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SCREENING AND SHORT-TERM CONTRACTS

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Titolo: SCREENING AND SHORT-TERM CONTRACTS

Screening and short-term contracts

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Abstract

This article studies the behavior of the firm when it is searching to fill a vacancy. The principal hypothesis is that the firm can offer two kinds of contracts to the workers, short-term or long-term contracts. We suppose that the worker's bargaining power over the wage is different according to the type of contract. We utilize this framework to study the firms' optimal policy choice and its welfare implications.

1 Introduction

In this paper, we study the behavior of a firm searching to fill a vacancy. The principal assumption is that they offers two kinds of contracts to the workers: a short-term contract (STC)[1] or a long-term one (LTC). Moreover we assume that the worker's bargaining power over the wage is different according to the type of contract.

Evidence shows that the share of temporary work in total employment has been increasing in Europe in recent years. At the end of the seventies, labor market regulations restricted temporary jobs to specific tasks, characterized by large variations in productivity. Those regulations have changed somewhat since the eighties, and it is now possible in a number of European countries to hire workers on a temporary basis even for jobs which are not subject to such variations in productivity.

For example, OECD study shows that in 1983 only 4% of the employees in the EC held temporary jobs, in 1991, while had rose to 10%.[2] After 1991, this amount is increasing in some countries (like Belgium and Italy) and decreasing in other ones (like Spain).

Temporary contracts are often regarded as a measure of labor market flexibility.

In periods of rapid technical change or demand volatility, temporary

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[1] The short-term contract (STC) and temporary contract will be used interchangeably throughout the paper.

[2] See OECD (2002) for a detailed description of xed-term contract regulation in OECD.

contracts allow firms to hire workers as they wish. In addition, the STC could be also viewed as a screen devise, that allows employers to observe the productivity of the job-worker pair. In this perspective, job matches are interpreted as "experience good", in the tradition of Jovanovic (1979, 1984). In that case they may seek to select the right workers into probationary jobs.

In a theoretical model, Wasmer (1999) Cahuc, Postel-Vinay (1999) and Paolini (2007) have introduced temporary jobs in matching models following the traditional equilibrium models of the labor market.

Wasmer (1999), in a model with exogenous job destruction, shows that in the periods of low growth the firms are more willing to make use of STCs, which is favorable to employment. Cahuc and Postel-Vinay (1999), in a model with endogenous job destruction, show that the combination of temporary jobs and firing restriction may be both inefficient in terms of aggregate welfare and an inadequate weapon to fight unemployment. This result comes from the fact that the share of temporary jobs transformed into permanent jobs is decreasing in the level of the firing cost. Paolini (2007) shows that it can be optimal for firms to start a working relationship with a short-term contract but that this policy has a negative impact on unemployment and welfare[3].

We show that, if the short-term wage is fixed endogenously, it can be optimal for firms to start a working relationship with a short-term contract, but that this policy has a negative impact on unemployment and welfare. On the contrary, if this wage is fixed exogenously, this policy could be optimal also from a welfare point of view.

In this article, we see the STC like a screening device. That is, we suppose that the only way to determine the quality of a particular match is "to form the match and experience it". In that case, firms may seek to select the right workers into the STC. Moreover, we suppose that firms and workers split productions according to different types of bargaining power arising from the type of contract. Search cost is captured by the discount factor. In order to preserve the stationarity of the distribution of types on the search market, we assume that the workers who filled a long-term work are replaced by workers of same quality. We utilize this framework to study the firms' optimal policy.

In this paper, we argue that firms may see temporary contracts as a measure to hire workers like they wish. This policy may not depend from demand volatility[4] but from the possibility to hire workers in a cheaper way, that comes from the lower level of short-term wages. Moreover, we show that firms may also use STCs like a screening device. The choice of the policy to follow depends crucially on the workers' relative bargaining power. If the relative power is sufficiently divergent firms prefer to engage workers directly with a long-term contract or short-term one, depending on the sign of this value. If, on the contrary, this value is not so much divergent, firms will discover the worker's type through the short-term contract and engage her only if her ability is above a threshold, endogenously determined.

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[3]See also Paolini (2000).

[4]See, for example, Wasmer (1999), who study STCs as a measure of labor market flexibility.

We also envisage a Social Planner whose objective is to maximize the sum of workers' utilities and firms' profits by intervening in the labor market by imposing transfers from the firms to the unemployed workers and/or by regulating the relative bargaining power.

The model is introduced in Section 2. Sections 3 and 4 present the main results. Section 5 analyzes the welfare. Section 6 concludes.

2 The Model

Workers are characterized by a real-value (the worker's type) distributed on $[0, 1]$. The worker's type is denoted x and is distributed according to atomless, continuous distribution function $F(x)$ with full support on $[0, 1]$. We denote its density by $f(x)$. The economy has a labor force of mass 1.

Firms are homogenous and the firms' number is $M \geq 1$. At each moment of time, a firm can have either a filled position, or a open vacancy, or be idle. An active firm with a filled position employs one worker and obtains a revenue from selling the output it produces. Idle firms pay no cost and earn no revenue. We assume M to be sufficiently large so that a positive measure of firms remain idle in any of the equilibria analyzed here. The renters do not work, and each of them holds a balanced portfolio of shares of all M firms. The income of renter consists of dividends (possibly negative, in which case he is liable for the losses) plus an endowment flow. This endowment is assumed to be sufficiently large to avoid limited liability issues[5].

The production function in each firm is the following:

$$Y = yx \tag{1}$$

where x is the type of the worker and y the technology of the homogenous firms. Throughout the article, we normalize the firm's technology to 1.

Time is discrete and runs as $t = 0, 1, ... + \infty$. At any t, firms can create a position at cost k that represents the firm job advertising. They are matched to the workers according to a simple random matching technology, α, that is assumed to be independent of the number of participants in the search market. The matching function exhibits constant returns to scale.

At each meeting in the search market the firms are not able observe the type of the worker. We allow the firm to offer a probationary contract to the worker. During this period the firm can learn the worker's type, and it can decide whether to offer a long-term contract after the short-term one.

We assume that if a long-term contract is signed, there is an incoming flow of workers of the same quality so that the workers' distribution is time-invariant.[6]

Firms have a discount factor denoted $\delta > 0$, and they obtain zero profit if there is no matching. The discount factor captures the search cost.

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[5] See Marimon and Zilibotti (1999).

[6] On the extension of the paper (section ??), we suppose that there is an exogenous incoming flow of workers, to study the steady state of the dynamic system.

We now define the optimal behavior for the representative firm. A policy for a firm is the choice of the contract to offer to the worker. We will make the hypothesis that there are no firing costs on the market.

The firm has two possibilities as to the policy it may pursue. Either it only offers long term contracts (L); or it may offer a short term contract to begin with, switching to a long term one after its completion (SL). The advantage of this last type of policy is that the firm, when offering the long term contract, has full information about the worker's type.

The firm seeks to maximize profit by choosing the optimal combination of contracts. So, a policy for a firm is $a \rightarrow \{L, SL(\sigma_x)\}$ where σ_x is measurable subset of $[0,1]$ corresponding to the set of workers the firm will accept after knowing the worker's type. If the subset σ_x is empty, it means that firm prefers to engage workers exclusively on short-term conditions.

In order to study the optimal policy choice we have to analyze before the firm's optimal strategy in the SL policy.

2.1 Bellman equations

Consider the Bellman equations characterizing the firm. Let $E\Pi^a$ denote the expected discounted payoff of a firm searching to fill a vacancy, with $a = SL, L$. A firm has a probability α to match a worker in the search market. With the probability $(1 - \alpha)$, this will not be the case, and it will try again to match next period. We denote X like the set of workers accepted by the firm. Then, the firm will maximize the following payoff:

$$Max\ E\Pi^a = -k + \alpha \left[\pi_0 + \sum_{t=1}^{+\infty} \delta^t (\int_{x \in X} xf(x)dx - w_a) + \delta(1 - \int_{x \in X} f(x)dx)E\Pi^a \right] +$$

$$\delta(1 - \alpha)E\Pi^a \tag{2}$$

where

$$\pi_0 = (\int_0^1 xf(x)dx - w_o) \tag{3}$$

In the first case, $a = SL$, when it chooses to begin with a STC, the long-term wage, starting in period 1, will be contingent on the worker's type, $w_a = w_{sl}(x)$. The domain X is in this case equal to σ_x. If the firm decides to offer exclusively STCs, refusing each work it meet, X is a empty set.

If the firm decides to offer directly a LTC $(a = L)$, it can not distinguish the worker's type, and hence the relevant X is the full support $[0,1]$ and $w_a = w_o = w_l$.

To simplify the analysis of the policy choice of the firm, we assume that the worker and the firm split the production according to their bargaining power. Moreover, we assume that the worker's bargaining power is different from one type of contract to the other.

The wages on the long-term informed contract (that is, after a trial stage) will be contingent to the worker's type:

$$w_{sl} = \gamma x_i \tag{4}$$

where γ is an index of the worker's bargaining power.

On the contrary, the wage on long-term contract (without trial stage) and short-term contract will be in expected value, because the firm doesn't know the worker's type before signing the contract. The wages will be respectively:

$$w_l = \gamma \left(\int_0^1 x f(x) dx \right) \tag{5}$$

$$w_o = \eta \left(\int_0^1 x f(x) dx \right)$$

where η denotes the bargaining power of the worker on STCs.

To simplify the analysis of the results, we define workers' relative bargaining power (RBP), the difference between the long-term bargaining power and the short-term one, i.e. $RBP = \gamma - \eta$.

3 Search equilibrium on the SL policy

The optimal strategy in the SL policy is σ_x corresponding to the set of workers accepted by firm after the STC.

Hence, for the stationary strategy profile (σ_x), we define the expected payoff of a firm as $E\Pi^{SL}(\sigma_x)$.

Definition 1 A search equilibrium in the policy SL is a stationary strategy profile (σ_x), if for all firms and for all strategies (σ'_x), $E\Pi^{SL}(\sigma_x) \geq E\Pi^{SL}(\sigma'_x)$.

Proposition 1 *The firm engages the worker on the interval* $(z, 1]$, *where* $z = \alpha \left(\frac{\delta}{1-\delta} \int_z^1 (x - z) f(x) dx + \frac{1-\eta}{1-\gamma} \int_0^1 x f(x) dx \right) - \frac{k}{1-\gamma}$.

Proof. See Appendix 1. ∎

Proposition 1 characterizes the search equilibrium of the SL policy. In this equilibrium, the firms partition the workers into two subintervals. Through the STCs, the firms can learn the workers' types, that they will engage in a LTC only if the worker's ability is above the threshold z. This value depends on relative bargaining powers (RBP).

To better understand this result, it is instructive to consider a simple example, where the abilities are uniformly distributed on $[0, 1][F(x) = x]$. Figure 1 depicts how the threshold z varies with the market power γ, taking $\gamma = \eta$, $k = 0.3$, and $\delta = 0.8$.
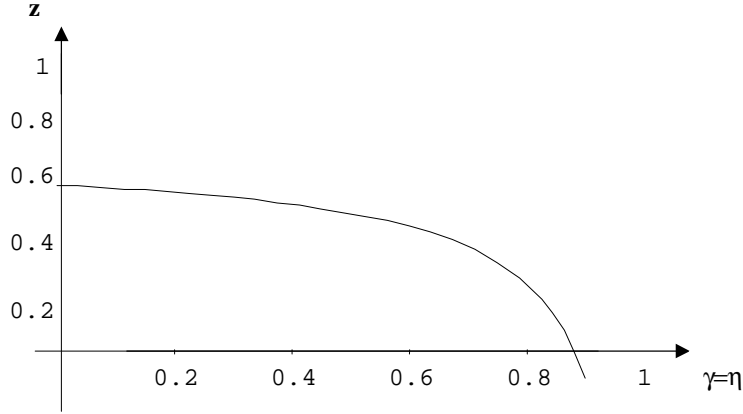
Figure 1: search equilibrium on the SL policy, with the $\gamma = \eta$

From figure 1, we see that when the workers' bargaining power is very high, the representative firm proposes to every worker a long-term contract following the short one. In the example this is will always be satisfied if $\gamma \geq 0.83$.

The next proposition summarizes some comparative statics properties of the equilibrium.

Lemma 2 *As the discount parameter δ increases, the threshold z increases. Similarly, as the matching (α) becomes more efficient. On the contrary, as the bargaining market power of the worker on the short-term contracts increases, or the advertising cost k increases, z decreases. An increase of the workers' market power on the long term contract increases z iff $\int\limits_0^1 x f(x) dx \geq k$.*

Proof. See Appendix 1.2 ∎

Lemma 2 establishes that as frictions on the search market are reduced (either through an increase in the efficiency of the matching technology or in the discount factor), the more qualified workers become more choosy, and only a smaller subset of workers will be engaged. The effect of an increase of workers bargaining power on the short-term contract (or in k) has an opposite effect: a bigger proportion of workers will be offered long term contracts. An increase in the workers bargaining power on the long-term contracts will involve an increase in z if and only if the expected output of the workers on the temporary market is bigger than the cost to re-open the vacancy.

4 Optimal contract

After the characterization of the SL contract, we can study the optimal contract choice of the firm. We will start the analysis with the hypothesis that $\gamma = \eta$; we relax this hypothesis later on.

Proposition 3 *If $\gamma = \eta$, SL policy is an equilibrium if $\eta \leq \eta^*$, otherwise we have an equilibrium L.*

Proof. See Appendix 2.1 ∎

If $\gamma = \eta$, the set σ_x is never empty. This means the firm will use the short-term contract always a screening device, so it not will choice to engage workers exclusively on short terms. The intuition of this result is very simple. If the two bargaining powers are the same on the two markets, the expected profit from offering only short-term contracts to the workers is the same as the one from offering directly a long-term contract (L) to the first worker that is matched. The only difference is that, if the firm engages workers exclusively on shot-term condition, the firm will have a supplementary cost that comes from the fact the it has to re-open the vacancy each period.

To better understand the result of the proposition 3, we take the same example as before. Figure 2 depicts the behavior of the firm on the contract choice.
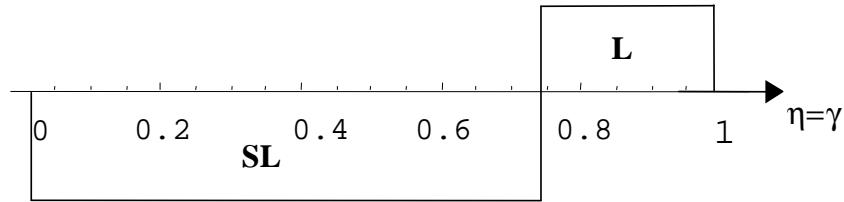

Figure 2: firms' optimal policy, with $\gamma = \eta$

When the firm has an important bargaining power, it prefers to be selective, screening the worker on the short-term contract and taking only the best qualified. In that case, the costs to re-open a position will be compensated with the surplus it can get in the following period.

As this power is lesser the surplus that it can get by waiting to find the good worker will be smaller, and therefore the cost to re-open the position each period will be difficultly compensated.

If we relax the hypothesis that the difference in workers bargaining powers is zero, we find that the firm could prefer to engage workers exclusively with short-term contracts.

Proposition 4 *Policy SL is an equilibrium if $\eta \leq \eta_1^*$, policy L otherwise.*

Proof. See Appendix 2.2 ∎

Taking the same example as before, with $\gamma = 0.8$, we have the following figure.

S   **L**

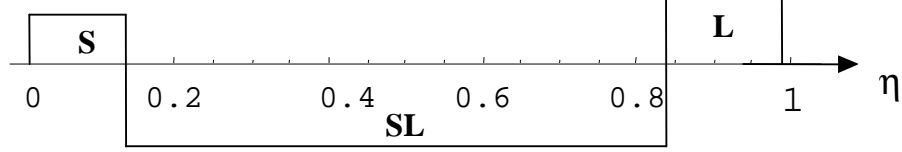0   0.2   0.4   0.6   0.8   1 η

SL

Figure 3: firms' optimal policy, with $\gamma \neq \eta$

From figure 3, if the RBP is negative and sufficiently high, the firm will prefer to engage the workers directly with a LTC. If these powers have the same value or become sufficiently divergent in the opposite sign, it will prefer to turn to a SL policy, in which it can screen the worker quality in the STC and eventually (if $\eta \leq 0.15$, the firm will engage workers exclusively with STCs) engage her with a LTC afterwards. Finally, if the RBP is positive and high , the firm will always prefer to engage in each period workers with temporary contracts.

Lemma 5 *As γ increases, the value increases if* $(1 - \delta F(z)) \int\limits_{0}^{1} x f(x) dx \geq$ $\delta \int\limits_{z'}^{1} x f(x) dx.$

Proof. Appendix 2.3 ∎

From the above Lemma, we conclude easily that if the workers' bargaining power on the LTCs (γ) decreases the firm is more likely to offer L policy if $(1 - \delta F(z)) \int\limits_{0}^{1} x f(x) dx > \delta \int\limits_{z'}^{1} x f(x) dx.$ On the contrary, if this condition is not satisfied, we are more likely to get a SL policy in equilibrium.

5 Welfare

To analyze the social efficient policy, we look first at the workers' welfare, defined as the sum of workers' utility in each possible policy[7]. Afterwards, we will look at the total welfare defined as the sum of workers' welfare and firms' expected profits in each possible policy a.

Workers' welfare can be written as

$$Max\ EU^a = F(z)EU^a_{x<z} + (1 - F(z))EU^a_{x \geq z} \tag{6}$$

where $a = sl, l$, moreover, $F(z)$ is the proportion of workers engaged exclusively on short-term conditions, and $(1 - F(z))$ is the proportion of workers that will be engaged with a LTC after having revealed their quality on the STC.

It is straightforward to compute the policy that maximizes the workers' welfare.

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[7] The workers' utilities are defined in Appendix 3.

Proposition 6 *If $\gamma = \eta$, the workers' welfare is maximum with $a = L$.*

Proof. Appendix 3.1 ∎

If the workers' bargaining powers are the same, the highest attainable workers' welfare is obtained with $a = L$.

If $\gamma = \eta$, a STC is never optimal for the workers. This is because after a STC they have to return to the labor market and look for another employment opportunity, which they will find with a probability α. The uncertainty of finding a new job could only be compensated by a bigger short-term market power.

A different kind of contract may maximize workers' welfare only if the market powers are sufficiently divergent.

Proposition 7 *The workers' welfare is maximum with policy SL if $\eta_1^{**} \leq \eta$, with policy L otherwise.*

Proof. Appendix 3.2 ∎

If we carry out the same numerical example as in section 4, we obtain again the result given in proposition 6.

Lemma 8 *As γ increases, the values η_1^{**} increase.*

Proof. Appendix 3.3 ∎

If the workers' bargaining power on the LTCs decreases, the interval of values of η where the workers' welfare will be at a maximum with the L policy is smaller.

In this example, the policy $a = SL$ will never maximize the workers' welfare. In fact, if η is small, the policy L will be clearly preferred to the policy SL. The higher this bargaining power, the policy SL becomes more interesting from a workers' welfare point of view. In this last case the increase in the aggregate workers' utility is driven mainly by the utility of the workers with type $i < z$, by the utility of the workers engaged only with STCs .

To complete the analysis, we should look at whether the workers welfare maximizing policy is also the one that is optimal for firms. However, the analytical expressions become cumbersome when we add both utilities and profits. We therefore briefly illustrate this point with the help of a few numerical simulations.

Consider the total welfare, defined by

$$S^a = E\Pi^a + EU^a \tag{7}$$

Making the same numerical example as before, we find that total welfare attains a maximum with policy SL, only if γ and η are sufficiently low. If γ is high, the firms' gains from screening do not compensate the workers' lost due to short-term contracts. On the contrary, if η is high, the gain from the screening activity of the firms will be partially canceled by the high wages they have to pay to workers. In all other cases policy L maximizes total welfare.

We now check whether there are labor market interventions that are welfare improving. In particular, we allow the Social Planner to design a transfer policy from the firms to the workers and to regulate bargaining power.

The firms prefer to engage workers exclusively with short-term contracts only in particular cases when the RBP is positive and high. In that case, the Social Planner has to pay more attention to the bargaining of the short-term wage, to incentive the firms to move from this policy to a different one[8].

If the firms adopt policy L, then their interests coincide with the workers' ones, and this does not leave room for intervention.

On the contrary, if firms prefer to screen among workers (policy SL) the Social Planner must begin by checking whether this is harmful for workers. If this is the case, a tax on firms' surplus might be envisaged, corresponding to the difference between $E\Pi^{SL}$ and $E\Pi^{L}$. This tax could be used to redistribute the firms' benefits stemming from the screening policy, without eroding the firms' interest in pursuing it. A welfare improving way to use this fiscal receipt would be to redistribute it directly to the disfavored workers engaged with short-term contracts and not retained for a long-term one. Or, equivalently, to let each firm pay a lump-sum amount to every worker it let go after the probationary contract. This transfer would amount to

$$[1 - F(z)] \left(E\Pi^{SL} - E\Pi^{L} \right)$$

where $[1 - F(z)]$ is the probability that the firm matches a worker with a type more than z[9].

To put things clear, we illustrate with an example. Say this probability $[1 - F(z)]$ is 0.25. Then the expected waiting time for the firm to fill a vacancy is four periods. Hence, in each of the four periods in which the firm does not hire the worker it gives this worker one fourth of its excess profit. When it will finally find the matching worker, it will already have given away all the difference between the profit it does out of the SL policy and the one it would make in the first best of the economy.

This policy could be coupled or replaced by a policy of bargaining regulation, to incite the firms to make use of a policy L, or to compensate, through a higher short-term wage, the workers with an unstable situation.

6   Conclusions

In this paper, we have supposed that the only way to determine the quality of a particular match is "to form the match and experience it". That is, firms view the initial STC as a probationary stage where they may seek to select the right workers.

In this framework, we have studied the optimal firm policy, where policy is defined as the choice of the contract to offer to the heterogenous workers.

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[8] This policy could take the form of a minum wage on the short contracts.

[9] The firms engages in long-term contract only workers with an aility above the treshold z. See Section 3.

The firm has two possibilities as to the policy it may pursue. Either it offers only short-term contracts; or it only offers long-term contracts; or it may offer a short-term contract to begin with, switching to a long-term one if it is satisfied about the productivity of the job-workers pair. We supposed that the worker's bargaining power is different according to the type of contract considered.

In this framework, we show that firms could view STCs a way to screen the workers, in order to engage for a long-term contract only workers above a given threshold. This policy is profitable only if the costs, that come from re-open the vacancy each period and the probability to be unmatched, are compensated by the surplus from the life matching. The higher is the workers' bargaining power, the less is the surplus that it can get by waiting to find the good worker. The workers' welfare[10] is hardly at the maximum with this policy.

Moreover, the firms could find it profitable to engage all workers exclusively with short-term contracts. This policy is profitable only if relative bargaining power[11] is positive and high. If that is the case, the costs of re-opening each period a vacancy are compensated by the higher surplus that comes from the temporary matching.

If this last policy could be profitable for firms, it is never optimal from the total welfare[12] point of view. That value achieves the maximum with a policy of workers' discrimination thought the temporary market only in particular cases. Otherwise only a policy in which all the workers are engaged directly with long-term contracts is optimal.

We show that the Social Planner could regulate the labor market making more attention to the short-term wages and more specifically by introducing a system of transfers from the firms to the workers engaged with short-term contracts.

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[10] We defined the welfare as the sum of the workers' utilities.

[11] We have defined relative bargaining power to be the difference between the long-term bargaining power and the short-term one, i.e. $\gamma - \eta$.

[12] We defined total welfare like the sum of the workers' utilities and firms' expected profits in each possible policy.

Appendix 1: **Search Equilibrium on the SL policy**

1.1 Proof of Proposition 1.

We start the proof of the proposition by proving a simple Lemma on the equilibrium search strategy of the firm.

Lemma 9 *The set X of workers the firm accepts in SL contract is an interval of the form $(z, 1]$.*

Proof. insider any worker x in X. Given the definition of equilibrium, $xy > \delta EU$. Hence for any $x' > x$, $x'y > \delta EU$. ∎

From Lemma 1, we know the set of workers accepted by the firm is an interval $(z, 1]$. The search problem faced by the firm may thus be rewritten as follows:

$$\Pi^{SL} = \frac{-k + \alpha \left[(1-\eta) \int_0^1 xf(x)dx + (1-\gamma) \sum_{t=1}^{+\infty} \delta^t (\int_z^1 xf(x)dx) \right]}{1 - \delta(1 - \alpha \int_z^1 f(x)dx)}$$

Taking first-order conditions with respect to z,

$$-zf(z) \frac{\alpha \delta (1-\gamma)}{1-\delta} \left[1 - \delta(1 - \alpha \int_z^1 f(x)dx) \right] +$$

$$\delta \alpha f(z) \left[-k + \alpha(1-\eta) \int_0^1 xf(x)dx + \frac{\alpha \delta (1-\gamma)}{1-\delta} \int_z^1 xf(x)dx \right]$$

or

$$z = \alpha \left(\frac{\delta}{1-\delta} \int_z^1 (x-z)f(x)dx + \frac{1-\eta}{1-\gamma} \int_0^1 xf(x)dx \right) - \frac{k}{1-\gamma} \qquad (8)$$

To check that this solution is unique, observe that the left-hand side of equation (8) is increasing in z, raising from 0 to 1. On the other hand, the right-hand side of equation (8) is decreasing in z, falling from $\left[\frac{\delta}{1-\delta} + \frac{1-\eta}{1-\gamma} \right] \int_0^1 xf(x)dx - \frac{k}{1-\gamma}$ to $-\frac{k}{1-\gamma}$. Hence there exists a unique solution to this equation.

1.2 Proof of Lemma 2

Consider the equation defining z:

$$z = \alpha \left(\frac{\delta}{1-\delta} \int_z^1 (x-z)f(x)dx + \frac{1-\eta}{1-\gamma} \int_0^1 xf(x)dx \right) - \frac{k}{1-\gamma} \qquad (9)$$

Implicit differentiation shows that

$$\frac{\partial z}{\partial \delta} = \frac{\alpha(1-\beta) \int_z^1 (x-z)f(x)dx \backslash [1-\delta]^2}{[1-\delta] + \alpha\delta[1-F(z)]} \geq 0$$

Similarly, we obtain

$$\frac{\partial z}{\partial \alpha} = \frac{\frac{\delta}{1-\delta} \int_z^1 (x-z)f(x)dx + \frac{1-\eta}{1-\gamma} \int_0^1 xf(x)dx}{[1-\delta] + \alpha\delta[1-F(z)]} \geq 0$$

We also obtain

$$\frac{\partial z}{\partial \eta} = -\frac{\alpha \int_0^1 xf(x)dx \backslash (1-\gamma)}{[1-\delta] + \alpha\delta[1-F(z)]} \leq 0$$

and

$$\frac{\partial z}{\partial \gamma} = \frac{\left[\alpha \int_0^1 xf(x)dx - k \right] \backslash (1-\gamma)^2}{[1-\delta] + \alpha\delta[1-F(z)]} \geq 0 \text{ iff } \int_0^1 xf(x)dx \geq k$$

Finally,

$$\frac{\partial z}{\partial k} = -\frac{1 \backslash (1-\gamma)}{[1-\delta] + \alpha\delta[1-F(z)]} \leq 0$$

Appendix 2: Optimal Contract

2.1 Proof of Proposition 3

Policy SL is an equilibrium if $E\Pi^{SL} > E\Pi^L$, we find:

$$E\Pi^{SL} > E\Pi^L \Leftrightarrow \eta \leq \eta^* = \frac{(1-(1-\alpha)\delta)\left(\int_z^1 xf(x)dx - \int_0^1 xf(x)dx \right) + F(z)\left[\alpha \int_0^1 xf(x)dx - k(1-\delta) \right]}{(1-(1-\alpha)\delta)\left(\int_z^1 xf(x)dx - \int_0^1 xf(x)dx \right) + \alpha F(z) \int_0^1 xf(x)dx}.$$

13

We define $E\Pi^S$ as the firm's expected profit if it engages workers exclusively on short-term conditions.

$$
\begin{aligned}
E\Pi^S &= -k + \alpha \left[\pi_0 + \delta E\Pi^S \right] + \delta(1-\alpha)E\Pi^S \Leftrightarrow \\
E\Pi^S &= \frac{-k + \alpha\pi_0}{1-\delta}
\end{aligned}
$$

The set σ_x is no empty if $E\Pi^S \geq E\Pi^L, E\Pi^{SL}$. If we look at the first condition, we find:

$$
E\Pi^S > E\Pi^L \Leftrightarrow -\frac{\alpha\delta \left[k + (1-\alpha)(1-\gamma) \int_0^1 xf(x)dx \right]}{(1-\delta)\left[1-\delta(1-\alpha) \right]} \geq 0
$$

this condition is never true.

2.2 Proof of Proposition 4

$E\Pi^{SL} \geq E\Pi^L \Leftrightarrow \eta \leq \eta_1^*$, where

$$
\eta_1^* = \alpha\delta F(x)(1-\gamma) + (\gamma-\delta)(1-\delta(1-\alpha))
$$

$$
\frac{\int_0^1 xf(x)dx + \delta(1-\gamma)(1-\delta(1-\alpha)) \int_z^1 xf(x)dx - kF(z)(1-\delta)\delta]}{\left[(1-\delta)(1-(1-\alpha)\delta) \int_0^1 xf(x)dx \right]} \tag{10}
$$

2.3 : Proof Lemma 5

$\frac{\partial \eta_1^*}{\partial \gamma} \geq 0$ iff $(1-\delta F(z)) \int_0^1 xf(x)dx \geq \delta \int_{z'}^1 xf(x)dx$, where η_1^* is given by equation (10).

Appendix 3: Welfare

To analyze the welfare properties, we look at the sum of the workers' utilities in each possible policy. To avoid mathematical complication, we assume that $M = 1$. With this hypothesis workers/firms have exactly the same probability to match a firms/workers. Total surplus can be written as

$$
\underset{a}{Max} S^a = F(z)EU_{x<z}^a + (1-F(z))EU_{x\geq z}^a \tag{11}
$$

where $a = sl, l$.

For policies $a = L$, the utility of all workers' type will be the same and they take the value:

$$
EU^L = \alpha \sum_{t=0}^{+\infty} \delta^t w_{sl} + \delta(1-\alpha)E\Pi^L \tag{12}
$$

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On the contrary, in the SL policy, firms will engage for life only workers with type more than z. Consequently, the sum of the workers utilities in policy $a = SL$ will be

$$EU^{SL} = F(z)EU^{SL}_{x<z} + (1 - F(z))EU^{SL}_{x \geq z}$$

where

$$EU^{SL}_{x \geq z} = \alpha(w_o + \sum_{t=1}^{+\infty} \delta^t w_{sl}) + \delta(1 - \alpha)EU^{SL} \qquad (13)$$

and $EU^{SL}_{x<z}$ is given by equation (14)

$$EU^{SL}_{x<z} = \alpha(w_o + \delta EU^L) + \delta(1 - \alpha)EU^S \qquad (14)$$

3.1 Proof of Proposition 6.

Policy L is maximal for problem (6) if $EU^L \geq EU^{SL}, EU$.

$EU^L - EU^{SL} \Leftrightarrow \dfrac{F(z)\int_{z}^{1} xf(x)dx + ((1 - F(z)) + cF(z)(1 - \alpha))}{(1 - c)[1 - (1 - \alpha)c]} \geq 0,$

This condition is always true.

3.2 Proof of Proposition 7.

$EU^{SL} \geq EU^L \Leftrightarrow \eta \geq \eta_1^{**}$ where

$$\eta_1^{**} = \frac{\gamma \left[\int_{0}^{1} xf(x)dx - \delta(1 - F(z)) \int_{z}^{1} xf(x)dx \right]}{F(z)\left[1 - (1 - \alpha)\delta\right] \int_{0}^{z} xf(x)dx + (1 - \delta)(1 - F(z)) \int_{z}^{1} xf(x)dx}; \qquad (15)$$

3.3 Proof of Lemma 8.

$\dfrac{\partial \eta_1^{**}}{\partial \gamma} = \dfrac{[1 - \delta(1 - F(z))] \int_{z}^{1} xf(x)dx + \int_{0}^{z} xf(x)dx}{(1 - \delta)[1 - F(z)] \int_{z}^{1} xf(x)dx + F(z)[1 - (1 - \alpha)\delta] \int_{0}^{z} xf(x)dx} \geq 0$ always, η_1^{*} is given

by equation (15).

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