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INDUSTRY PROFITS AND MARKET SIZE  
UNDER BILATERAL OLIGOPOLY

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**Abstract**

We show that, contrary to the key result of the standard Cournot-Nash oligopoly model, industry profits can increase with the number of firms if input prices are not exogenous but are determined by bargaining in bilateral oligopoly. The relationship between industry profits and market size is shown to depend on the relative bargaining power of the upstream and downstream agents.

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## 1. Introduction

It is a cornerstone result of the standard Cournot model of oligopoly that industry profits will decrease as the number of firms competing in the product market increases. The nature of this relationship influences, *inter alia*, the incentives of firms both to merge and to deter entry by new firms: it is a fundamental determinant of market structure. In this paper, we show that under bilateral oligopoly, when downstream firms costs are not exogenous but are determined through (Nash) bargaining with upstream agents, the relationship between industry profits and market size depends on the relative bargaining power of the downstream and upstream agents. If the former have sufficient bargaining power, then there is a range over which industry profits increase with the number of firms competing in the product market.

As far as we are aware, this is a new result. Dowrick (1989) considers a bilateral oligopoly - in which unions act as the upstream agent - and shows how the bargained wage varies with market size, but does not focus on the relationship between profits and the number of firms. Horn and Wolinsky (1988) examine a differentiated oligopoly with upstream agents (unions) and downstream firms, but assume a duopolistic market.<sup>1</sup>

The rest of this paper is organized as follows. In Section 2, we outline the basic model and in Section 3 we draw out the implications of the model for the relationship between industry profits and market size. Section 4 concludes.

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<sup>1</sup> Similarly, Naylor (1999) considers unionized oligopoly in the context of international trade and economic integration, but does not allow the number of firms to vary.

## 2. The Model

We follow Horn and Wolinsky (1988) in supposing that the upstream agents are firm-specific trade unions bargaining with firms over the wage rate. We analyze a non-cooperative two-stage game in which  $n$  identical firms produce an identical good. In the first stage (the labor market game), each firm independently bargains over its wage with a local labor union: bargaining is decentralized. The outcome of the labor market game is described by the solution to the  $n$  union-firm pairs sub-game perfect best-reply functions in wages. In the second stage (the Cournot product market game), each firm sets its output given pre-determined wage choices from stage 1 to maximize profits. We proceed by backward induction.

(i) *Stage 2: the product market game*

Let linear product market demand be written as:

$$p = a - bX, \quad (1)$$

where  $X = \sum_{i=1}^n x_i$ . Profit for the representative firm  $i$  can be

written as:

$$\mathbf{p}_1 = \left[ a - b \sum_{i=1}^n x_i - w_i \right] x_i, \quad (2)$$

where  $w_i$  is the outcome of the wage bargain for union-firm  $i$ . In this short-run analysis, we exclude non-labor costs. We also assume a constant marginal product of labor, and set this as a numeraire.

Under the Cournot-Nash assumption, differentiation of (2) with respect to  $x_i$  yields the first-order condition for profit maximization by firm  $i$ , from which it is straightforward to derive firm  $i$ 's best-reply function in output space as:

$$x_i = \frac{1}{2b} \left[ a - w_i - b \sum_{\substack{j=1 \\ j \neq i}}^n x_j \right]. \quad (3)$$

Solving across the  $n$  first-order conditions, the  $n$  best-reply functions can be re-written as sub-game perfect labor demand equations. From equation (3) for example, the expression for firm  $i$ 's labour demand is

$$x_i = \frac{1}{(n+1)b} \left[ a - nw_i + \sum_{\substack{j=1 \\ j \neq i}}^n w_j \right]. \quad (4)$$

It is useful to express firm  $i$ 's profits in terms of the vector of all firms wages. Substituting (4) in (2), we obtain

$$\mathbf{p}_i = \frac{1}{(n+1)^2 b} \left[ a - nw_i + \sum_{\substack{j=1 \\ j \neq i}}^n w_j \right]^2. \quad (5)$$

From (5), it follows that in symmetric equilibrium, with  $w_i = w$ ,

$$\mathbf{p}_i = \frac{1}{(n+1)^2 b} [a - w]^2, \quad \forall i, \quad (6)$$

where  $w$  is the outcome of the Stage 1 wage-bargaining game. It follows from (6) that, in equilibrium, industry profits are given by

$$\sum \mathbf{p} = \sum_{i=1}^n \mathbf{p}_i = \frac{n}{(n+1)^2 b} [a - w]^2. \quad (7)$$

We note that if  $w$  is given exogenously (or if unions have no bargaining power) then, with  $w = \bar{w}$  in (7), industry profits are falling in  $n$ , the number of firms in the industry, as

$$\frac{\partial(\sum \mathbf{p})}{\partial n} = -\frac{n-1}{(n+1)^3 b} [a - \bar{w}]^2 < 0, \quad (8)$$

for  $n > 1$ .

(ii) *Stage 1: the labour market game*

We assume that the representative trade union has the objective of rent-maximization. For union  $i$  bargaining with firm  $i$ , the union utility function is written as

$$U_i = [w_i - \bar{w}]x_i, \quad (9)$$

where  $\bar{w}$  denotes the wage which would obtain in a competitive non-unionised labour market. Under the assumption of a right-to-manage model of Nash-bargaining over wages, we write the maximand as:

$$B_i = U_i^b \mathbf{p}_i^{1-b}, \quad (10)$$

where we assume that disagreement payoffs are zero.  $\mathbf{b}$  represents the union's Nash-bargaining power in the asymmetric wage bargain.

Substituting (4), (6) and (9) in (10) yields

$$B_i = \frac{1}{(n+1)^{2(1-b)} b} [w_i - \bar{w}] \left[ a - nw_i + \sum_{\substack{j=1 \\ j \neq i}}^n w_j \right]^{2-b}. \quad (11)$$

The first order condition derived from the Nash maximand is

$$\frac{\partial B_i}{\partial w} =$$

$$\frac{1}{(n+1)^{2(1-b)}b} [w_i - \bar{w}]^{b-1} \left[ a - nw_i + \sum_{\substack{j=1 \\ j \neq i}}^n w_j \right]^{1-b} \quad (12)$$

$$\left\{ \mathbf{b} \left[ a - nw_i + \sum_{\substack{j=1 \\ j \neq i}}^n w_j \right] - (2 - \mathbf{b})n[w_i - \bar{w}] \right\} = 0$$

from which it follows that, in symmetric sub-game perfect equilibrium,

$$w = w_i = \bar{w} + \frac{\mathbf{b}[a - \bar{w}]}{2n - \mathbf{b}(n-1)}. \quad (13)$$

Substituting (13) in (7) gives equilibrium industry profits of

$$\sum \mathbf{p} = \frac{(2 - \mathbf{b})^2 n^3}{(n+1)^2 [2n - \mathbf{b}(n-1)]^2 b} [a - w]^2. \quad (14)$$

### 3. Industry profits and market size

We now investigate how industry profits vary with the number of firms in the market. Differentiating (14) with respect to  $n$ , we obtain

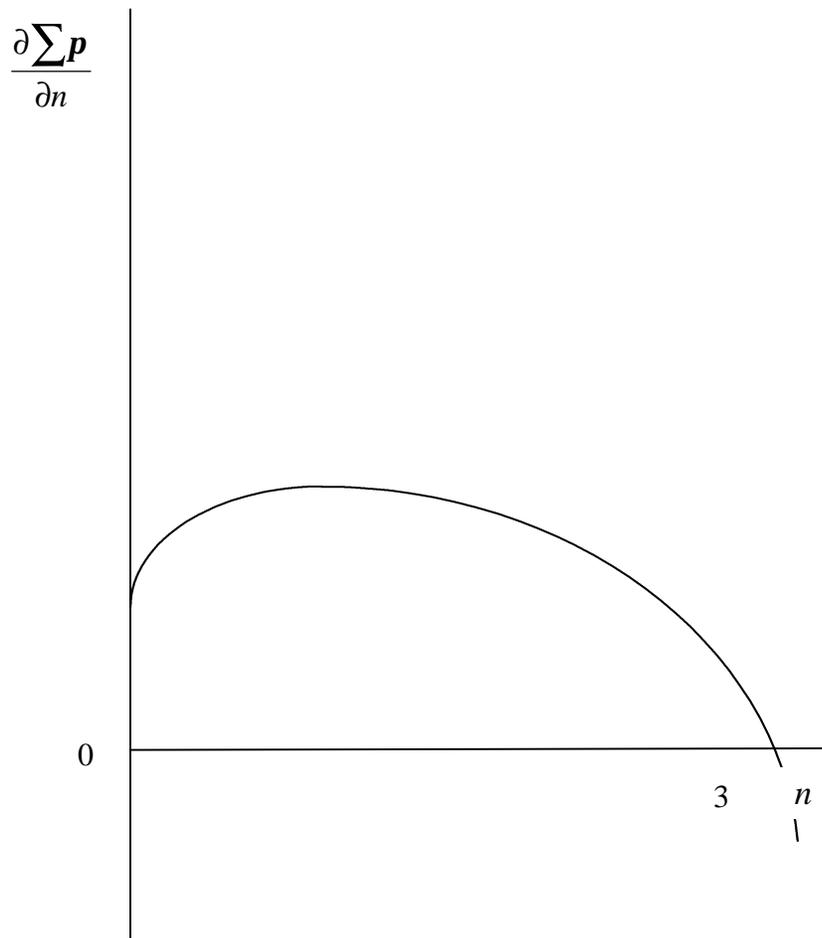
$$\frac{\partial(\sum \mathbf{p})}{\partial n} = \frac{(2 - \mathbf{b})^2 n^2}{(n+1)^3 [2n - \mathbf{b}(n-1)]^3 b} [-2(n-1)n + \mathbf{b}(3 + n^2)][a - w]^2, \quad (15)$$

which is positive implying that industry profits are non-decreasing in the number of firms if the following condition is satisfied:

$$(2 - \mathbf{b})n^2 - 2n - 3\mathbf{b} \geq 0. \quad (16)$$

Initially, consider condition (16) for the special case that  $\mathbf{b} = 1$ . In this case, the condition is satisfied for  $-1 \leq n \leq 3$ . It follows that for this monopoly union case industry profits are at a maximum when  $n = 1$  depicts (15) for this case of  $\mathbf{b} = 1$ .

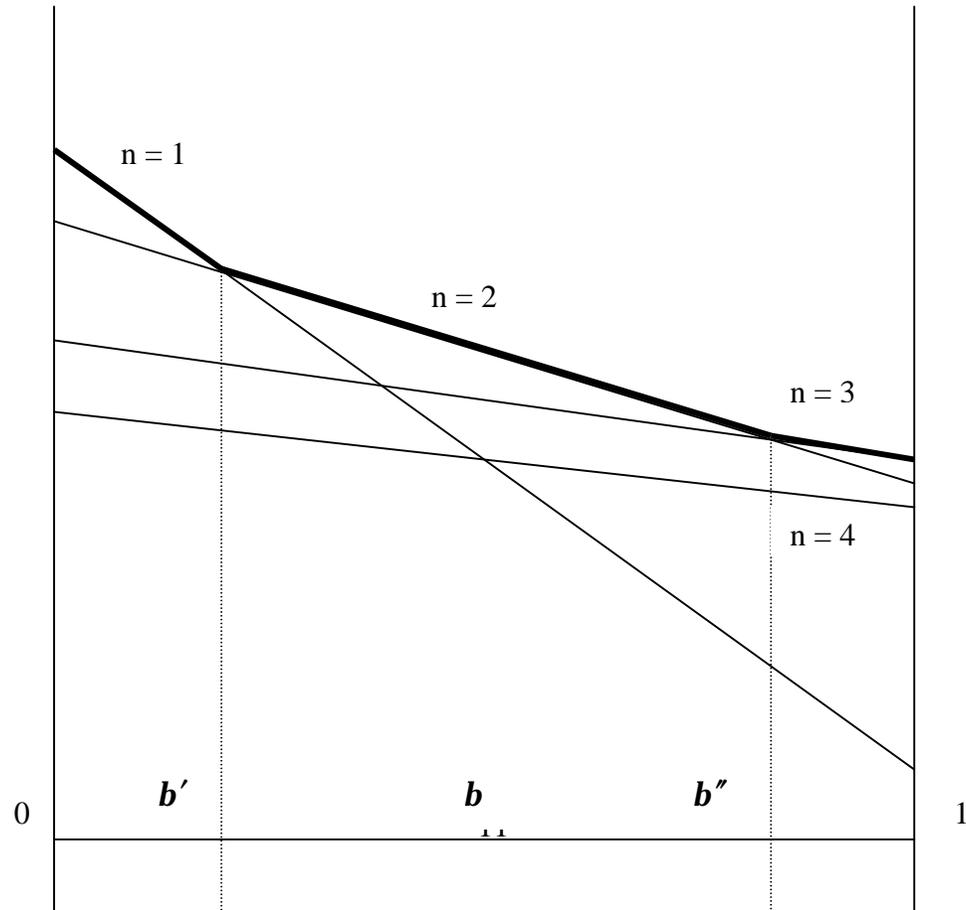
**Figure 1** The derivative of industry profits with respect to  $n$ ,  
for  $b = 1$ .



We now address the question of how the industry profit-maximising value of  $n$  varies with  $\mathbf{b}$ . We do this by evaluating equation (14) for different particular values of  $n$  and solving for the critical values of  $\mathbf{b}$  associated with intersections of the industry profit functions for the different values of  $n$ . The industry profit functions for  $n = 1, 2, 3$  and  $4$  are plotted against  $\mathbf{b}$  in Figure 2. In bold, we highlight that part of each profit function associated with maximum industry profits, given the value of  $\mathbf{b}$ .



**Figure 2** Industry profits and bargaining power for particular values of  $n$ .



From Figure 2, we can see that, in equilibrium, industry profits are at a maximum:

- (i) when  $n = 1$  if  $0 \leq \mathbf{b} < .25$
- (ii) when  $n = 2$  if  $.25 < \mathbf{b} < .8$
- (iii) when  $n = 3$  if  $.8 < \mathbf{b} \leq 1$

At the critical value  $\mathbf{b}' = .25$ , industry profits are equal for  $n = 1$  and  $n = 2$  and for the critical value  $\mathbf{b}'' = .8$ , industry profits are the same for  $n = 2$  and  $n = 3$ .

It follows that the industry profit-maximizing number of firms is increasing, up to a maximum of  $n = 3$ , in the extent of union bargaining power. The intuition for the result is straightforward. In the standard oligopoly model, an increase in the number of firms unambiguously reduces industry profits through increased product market competition. For the bilateral oligopoly case developed in the current paper, this profit-reducing product market demand effect still operates, but is offset by a profit-enhancing effect within the labour market. The increase in  $n$  has the effect of increasing the elasticity of the derived demand for labour and this leads unions to bargain for lower wages. Notice from (13) that, in equilibrium, the bargained wage is decreasing in  $n$ . If  $\mathbf{b}$  is small or if  $n$  is large then this effect is relatively insignificant. But if  $\mathbf{b}$  is sufficiently large, and  $n$  sufficiently small, then the profit-enhancing labour market effect dominates and profits are increasing in  $n$ .

#### 4. Conclusions

We have shown that in a unionized bilateral oligopoly with decentralized bargaining, industry profits are initially increasing in the number of firms,  $n$ , in the product market if unions have sufficient bargaining power,  $\mathbf{b}$ . The standard oligopoly result is turned round because an increase in  $n$  causes a profit-enhancing fall in bargained

wages and this dominates the standard profit-reducing effect of an increase in  $n$  if  $\mathbf{b}$  is sufficiently large and  $n$  is sufficiently small. As we have focused exclusively on the case of the rent-maximizing union,<sup>2</sup> it can be shown that the results also obtain in a standard upstream firm/downstream firm setting.

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<sup>2</sup> If instead we allow for a more general Stone-Geary utility function, it can be shown that individual firms profits are also increasing in  $n$ , if unions place sufficient weight on the wage argument in their utility function (see Naylor, 2001).

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