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EXAMINING PREDICTIVE ACCURACY AMONG DISCOUNTING MODELS

Abstract

Both descriptive and normative arguments claim that the discount rate to be applied to public projects should be elicited from individual intertemporal preferences. We present a methodology to analyze data from experimental surveys on intertemporal preferences. Focusing on the exponential and the hyperbolic discounting models, we model the experimental data published by Thaler (1981) by means of different specifications. Standard measures of goodness of prediction are then applied to fitted data to select among alternative specifications. We first present our approach by applying it to simulated data. We then present a procedure for statistical estimation of the sample discount rate, testing four specifications. The estimation procedure we adopt can be easily extended to other functional forms, allowing for non-linearity of the valuation function, inclusion of socio-economic individual characteristics as regressors, and different specifications for the discounting model.

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Examining predictive accuracy among discounting models

Evaluation of public projects and policies relies on some criterion of economic efficiency, either in the form of cost-benefit analysis, or, when it seems more suitable, of cost efficiency analysis. Both tests require that the option with higher net value should be selected. Unfortunately, computation of costs and benefits is often problematic: one reason is that many public projects involve costs and benefits that belong to different outcome domains. This problem is typical of projects that deal with risks to the environment or public health: for example, financial benefits may be achieved by incurring environmental costs, or health benefits are achieved by losses in financial terms. In these circumstances the usual strategy is to translate all costs and benefits into a single domain, normally the monetary domain, so that projects can be effectively compared. Several techniques are currently available to implement this "translation" procedure: the most widely used are the hedonic pricing and the contingent valuation methods. The former can be used when it is possible to refer to some parallel market prices for the good to be evaluated: for example, insurance prices for outcomes in the health domain, or residential housing prices for outcomes in the environmental domain. When it is not possible to refer to any existing market, analysts apply the contingent valuation method: it is a procedure that requires the direct elicitation of the value that individuals attach to the public good of interest. This procedure is analogous to the elicitation of preferences used in decision analysis and experimental economics to investigate the patterns of behavioral decision making: its reliability rests in large part on the accuracy of the experimental setting (cfr. Arrow et al., 1993).

Besides the level of costs and benefits, timing of implementation and duration of the effects of the projects are another important element of the decision. For example, suppose that project A and project B give rise to the same costs and benefits: the only difference is that benefits produced by project A are available before those produced by project B. Then project A would be preferred. Conversely, if, ceteris paribus, costs of project A are to be borne before the (same amount of) costs of project B, then project B would be preferred. Even after all costs and benefits arising from a specific project are expressed in the same monetary terms, it is still necessary to use another conversion procedure to reduce cash flows spanning different time periods. This procedure is called discounting: the way it actually operates depends on behavioral assumptions that will be explored more thoroughly in the next section. Here we only observe that the standard discounting procedure implies application of the same discount rate, usually the official rate of discount, for different variable dimensions, for gains or losses, and for the short or the long run.

The validity of a single discount rate is questioned from a descriptive point of view. The rationale behind using the official rate of discount is that, assuming perfect capital markets, everyone should behave the same way at the margin. Firms and individuals should borrow and lend until their marginal rate of substitution (MRS) between present and future consumption is equal to the interest rate. If a consumer failed to act as the theory predicts, there would be some way to rearrange his consumption plan to make him better off. For example, if his MRS is higher than the interest rate, the individual would find it attractive to trade some future consumption with present consumption– while the opposite holds if his MRS is lower. So, the market interest rate should reflect perfectly the intertemporal preferences of individuals. Yet, as pointed out by Lind (1990), we can observe that individuals trade at very different interest rates: for example, people may at the same time save at some interest rate, and charge consumption on credit cards at a much higher interest rate. The reason may not just be a matter of transaction costs (easy access to one's own funds is obviously the basic motivation for using credit cards) which invalidate the assumption of perfect capital markets, but it can also depend on the individuals' desire to maintain separate budgets, as a means of control on their spending. A typical example may be the limited amount that people may decide to carry with them when going to the horse races. If "people adopt rules and divide assets into separate budgets to facilitate actions that require self-control, then it also follows that individuals do not necessarily change levels of present and future consumption to equalize their marginal rates of substitution with the marginal rate (i.e., the interest rate) at which they can transform present into future income"(Lind, cit., p. S20). "Therefore, market rates that determine consumers' potential rates of transformation may tell us nothing about people's rates of time preference" (*ibidem).* Lind suggests use of the consumer's rate of time preference, that may be context dependent, rather than use of the official rate of discount in the capital market. This position is also supported by Arrow et al. (1996), who argue that discount rates should be based on how individuals trade off present to future consumption, and admit that discount rates can change with the time horizon to reflect the judgement and behavior of individuals. Given uncertainties in identifying the correct rate of discount, they also suggest that it is appropriate to use a range of discount rates, and that this range should be applied to all analyses on (similar) public projects. Just as present preferences for non-market goods are elicited with experimental methods, the same can be done for intertemporal preferences: so again, experimental methods may help to define a range of discount rates in the relevant setting.

Furthermore, application of the discounting technique to projects that produce effects in non-monetary domains has led some authors to claim even a normative shortcoming of the standard discounting procedure, when public projects have a high impact on health or the environment. According to this view, discounting *should* depend on the problem that is being analyzed: different circumstances would require not only different discount rates, but also different procedures. For example, the standard discounting technique implies that flows in the distant future are so heavily discounted that even huge amounts result in a negligible discounted present value. This would unduly penalize (promote) those projects that present extremely high benefits (costs) that are delayed into the distant future. Descriptive experiments have shown differing temporal preferences across monetary and non-monetary domains of health (Chapman, 1996), and air and ocean shore water quality (Guyse, Keller, and Eppel, 1999), and for different time horizons for protective investments (Kunreuther et al. 1998). These descriptive findings and the normative argument against a single discount rate have led to the proposition of alternative discounting models, that will be briefly reviewed in section 1.

In section 2 we present a methodology to analyze data from experimental surveys on intertemporal preferences. Using a published dataset, we examine exponential and hyperbolic discounting models fit exactly to different certainty equivalent judgments from aggregated data. Section 3 contains measures for selecting the best performing model. In section 4 we generate a simulated dataset generated from the discount rates elicited by applying the exponential and hyperbolic models in section 2. In section 5 we examine the predictive accuracy of the models for the simulation dataset. In section 6 we test different econometric specifications to estimate the sample discount rate, and select the model with best predictive accuracy. Section 7 concludes the paper.

1. Discounting models

Most discounting models are based on the behavioral assumption that people prefer to experience pleasurable experiences as soon as possible, and to delay painful experiences.

While the first hypothesis, impatience, seems fairly robust to empirical observation and experimental tests, the second one, procrastination of pain, is more controversial. In fact, it can often be observed that people may prefer to get rid sooner of some unpleasant experience, rather than wait (it may be argued that in so doing, they are avoiding the unpleasant experience of anticipating the future unpleasant experience.)

We will see in the following that different sets of behavioral assumptions generate different types of discounting models. We will refer to the general approach taken by Fishburn and

Rubinstein (1982) in examining the effect of the time of realization of an outcome to the relative desirability of the outcome. They study the implications of various axioms for a (weak) preference relation. They start from a simple axiomatic structure: given a non-degenerate real interval X (the set of outcomes); and either a set T of successive non-negative integers, or an interval T of non-negative numbers (the set of time points), consider the topological space X×T (the dimensions of outcomes and time). Consider the axioms:

A1. ³ is a weak order on X´T;
A2. If x > y then (x, t) > (y, t);
A3. {(x, t): (x, t) ³ (y, s)}, and {(x, t): (x, t) £ (y, s)}
are closed in the product topology on X´T.
A4:If s < t then x > 0 ® (x, s) > (x, t); x = 0 ® (x, s) ~ (x, t);
x < 0 ® (x, s) < (x, t).

The first three axioms ensure continuity, monotonicity, and ordering of outcomes in the space X×T; the fourth is the behavioral assumption of impatience for positive outcomes, and procrastination for negative outcomes. Fishburn and Rubinstein show that this axiomatic structure implies the existence of a real valued function u on X×T that is monotonic in x and t; continuous, and increasing in x; continuous in t if t is continuous; decreasing (constant, increasing) in t if x is greater (equal, less) than zero.

Fishburn and Rubinstein do not present a specific functional form associated with the general set of axioms A1-A4. A representation function is instead provided when an axiom of stationarity is added to the previous set of axioms:

A5. If (x, t) R (y, t+d) then (x, s) R (y, s+d).

The model implied by this axiomatic structure assumes the form

E1. $a^t f(x)$,

known as the *exponential discounting model* when f is linear on x. So receiving \$10 with a t-period delay would be equivalent to $(1/1+\delta)^t$ \$10 today, where δ is the discount rate.The function f need not necessarily be linear, though. Fishburn and Rubinstein show that the representation holds with f concave as well, as when f is a risk averse von Neumann-Morgenstern utility function.

Relaxing the axiom of stationarity, and substituting it with a weaker axiom of separability (*Thomsen condition*), Fishburn and Rubinstein obtain another functional form, that still allows one to separate the effect of time preference from outcome preference:

E2. $r(t) f(x)$,

with ρ, the preference functional over time, continuous, positive and decreasing in t; and f, the preference functional over outcomes, continuous and increasing in x, with f(0)=0. This formulation allows the functional form for costs to be different from that for benefits.

This representation is compatible with the market segmentation approach (cfr. Benzion et al., 1989), that assumes different discount rates for different types of moves from some equilibrium position. In this approach discounting depends not only on the sign of the monetary amount (i.e., a payment or a receipt), but also on the sign of the temporal movement (i.e. an anticipation or a postponement of the cost or the benefit). This gives four different discount rates depending on four possible changes from the equilibrium position. The market segmentation hypothesis is empirically supported by observation of discrepancies between borrowing and lending rates, and can be modeled by means of a concave utility function. It is also consistent with the Anticipated Discount model introduced by Loewenstein (1987), that substitutes a Prospect Theory (cfr. Kahneman and Tversky, 1979) type of utility function for the standard von Neumann-

Morgenstern function in the discounting model; we will describe this model later.

In the market segmentation approach the discount rate is dependent on the direction of the movement (gain or loss, anticipation or delay), but the absolute level of the variation is assumed not to affect discounting. This assumption is dropped in another approach, defined by Benzion et al. as the *Added Compensation* approach, that allows the discount rate to vary with both the sign and the actual level of the change from the equilibrium position.

The Fishburn and Rubinstein model with the *Thomsen condition* holds in the continuous time setting, but it does not hold in a discrete time setting without invoking a more complex set of axioms. Another approach is to adopt another separability axiom, that can be defined as a utility independence condition:

A6. If (x, s) R (y, s), then (x, t) R (y, t).

According to this assumption, preferences over outcomes are independent of preferences over time periods. It can be considered as an intertemporal consistency axiom.

A stronger independence axiom, proposed by Harvey (1986), is the stretching axiom: it states that the ordering of outcomes in two periods depends on the relative difference (the ratio) between two periods.

A7. *If (x, s) R (y, t), then* $(x, d^x s)\ R\ (y, d^x t)$.

The set of axioms A1-A4 plus the axiom A7 supports the following representation, known as the *hyperbolic model*:

E3. $[1/(1+t)^g]\ f(x)$,

where $g > 0$ is a parameter that represents individuals' intertemporal preferences. The hyperbolic model was first axiomatized by Harvey (1986) to provide a solution to the problem of the excessive discounting of distant future flows implied by the exponential discounting model.

2. Test of the models

We now present a methodology to analyze experimental data from surveys on intertemporal preferences. Since this demonstration is to be considered as illustrative of the method, we chose to apply it to the dataset published by Thaler (1981).

The standard approach used in decision theory to analyze this type of data has been to apply some statistical test (usually non-parametric, but also some parametric models have been applied, see Benzion et al. (1989) to test the validity of specific assumptions of different models). The approach we will use here instead is more general, in that it considers different models as estimators of the data drawn from the elicitation procedure of the experiment. In the review presented in the preceding section we have examined two main discounting models: the exponential model and the hyperbolic model. In the present application we analyze the performance of these two models, assuming $f(x)$ is linear in the monetary outcome.

There are different procedures to elicit individuals' intertemporal preferences. A choice procedure requires the decision maker to choose between an amount to be received (or paid) now, and another specified amount to be received (paid) some specified time later. Another procedure, called the matching method, requires the individual to assess the amount that would make her indifferent between getting some given amount now, and getting that amount at some specified later time period. Drawing from results in the contingent valuation literature, we can infer that the former procedure is the easiest for the respondent, but it is also less informative. While the matching method produces precise data points, the choice method only generates dichotomous ordering type of data, and requires more observations for an efficient statistical analysis.

Also, the amounts may be stocks, to be received or paid at a point in time, or flows, to be received or paid along a time stream.

The latter setting has been analyzed by Loewenstein and Prelec (1993), who argue that preferences over sequences of outcomes may also be affected by the distribution of the outcomes along the time dimension, in addition to the absolute level of the amounts to be received or paid. A descriptively valid analysis of preferences over sequences of outcomes would require more complex behavioral assumptions than those we considered in the preceding section. To simplify the analysis, in this paper we only consider intertemporal preferences over stocks, rather than flows of outcomes.

The data published by Thaler (1981) are medians of amounts elicited using the matching method: therefore, we have data point observations, that we can use for our illustrative purpose. We have four subsamples, each of them was presented with a given amount now to be matched with some amount in three months, one year, and three years for a total of nine matching points for each subsample. The dataset is represented in Table 1: the left column contains the M_0 present amounts proposed to each subsample. The other columns contain the (median) amounts expressed by respondents when asked the outcome M_t that would have made them indifferent between getting a given M_0 now, or getting M_t later. For example, scenario A required subjects to consider amounts to be gained now and state an indifferent amount to be received in three months (M3), one year (M12), and three years (M36), respectively. The first three subsamples were presented with gains, i.e., amounts to be received, now or later; the last group was instead presented with a loss, i.e., a payment to be sustained now or later.

Table 1. Median Amounts Matching Monetary Gain/Loss M_0 Now with Delayed Amount M_t (from Thaler, 1981)

	Amount Now, M_0	***Matching Amount, M_t, Delayed by t Months***		
Scenario A	***Now***	***3 Months***	***12 Months***	***36 Months***
(gains)	15	30	60	100
	250	300	350	500
	3000	3500	4000	6000
Scenario B	***Now***	***6 Months***	***12 Months***	***60 Months***
(gains)	75	100	200	500
	250	300	500	1000
	1200	1500	2400	5000
Scenario C	***Now***	***1 Month***	***12 Months***	***120 Months***
(gains)	15	20	50	100
	250	300	400	1000
	3000	3100	4000	10000
Scenario D	***Now***	***3 Months***	***12 Months***	***36 Months***
(losses)	-15	-16	-20	-28
	-100	-102	-118	-155
	-250	-251	-270	-310

If the individual is indifferent between M_0 and M_t, the discounted present value of M_t must be equal to M_0. Assuming that a particular model (exponential or hyperbolic) holds, it is then possible to calculate the implicit discount rate. For example, if individuals are indifferent between \$15 now and \$30 in three months, the implicit monthly rate of discount for the exponential model is the ***d*** that solves the following equation:

$\$15 = \$30/(1+d)^3$,

i.e., $d = 0.260$.

The implicit discount rate for the hyperbolic model is instead the γ that solves the following equation:

$\$15 = \$30/(1+3)^g$,

i.e., $g = 0.500$.

Thaler observed that the implicit monthly discount rates calculated from the exponential discounting model from the elicited matching values present a pattern far from uniform: generally they decrease as the time length and the amount levels increase. When instead the hyperbolic discounting model is applied to the same data, we do not observe a clear pattern. Table 2 shows the implicit (monthly) discount rates inferred from each model:

Table 2A. Implicit (Monthly) Discount Rates δ from Thaler's Data (Exponential)

0 Month\s	**1 Month**	**3 Months**	**6 Months**	**12 Months**	**36 Months**	**60 Months**	**120 Months**
Scenario A Gains							
		.260		.063	.053		
		.122		.028	.024		
$		.054		.019	.019		
Scenario B Gains							
$75			.101	.063		.077	
$250			.085	.059		.059	
$1200			.032	.023		.024	
Scenario C Gains							
$15	.101			.063			.011
$250	.106			.040			.024
$3000	.016			.012			.010
Scenario D Losses							
-$15		.022		.007	.001		
-$100		.024		.014	.006		
-$250		.017		.012	.006		

Table 2B. Implicit (Monthly) Discount Rates γ from Thaler's Data (Hyperbolic)

0 Months	1 Month	3 Months	6 Months	12 Months	36 Months	60 Months	120 Months
Scenario A Gains							
$15		.500		.132	.111		
$250		.540		.131	.112		
$3000		.525		.192	.192		
Scenario B Gains							
$75			.148	.094		.115	
$250			.382	.270		.270	
$1200			.461	.337		.347	
Scenario C Gains							
$15	.415			.263			.047
$250	.469			.183			.112
$3000	.396			.289			.251
Scenario D Losses							
-$15		.047		.014	.003		
-$100		.112		.065	.030		
-$250		.173		.121	.060		

A quick look at the implicit rates of discount reported in Tables 2A and 2B would lead us to think that the hypothesis of a unique discount rate, to be applied to all projects, independently of their time horizon, and the level of the outcomes involved, should be rejected. This is in fact the conclusion reported by Thaler. A number of techniques for statistical analysis can be used to test the hypothesis in a more rigorous manner. A standard practice is to apply an analysis of variance to the implicit discount rates.

The method we propose here is to test if the values obtained from a particular model are a good predictor of the actual values. We want to test the hypothesis that the discount rate is independent of the time horizon and the outcome levels of the project. If the values obtained by applying some constant discount rate to the present values M_0 (by using that rate as if compounding interest over time on the base amount M_0) can be considered an acceptable prediction of the actual values M_t , the hypothesis can be accepted. If a constant discount rate yields an acceptable model, then this parameter value could be used in practical settings for guiding decision making or describing people's preferences.

In the following, we first show an application to simulated data, and then we will apply the method to values obtained through Maximum Likelihood estimation. In the simulation exercise, we multiply the 36 values M_0 by the implicit discount rates obtained from the elicited matching values: we obtain 36 vectors of simulated matching values, "predicted" by a specific discounting model (exponential or hyperbolic) given a specific constant discount rate. The model that gives the best prediction would be selected. Of course, best prediction does not mean good prediction: it may well be that even the best is so bad that we will anyway wish to reject the hypothesis of a constant rate of discount, at least under the exponential or the hyperbolic model, i.e., the two discounting models under analysis. Nevertheless, in practice, it may be necessary to specify a constant discount rate for analysis due to regulatory or administrative requirements, which our method will do. In the next section we will briefly describe the statistical criteria we will apply in our analysis.

3. Model selection

A commonly used criterion for goodness of prediction is the Mean Square Error:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(p_i - a_i)^2 ,$$

i.e., the average of the squared differences between predicted and actual values, or its root, which is often preferred because its value level is the same as that of the data. The model that minimizes this measure can be considered as the best predictor, and different models can be ranked according to this measure. However, it may also be useful to indicate if the prediction is good or bad in absolute terms. For this purpose, Theil (1966) suggested the U-statistic defined as:

$$U = \sqrt{\frac{MSE}{\sum a_i^2 / n}}\ .$$

This measure is zero for perfect predictions, and values close to zero can be considered as good predictors; however, it does not have an upper bound. Another measure, previously suggested by Theil, is the following:

$$U_b = \sqrt{\frac{MSE}{(\sum a_i^2 / n) + (\sum p_i^2 / n)}}\ .$$

It is well known (cfr. Maddala, cit., p. 346) that this measure may not give a correct ranking of the models; the problem is that, as also pointed out by Theil (1966), the coefficient is not uniquely determined by the MSE. However, it has the virtue of producing an index ranging in the bounded interval [0,1], where values close to one indicate worse predictors, and values close to zero better predictors. This feature makes the comparison of estimators easier, and we will therefore employ also this measure in our analysis, in addition to the other two.

The mean square error, and the other related measures described above, account for all deviations of the predicted values from the actual values. It can be useful to analyze more thoroughly the nature of these deviations. As suggested by Theil (1966) the MSE can be decomposed into three parts:

$$U^M = \frac{(\bar{p} - \bar{a})^2}{MSE} = \qquad \textit{Bias proportion}$$

$$U^R = \frac{(s_p - rs_a)^2}{MSE} = \qquad \textit{Regression proportion}$$

$$U^D = \frac{(1 - r^2)s_a{}^2}{MSE} = \qquad \textit{Disturbance proportion}$$

The first two measures indicate a systematic error in the prediction, while the last measure indicates a disturbance. As an alternative to the criterion of minimization of the whole MSE, the analyst may decide to adopt the criterion of minimization of the systematic error: both U^M and U^R tend to zero for the optimal predictor (cfr. Maddala (1977, p.345)).

An alternative measure is to consider the correlation coefficient *r* between predicted and actual values. As pointed out by Maddala (cit.), the disadvantage of this measure is that it does not penalize the predictor for systematic linear bias: so, for example, a model that always underpredicts actual values by 50%, receives a perfect score. However, if we combine the two criteria of maximizing the correlation and minimizing the systematic error in a sort of multicriteria analysis framework, we may overcome this problem. We will carry out this test too, and compare the results obtained with the other tests based on the minimization of MSE criterion.

Finally, it is also often recommended in the literature that the evaluation of models should be always supported by the analysis of the plots of the prediction error series. This analysis will serve as a "tie breaker" if the two classes of criteria described above were to give different results.

4. The simulation procedure

We generate a new dataset from the discount rates elicited by applying the exponential and the hyperbolic model to the data. The procedure consists of applying each rate of discount in Tables 2A and 2B to the whole series of 36 proposed M_t values which match present amounts M_0: this produces a vector of 36 elements of simulated matching values for each of the 36 discount rates. The set of simulated data for each model is therefore a 36×36 matrix. Tables 3A and 3B report the mean and standard deviation of the simulated values produced by each model. For example, in the upper left-hand corner of Table 3A, Thaler's subjects gave an actual median response of $30 for a three month delay to match

$15 now. When the 36 discount rates δ from Table 2A are each used to predict the actual amount, by the exponential formula:

$$\$15 = \left(\frac{1}{1+d}\right)^3 \text{ prediction},$$

the mean prediction was $18 and the standard deviation was $4. These measures are obviously too rough to provide an indication of the goodness of either model: the statistics refer to the data generated through a wide range of discount rates, and even a perfect prediction with one of the discount rates would be unrevealed by these data. Yet, it is worth observing that the hyperbolic model gives on average simulated values closer to the real ones, and especially that it avoids the problem of "explosive" results obtained by the exponential model, especially when the procedure involves long time periods.

Table 3A. Mean and Standard Deviation of Simulated Amounts: Exponential Model

Now	***Actual Values and Mean and Standard Deviation of Predicted Values***					
Scenario A Gains	***3 Months***		***12 Months***		***36 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
15	30	18 *(4)*[a]	60	45 *(84)*	100	15201 *(78975)*
250	300	297 *(71)*	350	755 *(1401)*	500	253357 *(1316250)*
300	3500	3559 *(855)*	4000	9066 *(16818)*	6000	3040292 *(1579499)*
Scenario B Gains	***6 Months***		***12 Months***		***60 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
75	100	111 *(69)*	200	227 *(420)*	500	67631271 (→+¥[b])
250	300	372 *(229)*	500	755 *(1401)*	1000	→+∞ (→+¥)
1200	1500	1783 *(1098)*	2400	3626 *(6727)*	5000	→+∞ (→+¥)
Scenario C Gains	***1 Month***		***12 Months***		***120 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
15	20	16 *(1)*	50	45 *(84)*	100	→+∞ (→+¥)
250	300	263 *(18)*	400	755 *(1401)*	1000	→+∞ (→+¥)
3000	3100	3161 *(215)*	4000	9066 *(16818)*	10000	→+∞ (→+¥)
Scenario D Losses	***3 Months***		***12 Months***		***36 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
-$15	-16	-18 *(4)*	-20	-45 *(84)*	-28	-15201 *(78975)*
-$100	-102	-118 *(28)*	-118	-302 *(560)*	-155	-101343 *(526500)*
-$250	--251	-297 *(71)*	-270	-755 *(1401)*	-310	-253358 *(131250)*

[a]Values in parentheses are standard deviations.
[b]Values greater than 10^7 are indicated as approaching infinity.

Table 3B. Mean and Standard Deviation of Simulated Amounts: Hyperbolic Model

	Actual Values and Mean and Standard Deviation of Predicted Values					
Scenario A Gains	***3 Months***		***12 Months***		***36 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
	30	21 *(5)*[a]	60	29 *(13)*	100	39 *(25)*
	300	347 *(81)*	350	478 *(214)*	500	656 *(425)*
	3500	4169 *(969)*	4000	5740 *(2571)*	6000	7873 *(5094)*
Scenario B Gains	***6 Months***		***12 Months***		***60 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
	100	120 *(40)*	200	144 *(64)*	500	231 *(172)*
	300	402 *(134)*	500	478 *(214)*	1000	771 *(573)*
	1500	1932 *(644)*	2400	2296 *(1029)*	5000	3701 *(2751)*
Scenario C Gains	***1 Month***		***12 Months***		***120 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
	20	18 *(2)*	50	29 *(13)*	100	58 *(51)*
	300	293 *(33)*	400	478 *(214)*	1000	972 *(851)*
	3100	3515 *(398)*	4000	5740 *(2571)*	10000	11670 *(10206)*
Scenario D Losses	***3 Months***		***12 Months***		***36 Months***	
	actual	***predicted***	***actual***	***predicted***	***actual***	***predicted***
	-16	-21 *(5)*	-20	-29 *(13)*	-28	-39 *(25)*
	-102	-138 *(32)*	-118	-191 *(86)*	-155	-262 *(170)*
	--251	-347 *(81)*	-270	-478 *(214)*	-310	-656 *(425)*

[a]Values in parentheses are standard deviations.

5. Predictive accuracy of models for the simulated dataset

The hypothesis to be tested is that there exists a unique discounting model (i.e., a specific mathematical discounting procedure applied to a specific discount rate) that can predict reasonably well the data. To test the goodness of fit, or prediction, of the model, we apply the MSE criterion, and the related indexes U and U_b described in section 3; for all these measures lower values are better. In addition, we will consider the component U^D to measure the disturbance or non-systematic term (complementary of U^M+U^R to unity), and the correlation between predicted and actual values. For these alternative criteria, higher values are better for the best predictor. Tables 4A and 4B report some summary statistics on the tests respectively performed on the 36 exponential specifications and the 36 hyperbolic specifications. Cells shaded in the minimum column are for measures which are better when low, cells shaded in the maximum column are for measures which are better when high.

Table 4A: Summary Statistics on Tests: Exponential Model

	Minimum	*Maximum*	*Mean*	*Std. Dev.*
Mean Square Error MSE	617.454	$\to+\infty$[a]	$\to+\infty$	$\to+\infty$
U-Statistic	0.246	$\to+\infty$	$\to+\infty$	$\to+\infty$
Bounded U_b	0.132	1	0.667	0.327
Bias Proportion U^M	0.002	0.196	0.052	0.056
Regression Proportion U^R	0.042	0.968	0.797	0.282
Disturbance Proportion U^D	$\to0$[b]	0.876	0.150	0.249
Correlation r	0.675	0.971	0.795	0.189

[a]Values greater than 10^7 are indicated as approaching infinity.
[b]Values less than 10^{-7} are indicated as approaching zero.

Table 4B: Summary Statistics on Tests: Hyperbolic Model

	Minimum	*Maximum*	*Mean*	*Std. Dev.*
Mean Square Error MSE	585.082	6153.078	1659.368	1425.227
U-Statistic	0.233	2.453	0.661	0.568
Bounded U_b	0.111	0.560	0.273	0.121
Bias Proportion U^M	0.026	0.153	0.111	0.037
Regression Proportion U^R	0.041	0.860	0.424	0.279
Disturbance Proportion U^D	0.009	0.923	0.465	0.299
Correlation r	0.867	0.984	0.946	0.034

It is quite clear from these data that in general the hyperbolic model outperforms the exponential: the statistics for all measures, with the exception of the U^M test, are better for the hyperbolic model. It can also be observed that the exponential model is much riskier than the hyperbolic model: applying incorrectly the exponential model may produce very bad results, as testified by the extreme values reached by some of the measures.

However, one particular exponential specification might turn out to fit the data better. We therefore proceed to select the optimal specifications for each model according to the two criteria defined in section 3:

minimization of the MSE, or one of the related indexes; or

minimization of systematic bias combined with maximization of correlation between predicted and actual values.

Minimization of the systematic bias, i.e. the components U^M+U^R, is equivalent to maximization of the complementary value U^D. Therefore we now consider this value only. In addition, for the satisfaction of the second criterion, we should consider the correlation coefficient. For the minimization of the MSE we consider now the U-test. The best values for these three tests are reported in Table 5, referring to five specifications: two of them are specifications of the exponential model, and the other three are specifications of the hyperbolic model. The rates of discount producing these different specifications are also reported in Table 5.

Table 5: Tests on Selected Specifications

Model Specification	***Discount Rates***	***U-Statistic***	***Disturbance Proportion U^D***	***Correlation r***
E1	d = 0.010	0.246	0.731	0.971
E2	d = 0.012	0.263	0.876	0.961
H1	g = 0.192	0.255	0.923	0.961
H2	g = 0.251	0.233	0.712	0.975
H3	g = 0.347	0.605	0.068	0.984

If the decision rule adopted is a), the hyperbolic discounting model specification H2 (with discount rate ã = 0.251) should be selected, since its U measure is the lowest. If instead the criterion b) is adopted, the ranking is not so straightforward. The hyperbolic model specification H3 has the highest correlation coefficient; however, the bad score in the other two tests suggests that the model is affected by a serious systematic bias problem, and we can safely reject it. The choice over the other specifications is more problematic, since there is not a clear pattern. The hyperbolic model specification H1 (with ã = 0.192) might be preferred, because it is considerably better than the others in terms of absence of systematic bias, while it is just a little bit worse than the other with respect to the correlation measure; however, other rankings may be justifiable as well.

Our method shows how to find the discounting model which best fits the data, without explicitly assuming error in a person's judgments. We consider error in the next section.

6. Estimation of discount rates assuming an error term

Analogously to the experimental procedures to assess non-market values, that involve the estimation of the sample valuation from elicited individual values (which may be distorted due to judgment errors), a sample discount rate could be estimated from the elicited individual rates. In order to obtain a (parametric) estimate of the discount rate from our sample observations, we should first specify a statistical model with an error term. For each given value M_0, we assume that the matching value M_t is functionally related to M_0 according to the model:

$$M_0 = f \cdot M_t \cdot e \, .$$

Apart from specifying a multiplicative error term, the model above is still very general, since both the functional form *f* and the distributive properties of the error term are implicit. When individual socio-economic characteristics are available, the functional form *f* may be specified in order to include these characteristics as explanatory variables. In our example we do not include any regressors in the model; furthermore, we maintain the usual assumption of linearity of the valuation function, that can be easily relaxed just by applying an appropriate transformation to the amounts *M*. We will make two hypotheses on the functional form of the discounting model, specifying the function in terms of the exponential model or the hyperbolic model. Two alternative hypotheses will be tested also for the error term: a) the distribution of the error term is *Normal*, with mean zero and variance s^2; b) the distribution of the error term is *Log-normal*, so that *ln(e)* is *Normal*, with mean zero and variance s^2. These hypotheses give rise to four different specifications:

$$M_0 = \frac{M_t}{(1+d)^t} \cdot e$$, that can be also expressed as:

$$M_0 = M_t \cdot q^t \cdot e$$ ***(exponential normal specification)***. (1)

If the error term in (1) is Normal, the model is non-linear; if the error term is Log-normal, the model can be linearized through a transformation:

$$\ln \frac{M_t}{M_0} = t \cdot \ln(1+d) + \ln e$$, that can also be expressed as.

$$\ln \frac{M_t}{M_0} = t \cdot q + \ln e$$ (exponential log-normal specification). (2)

If the discounting model to be applied is the hyperbolic, the specification is:

$M_0 = \frac{M_t}{(1+t)^g} \cdot e$, that can also be expressed as:

$$\frac{M_0}{M_t} = \frac{1}{(1+t)^q} \cdot e \quad \text{(hyperbolic normal specification).} \qquad (3)$$

If the error term is Normal, the model is non-linear; if it is Log-normal, the model can be transformed into a linear one:

$$\ln \frac{M_t}{M_0} = q \cdot \ln(1+t) + \ln e$$

(hyperboliclog-normal Specification). (4)

All models will be estimated through Maximum Likelihood procedures, according to the general specification:

$$\ell = -\frac{n}{2} \ln 2p - n \ln s - \frac{1}{2s^2} \sum \left[y_i - h(q) \right]^2 ,$$

where the values inside the brackets are the dependent and independent variables, that depend on the specification selected. The parameter estimates and log-likelihood for the four models are shown in the following table:

Table 6: Log-Likelihoods and Parameter Estimates

	Exponential Normal	Exponential Log-normal	Hyperbolic Normal	Hyperbolic Log-normal
ℓ	6.527	-24.616	8.671	-19.238
q	0.9737 (0.0043)[a]	0.0169 (0.0019)	0.2059 (0.0212)	0.2597 (0.0238)
s	0.2018 (0.0238)	0.4794 (0.0565)	0.1902 (0.0224)	0.4129 (0.0487)

[a]Values in parentheses are standard errors

The discount rates for each model are obtained after the appropriate transformation of the estimated parameter theta: e.g., for the Normal exponential model, $d = \frac{1}{q} - 1$; for the Log-normal exponential model, $d = (\exp(q)) - 1$; while for the hyperbolic model, g º q, i.e. the estimated parameter is exactly the hyperbolic discount rate.

Table 7: Tests on Estimated Specifications

Model Specification	***Discount Rates***	***U-Statistic***	***Disturbance Proportion U^D***	***Correlation r***
EN	d = 0.027	4.316	0.017	0.779
EL	d = 0.017	0.876	0.205	0.894
HN	g = 0.206	0.235	0.979	0.965
HL	g = 0.256	0.248	0.594	0.976

It is quite clear from the statistics reported in Table 7 that the hyperbolic model dominates the exponential. The ranking between the two specifications of the hyperbolic model is not so well defined, but the hyperbolic normal has both a better *U-Statistic* and a lower systematic error than the hyperbolic log-normal specification. Indeed, the hyperbolic normal model seems to perform quite well: the values obtained with either criterion support the hypothesis that the elicited values are generated by a unique discount rate and the differences in elicited values are due to a multiplicative error term which follows a Normal distribution. The intertemporal preferences of Thaler’s sample are then characterized by the hyperbolic discount rate g = 0.206.

7. Conclusions

A method for characterizing intertemporal preferences by selecting the discounting model which best fits data on people’s preferences is presented. Such an approach can be useful when analyzing experimental data or in policy making when a discounting model to characterize residents’ temporal discounting preferences is needed. Standard measures of goodness of prediction are applied to fitted data to select among alternative specifications of discounting models. We limited our analysis to the exponential and the hyperbolic discounting models. We first presented our approach by applying it to simulated data, that we obtain by manipulating a matrix of experimental data on intertemporal preferences published by Thaler (1981). We then proceeded to estimate the sample discount rate, testing four different specifications: exponential or hyperbolic discount models, modeled with a Normal or log-Normal distribution of the error term. We found that the hyperbolic discounting model with a Normal error term applied multiplicatively provided the best fit. Furthermore, in contrast to Thaler's conclusions, we found that its predictive accuracy is good enough to warrant acceptance of the hypothesis that the data are expressed by a unique discount rate: i.e. the hyperbolic discount rate g = 0.206. As Camerer (1998) notes, the economics profession has been slow to accept the hyperbolic discounting model. Our method provides an approach for determinining when it is most appropriate.

The estimation procedure we adopted can be easily extended to other functional forms, allowing for non-linearity of the valuation function, socio-economic individual characteristics included as regressors, and different specifications for the discounting model.

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