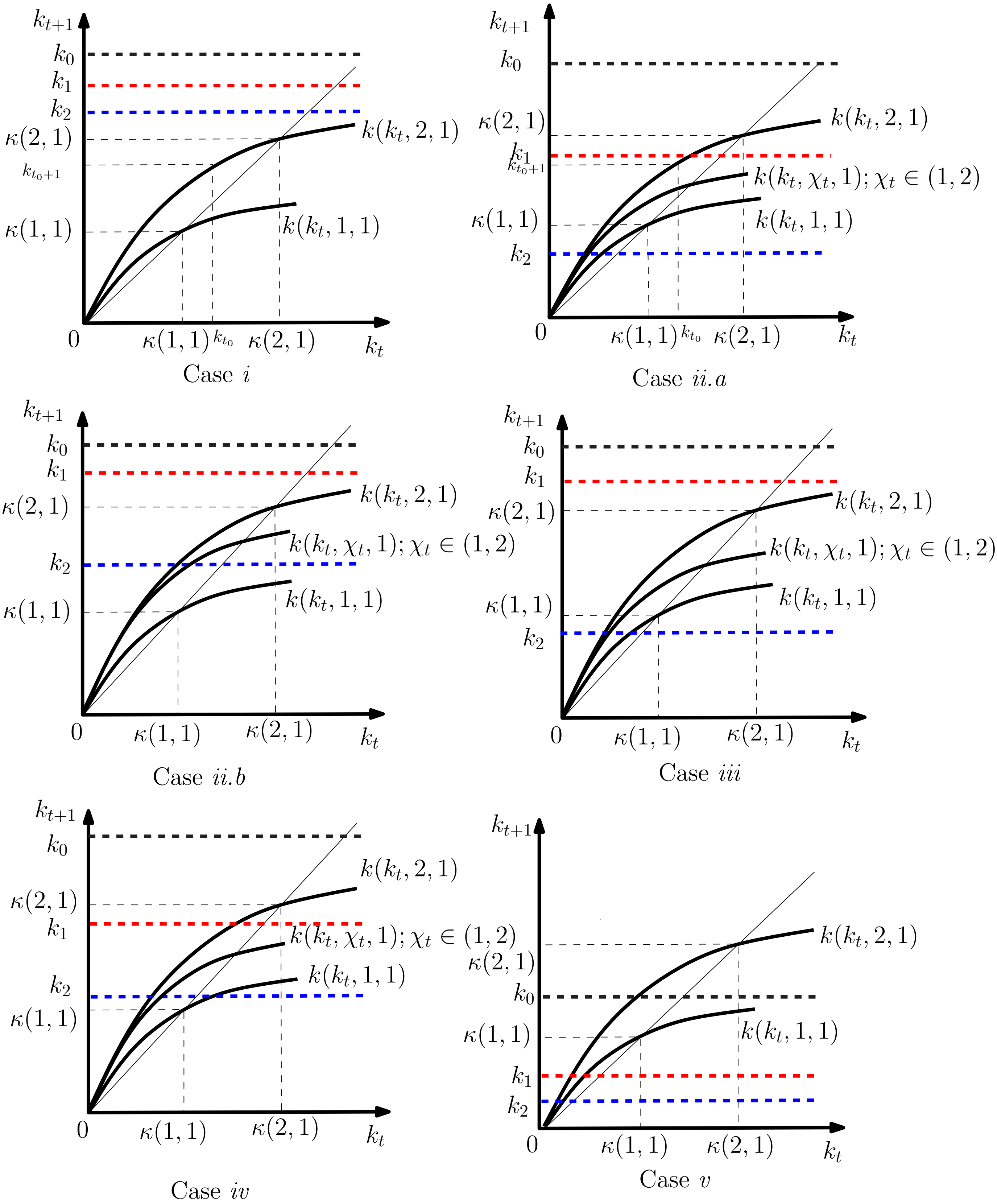
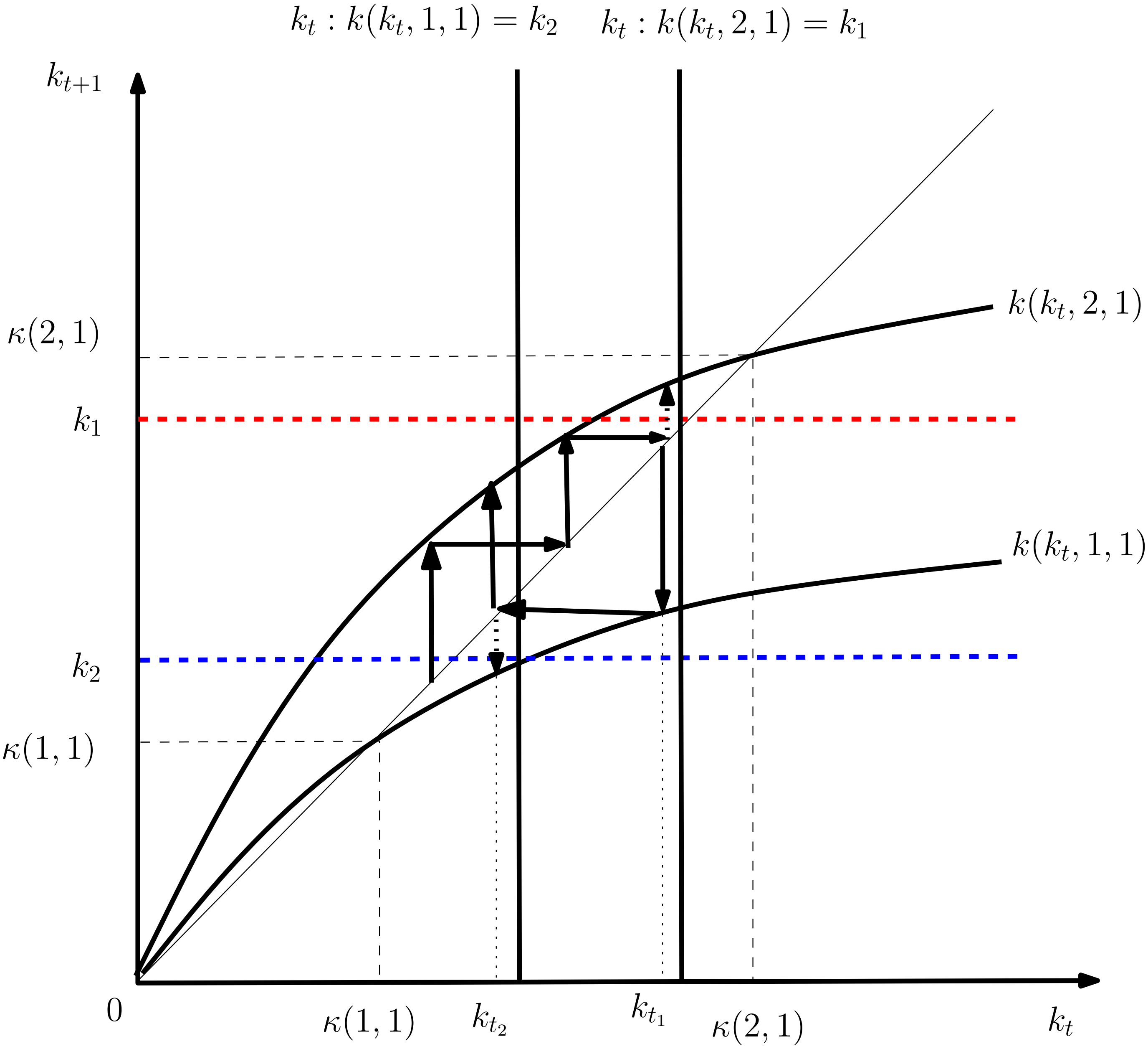


Figure 3: Endogenous business cycles



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Via Torre Tonda 8 – Tel. 079.200395 – Fax 079.4360444
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