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STRUCTURAL CHANGE AND GROWTH IN A NEG MODEL

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Structural Change and Growth in a NEG model *

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Abstract

This paper presents a New Economic Geography model of structural change, agglomeration and growth. By assuming the same non-homothetic preference structure as Murata (2008), we obtain similar results in that a progressive reduction of trade costs allows the economy to pass from a pre-industrialized to an industrialized stage and then, within the latter, from a dispersed to an urbanized regime. However, the introduction of capital accumulation and the dynamic setting of our model open the door to a richer set of implications. First, an additional stage is introduced as, for some intermediate values of trade costs, a multiple equilibria regime emerges with the symmetric and the core-periphery equilibria stable at the same time. Second, the introduction of non-homotheticity introduces a new channel through which growth is affected by trade costs and agglomeration. In particular, integration is always growth-enhancing while agglomeration is growth-detrimental.

Key words: structural change; non-homothetic preferences; agglomeration; growth.
JEL Classifications: O11; O18; 041; R11; R12

1 Introduction

Structural change – defined as a shift in labour and expenditure allocation from agriculture to non-agricultural sectors and, within the latter, from manufactures to services – is a well established stylized fact for developed economies. The recent literature has investigated two main explanations for this fact. One is based on supply and suggests that the evolution of expenditure and labor shares is due to differences in sectoral TFP growth (among others Hansen and Prescott, 2002; Acemoglu and Guerreri, 2008). The other is based on demand and explains the evolution of expenditure and labor shares as the result of non-homothetic preferences (among others Matsuyama, 1992; Kongsamut, Rebelo and Xie, 2001)[1]. Among the latter, Murata (2008) introduces non-homothetic preferences and decreasing returns to scale in the agricultural sector in a static model of New Economic Geography (NEG) in order to show that integration can explain both structural change and the urbanization process.

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[1]The concept of structural change has been pioneered by Fisher (1939) and Clark (1940). The two explanations of structural change have already been discussed by Simon Kuznets (1966).

Our model might be seen as an attempt to integrate the above approaches. By introducing non-homothetic preferences and allowing for productivity growth differences within sectors at the same time, it encompasses the demand and the supply-based views. Moreover, it accounts for the spatial and the dynamic dimensions of the phenomenon: by introducing non-homothetic preferences in a NEG and Growth model *a la* Baldwin and Martin (2004), we replicate the economic mechanism introduced by Murata (2008) within a dynamic framework of endogenous growth, which enables us to fully explore the long run process of structural change.

In the model, structural change is led by the endogenization of sectoral expenditure shares which, in equilibrium, depend on trade costs and spatial concentration of economic activities. In particular, a reduction in trade costs leads to a reallocation in the sectoral expenditure (*Engel's law*) and labor shares (*Petty's law*) in both regions. When trade costs are high enough, the typical demand-linked and self-reinforcing agglomeration force of Baldwin et al. (2001) - stemming from the birth of a new firm whose profits are spent locally thereby increasing the regional market size - is weaker than the usual congestion effect, which is a dispersion force working on the opposite direction. But, due to the endogenous expenditure shares, our model displays an *additional* demand-driven self-reinforcing mechanism of agglomeration: by increasing the northern real income through the home market effect, a firm locating in the North will lead to an increase in northern expenditure *shares* in the industrial good, thereby providing the incentive for another firm to invest in the North. This additional agglomeration force, which we call *Expenditure Shares Effect* (as in Cerina and Mureddu, 2012) drives most of the theoretical outcomes. These can be divided in two groups: 1) the dynamic properties of equilibrium allocation; 2) the equilibrium growth prospect.

As for the first set of results, non-homotheticity introduces a nonlinear element in the set of equilibrium conditions which leads to the emergence, for some intermediate values of trade costs, of a multiple equilibria regime with the symmetric and the CP equilibria stable at the same time. More precisely, among the many possible cases, there is a wide range of plausible parameters' values such that the stability map with respect to (exogenous) movements in the trade costs seems to resemble historical data.

For almost prohibitive trade costs, prices can be so high that real income is low enough for the representative consumer to purchase only the necessary agricultural good where no technological progress is allowed. As a consequence, consistently with the so-called *Malthusian era*[2], growth is nil. Following Murata (2008) we call this a *pre-industrialized* stage of the economy.

As trade costs decrease, real income increases up to a level which allows the industrial (and luxury) good to be demanded. This creates an incentive for industrial firms to set-up and invest in R&D. Hence the economy enters an *early industrialized pre-urban* stage with positive but low growth (trade costs are not low enough to allow for massive consumption of the technology-intensive good) and with a low degree of agglomeration (trade costs are not low enough to activate self-reinforcing mechanisms of agglomeration).

A further reduction of trade costs leads to multiple equilibria: a small perturbation for an economy located in one of the two emerging unstable equilibria may lead to self-reinforcing mechanisms of both dispersion or agglomeration according to how firms are allocated in the economy. This stage - which is not present in Murata (2008) - is defined as an *intermediate industrial economy with either dispersion or agglomeration.*

Finally, if trade costs decrease even further so to reach the so-called *break-point*, the symmetric equilibrium looses its stability and there is a strong incentive for industrial firms to concentrate in space to fully exploit increasing returns to scale. We then finally

[2]Hansen and Prescott (2002)

enter a *modern industrialized urban economy* with a urbanized and industrialized core and a rural periphery.

As for the second set of results, in contrast with standard models, non-homothetic preferences allow growth to depend both on integration and agglomeration *even* in the absence of localized spillovers. Integration, by increasing both regional expenditure shares on industrial goods, creates an additional incentive for industrial firms to invest in R&D and, therefore, it is always good for growth. This feature represents an alternative channel through which integration boosts growth[3] with respect to the traditional explanations based on comparative advantage, technology flows and efficiency gains. We believe this mechanism to be relevant in real economies and deserving to be empirically tested.

By contrast, agglomeration turns out to be always growth-detrimental because it reduces the aggregate expenditure in industrial goods. This provides an explanation (alternative to Cerina and Mureddu, 2009) for the recent empirical evidence according to which agglomeration boosts GDP growth only up to a certain level of development (Bruhlardt and Sbergami, 2009). However, we find that an hypothetical central planner willing to maximize growth will always choose the maximum level of integration even if the latter implies a maximum level of agglomeration. Hence, the positive effect of integration more than compensates (in absolute terms) the negative effect of agglomeration on growth.

The rest of the paper is organized as follow. Section 2 presents the analytical framework; section 3 deals with the dynamic properties of equilibrium allocations; section 4 is dedicated to the analysis of the rate of growth and section 5 concludes.

2 The analytical framework

The model's structure can be viewed as a mix of Baldwin et al. (2001) and Murata (2008).

2.1 Production side

There are two regions, North and South, symmetric in terms of production factors (labour L and capital K), preferences, technology, trade costs and labour endowments. In both regions three sectors are active: traditional (agricultural) T, manufacturing (industrial) M and innovation I. Labour is immobile across regions[4] but mobile across sectors.

The traditional good T is produced under perfect competition and constant returns to scale so that its production function is $T = L$. Moreover it is freely traded, while the manufacture good M is subject to iceberg trade costs. Manufactures are produced under Dixit-Stiglitz monopolistic competition (Dixit and Stiglitz, 1977) and enjoy increasing returns to scale: firms face a fixed cost in terms of knowledge capital and a variable cost a_M in terms of labour. Thereby the cost function is $\pi + wa_M x_i$, where π is the rental rate of capital, w is the wage rate and a_M are the units of labour necessary to produce a unit of output x_i. Since a unit of capital K is required in order to set up the production of a new variety n, we have $K + K^* = K^w = n + n^* = n^w$. When capital is immobile, a unit of K might be interpreted as a mix of physical and human capital (Baldwin and Martin, 2004) and then once a firm is set-up in a region, the owner is forced to spend

[3]The positive relationship between integration and growth is widely accepted. See Frankel and Romer (1999).

[4]This is not the case in Murata (2008), where labor is mobile. From this viewpoint, our model is more suitable to describe the European case, where only a very limited fraction of the labor force works in a foreign country.

its capital income in the same region, therefore increasing the local market-size[5]. In this case: $n = K$ and $n^* = K^*$. By defining $s_n = \frac{n}{n^w}$ and $s_K = \frac{K}{K^w}$, we also have $s_n = s_K$.

Each region's K is produced by its own I-sector, so the production and the cost functions of innovation are respectively $\dot{K} = \frac{L_I}{a_I}$ and $F = w_I a_I$. To individual I-firms, the innovation cost a_I is a parameter, thereby our model enjoys endogenous growth assuming that the labour unit requirements a_I declines as the cumulative output rises. In this model learning spillovers are assumed to be global so $a_I = \frac{1}{K^w}$. The growth rate of capital, firms and varieties is then given by $g \equiv \frac{\dot{K}}{K}$ and $g^* \equiv \frac{\dot{K}^*}{K^*}$. In the following analysis, we will focus on the northern region, as the southern expressions and definitions are isomorphic.

2.2 Households' behaviour

As for the demand-side, an infinitely-lived representative household maximizes:

$$U(t) = \int_{t=0}^{\infty} e^{-\rho t} \ln Q(t) dt; Q = (C_T(t))^{1-\mu} \left(n(t)^{w^{\frac{1}{1-\sigma}}} C_M(t) + \gamma \right)^{\mu}; C_M(t) = \left(\int_{i=0}^{K+K^*} c_i(t)^{1-\frac{1}{\sigma}} di \right)$$

Where $\rho > 0$ is the time-preference rate, $\gamma > 0$ is the non-homotheticity parameter, $Q(t)$ is the consumption bundle at time t which is a Stone-Geary non-homothetic combination of the manufacture bundle $C_M(t)$ and of the agricultural good $C_T(t)$. Finally, the manufacture bundle $C_M(t)$ is a combination of the n^w industrial varieties[6].

The preference structure is basically the same as in Murata (2008) except for the intertemporal dimension which is not present in the latter. The non-homothetic element in the utility function, by making the industrial good a *luxury* good and the agricultural one a *necessary* good, drives all the novel results of this paper. Also notice that, as in Murata (2008) in the context of a NEG model and Blanchard and Kiyotaki (1987) in a macroeconomic context, we neutralize agents' love for variety by setting to zero its parameter. The analytical consequence is the emergence of the term $n^{w^{\frac{1}{1-\sigma}}}$ in the second-stage utility function: this normalization neutralizes the dependence of the price index on the number of varieties allowing us to concentrate the analysis on the influence of firms' location and transport costs on the expenditure shares.

The infinitely-lived representative consumer's optimization is carried out in three stages. In the first stage the agent intertemporally allocates consumption between expenditure and savings. In the second stage, in any t, she allocates expenditure between industrial and traditional goods, while in the last stage she allocates industrial expenditure across varieties. As a result of the intertemporal optimization program, the path of consumption expenditure E across time is given by the standard Euler equation $\frac{\dot{E}}{E} = r - \rho$. In the second stage the agent maximizes Q under the constraint that $P_M C_M + p_T C_T = E$, where p_T is the price of the traditional good and $P_M = \left[\int_{i=0}^{K+K^*} p_i^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$ is the Dixit-Stiglitz perfect price index. Getting rid of the time notation and setting $p_T = 1$, second-

[5]This is not the case when capital is mobile. In this case knowledge capital is interpreted as a blueprint, an idea, a new technology, a patent, or a machinery. Hence a production shift need not being associated to a demand shift. In this paper we will assume capital immobility but the case of capital mobility, being analytically a special case of the former, is briefly analysed in the downloadable appendix.

[6]Notice that only the second-stage (intratemporal) optimization problem has a non-homothetic nature, while the *intertemporal* optimization is intrisecally homothetic as there is no minimum level of consumption to be satisfied. For this reason, the optimal "global" saving rate in each date is constant and does not depend on real income. This is not the case in works focusing on convergence patterns with subsistence consumption which is a related but different form of non-homotheticity (see Steger, 2000; Chattarjee, 1994; Bertola et al., 2007).

stage optimization leads to the following optimal expenditure shares:

$$\frac{P_M C_M}{E} = \max\left(\mu - (1-\mu)\,\gamma\frac{P_M}{E n^{w\frac{1}{1-\sigma}}}\ ;0\right) = \max\left(m\left(\frac{n^{w\frac{1}{\sigma-1}}}{E/P_M}\right);0\right) \tag{1}$$

$$\frac{C_T}{E} = \min\left(1-\mu + (1-\mu)\,\gamma\frac{P_M}{E n^{w\frac{1}{1-\sigma}}};1\right) = \min\left(1-m\left(\frac{n^{w\frac{1}{\sigma-1}}}{E/P_M}\right);1\right) \tag{2}$$

These expressions deserve some comments. First, notice that once γ is set to zero, the expenditure shares collapse to the usual Cobb-Douglas ones. Second, it is easy to see that when γ is large enough and/or E/P_M are low enough, then $m\left(\frac{n^{w\frac{1}{\sigma-1}}}{E/P_M}\right)$ can be negative so that consumption of industrial goods is set to zero and, consequently, expenditure shares in the agricultural good are equal to one. Finally, notice we have $\frac{\partial m}{\partial P_M} < 0$ and $\frac{\partial m}{\partial E} > 0$ so that regional industrial expenditure share rises with regional total real expenditure.

In the third stage, whenever $m\left(\frac{n^{w\frac{1}{\sigma-1}}}{E/P_M}\right)$ is positive, the amount of $M-$ goods expenditure is allocated across varieties according to the a CES demand function for a typical $M-$variety $c_j = \frac{p_j^{-\sigma}}{P_M^{1-\sigma}} m\left(\frac{n^{w\frac{1}{\sigma-1}}}{E/P_M}\right)E$, where p_j is variety j's consumer price.

2.3 Expenditure shares, integration and agglomeration

Due to perfect competition in the T-sector, the price of the agricultural good must be equal to the wage of the traditional sector's workers: $p_T = w_T$. Moreover, *as long as both regions produce some* T the assumption of free trade in T implies that not only price, but also wages in agriculture are equalized across regions[7]. But since labour is mobile across sector, wages of the three sectors cannot differ. Hence, $M-$ sector and I-sector wages are tied to T-sector wages and remain fixed at the level of the unit price of T-good:

$$p_T = p_T^* = w_T = w_T^* = w_M = w_M^* = w_I = w_I^* = w = 1 \tag{3}$$

Finally, since wages are uniform and all varieties' demands have the same constant elasticity σ, firms' profit maximization yields local and export prices that are identical for all varieties no matter where they are produced: $p = w a_M \frac{\sigma}{\sigma-1}$. Then, imposing the standard normalization $a_M = \frac{\sigma-1}{\sigma}$ and using (3), we finally obtain $p = w = 1$. As usual, since trade in the $M-$good is impeded by iceberg import barriers, prices for markets abroad are higher: $p^* = \tau p$; $\tau \geq 1$. By labeling as p_M^{ij} the price of a particular variety produced in region i and sold in region j (so that $p^{ij} = \tau p^{ii}$) and by imposing $p = 1$, the $M-$goods price indexes might be expressed as follows:

$$P_M = \left[\int_0^n (p_M^{NN})^{1-\sigma} di + \int_0^{n^*} (p_M^{SN})^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} = (s_K + (1-s_K)\,\phi)^{\frac{1}{1-\sigma}}\, n^{w\frac{1}{1-\sigma}} \tag{4}$$

$$P_M^* = \left[\int_0^n (p_M^{NS})^{1-\sigma} di + \int_0^{n^*} (p_M^{SS})^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} = (\phi s_K + 1 - s_K)^{\frac{1}{1-\sigma}}\, n^{w\frac{1}{1-\sigma}} \tag{5}$$

[7] An assumption is actually needed in order to avoid complete specialization: a single country's labour endowment must be insufficient to meet global demand. The purpose of making this assumption, which is standard in most New Economic Geography and Growth models, is to maintain the M-sector and the I-sector wages fixed at the unit value. See Bellone and Maupertuis (2003) for an analysis of the implications of removing this assumption.

where $\phi = \tau^{1-\sigma}$ is the so called "phi-ness"or *freeness* of trade which ranges from 0 (prohibitive trade) to 1 (costless trade). To close the model, we need to express regional expenditures as function of trade costs (freeness of trade ϕ) and the degree of spatial agglomeration s_K. We obtain this expression by exploiting the labour market clearing conditions in the three sectors and at the world level. Hence we can write[8] $E = L + \rho s_K$ and $E^* = L + \rho(1 - s_K)$. Substituting for this two values in (2) and (3) we finally obtain the expressions for regional expenditure shares in industrial goods as functions of trade costs ϕ and industrial allocation s_K only:

$$\frac{P_M C_M}{E} = \max\left(\mu - (1-\mu)\gamma\frac{(s_K + (1-s_K)\phi)^{\frac{1}{1-\sigma}}}{L + \rho s_K}\ ; 0\right) = \max\left[m(s_K, \phi), 0\right] \quad (6)$$

$$\frac{P_M^* C_M^*}{E^*} = \max\left(\mu - (1-\mu)\gamma\frac{(\phi s_K + (1-s_K))^{\frac{\sigma}{1-\sigma}}}{L + \rho(1-s_K)}\ ; 0\right) = \max\left[m^*(s_K, \phi), 0\right] \quad (7)$$

We now focus on this set of expressions and we first analyse the role of industrial allocation s_K in determining the equilibrium regional industrial expenditure shares. It is easy to see that regional expenditure shares are identical in the symmetric allocation $s_K = 1/2$ but unlike models with homothetic second stage-utility, regional expenditure shares in manufacture can differ when $s_K \neq 1/2$. In particular $\frac{\partial m}{\partial s_K} > 0$ and $\frac{\partial m^*}{\partial s_K} < 0$ so that an increase in the share of industrial firms located in the North increases northern expenditure share and reduces southern one.

This is due to a twofold outcome. The first effect works through the regional price indexes P_M and P_M^*: by home market effect, an increase in s_K allows northern households to purchase a lower amount of goods from the other region so that they are less hurt by trade costs. This increases northern real income which we know has a positive effect in industrial expenditure shares (equation (1)). The opposite, of course, happens in the South. The second effect works through regional expenditures E and E^*: an increase in s_K rises northern profits which themselves rise northern expenditure. But manufactures are luxury goods, this movements to an increase in northern industrial expenditure *share* and a reduction in southern one.

As for the role of trade costs, their effects are more symmetric in the two regions: a higher degree of integration (larger ϕ) leads to lower prices in both regions and then, ceteris paribus, to an increase in North and South purchasing power. This increase - whose exact amount will differ in the two regions as long as $s_K \neq 1/2$ - allows the representative household to allocate a larger fraction of expenditure in the luxury good so that $\frac{\partial m}{\partial \phi} > 0$ and $\frac{\partial m^*}{\partial \phi} > 0$. Hence a reduction in trade costs is also able to explain *Engel's law*.

As in Murata (2008), agglomeration and integration are also able to explain the so-called *Petty's law* - i.e. the observed shift of the labour force from agricultural to non-agricultural sectors. Using the market clearing conditions in the traditional and industrial sector, we have:

$$L_T + L_T^* = (1 - m(s_K, \phi))(L + \rho s_K) + (1 - m^*(s_K, \phi))(L + \rho(1 - s_K))$$

$$L_M + L_M^* = \frac{\sigma - 1}{\sigma}\left(m(s_K, \phi)(L + \rho s_K) + m^*(s_K, \phi)(L + \rho(1 - s_K))\right)$$

The latter is clearly increasing in ϕ. Hence, considering that the both northern and southern labor forces are constant at, respectively, L and L^*, we conclude that the integration explains *Petty's law* both at the regional and aggregate level. The same cannot be said

[8]The derivation is standard and it is available at request

about agglomeration as we have $\frac{\partial L_M}{\partial s_K} > 0$ but $\frac{\partial L_M^*}{\partial s_K} < 0$ so that agglomeration in the North has an opposite effect on regional industrial labor force. Clearly the net aggregate effect of agglomeration on the global industrial labour force depends on the relative intensity of the two regional effects.

3 Integration, Industrialization and Agglomeration

In this section, we will perform a detailed analysis of the equilibrium dynamics of industrial allocation. As the only source of the dynamics in our model is the birth of new industrial firms, the dynamic analysis can only be undertaken if the industrial sector exists. However, this is not always the case as with non-homothetic preferences there is a wide range of parameters values such that the consumer demand for industrial goods is nil and therefore there is no incentive for industrial firms to set a new investment. The first part of this section is dedicated to the analysis of the conditions under which a pre-industrial economy is a steady state.

3.1 A pre-industrial economy

As we can easily see from (6) and (7) regional expenditure shares in manufactures might not be strictly positive. When this is the case, given the lack for demand for manufactures, the industrial sector simply does not exist and the regional workforce is wholly allocated to the agricultural sector. As it's straightforward from (6) and (7), that could happen when: 1) the importance of industrial goods in the utility function μ is low enough; 2) the non-homotheticity parameter γ is large enough; 3) the number of workers L and the interest rate ρ are small enough and, most importantly, 4) trade costs are large enough. Therefore, industrialization can be triggered by an exogenous change of one of the previous parameters.

Following the NEG tradition, we concentrate on the effect of an exogenous reduction in trade costs. When trade costs are sufficiently high, there might be no demand for industrial goods and therefore no incentive for an industrial firm to set-up. In this case, the economy is in a pre-industrial stage of development and the level of ϕ should be such that:

$$\phi < \phi_I : m(s_K, \phi) = \max\left(\mu - (1-\mu)\gamma\frac{(s_K + (1-s_K)\phi)^{\frac{1}{1-\sigma}}}{L + \rho s_K}; 0\right) = 0$$

$$\phi < \phi_I^* : m^*(s_K, \phi) = \max\left(\mu - (1-\mu)\gamma\frac{(\phi s_K + (1-s_K))^{\frac{1}{1-\sigma}}}{L + \rho(1-s_K)}; 0\right) = 0$$

where ϕ_I and ϕ_I^* are the level of freeness of trade below which industrialization is not triggered in the North and in the South respectively.

Given the perfect symmetry of the two regions in terms of technology, endowments and preferences, industrialization should be triggered simultaneously in the two regions because, in a rural economy, the incentive to set-up a new firm cannot differ across regions. There are two consequences for this observation: first $\phi_I = \phi_I^*$ because trade-costs are symmetric; second, at the very beginning of the industrial age (i.e. when $\phi = \phi_I$) the industrial sector should be equally divided among the two regions so that the "first"industrial steady-state (stable or not) is a symmetric one. As a consequence, $\phi = \phi_I$

implies $s_K = \frac{1}{2}$. Therefore we have:

$$m\left(\frac{1}{2},\phi^I\right) = m^*\left(\frac{1}{2},\phi^I\right) = \mu - 2(1-\mu)\gamma\frac{\left(\frac{1+\phi_I}{2}\right)^{\frac{1}{1-\sigma}}}{2L+\rho} = 0$$

From this expression we can find an explicit value for ϕ_I, i.e., the industrialization-triggering level of freeness of trade which is $\phi_I = 2\left(\frac{2(1-\mu)\gamma}{\mu(2L+\rho)}\right)^{\sigma-1} - 1$. Notice that there are ranges of parameters' values such that ϕ_I is either negative or larger than 1. In the first case, the economy is always industrialized for whatever values of trade costs. In the second case, industrialization is ruled out for any values of trade costs. Clearly, if industrialization is impossible, there is no room for agglomeration. On the other hand, when $\phi_I < 0$, industrialization trivially exists for any level of trade costs. For this reason, in the rest of the paper we concentrate on the case when ϕ_I belongs to the interval $(0,1)$. A rural economy is then a steady state when $\phi < \phi_I$.

3.2 An Industrial Economy: Steady-State Allocations

In the next subsections, we will assume that $0 < \phi_I < \phi < 1$ so that both regional demands for industrial goods are strictly positive and industrial firms have the incentive to set-up new investments. What does an equilibrium look like in such an economy? In any equilibria the growth rate of the world capital stock will be constant and will either be common ($g = g^*$ in the interior case) or North's ($g > g^* = 0$ in the Core-Periphery case)[9]. In any case, the value of investing in a new unit of capital in the North and in the South is respectively $V = \frac{\pi}{\rho+g}$ and $V^* = \frac{\pi^*}{\rho+g}$. The expressions for profits stemming from firms' profit maximization are:

$$\pi = B(s_E, s_K, \phi)\frac{E^w}{K^w\sigma} = \left[\frac{s_E}{(s_K+(1-s_K)\phi)}m(s_K,\phi) + \frac{\phi(1-s_E)}{(\phi s_K+1-s_K)}m^*(s_K,\phi)\right]\frac{E^w}{K^w\sigma}$$

$$\pi^* = B^*(s_E, s_K, \phi)\frac{E^w}{K^w\sigma} = \left[\frac{s_E\phi}{(s_K+(1-s_K)\phi)}m(s_K,\phi) + \frac{1-s_E}{(\phi s_K+1-s_K)}m^*(s_K,\phi)\right]\frac{E^w}{K^w\sigma}$$

By using the labour market clearing condition and the expressions for the profits we are able to find the equations representing the Tobin's q in the two regions:

$$q = \frac{V}{F} = B(s_E, s_K, \phi)\frac{E^w}{(\rho+g)\sigma}; q^* = \frac{V^*}{F^*} = B^*(s_E, s_K, \phi)\frac{E^w}{(\rho+g)\sigma} \quad (8)$$

3.2.1 Interior steady states

Each firm will invest in the more profitable regions, i.e. where the Tobin's q is higher. Since firms can be created both in the North and in the South, in any interior equilibria $q = q^* = 1$. The first equality is a no-arbitrage condition ($q = q^*$), stating that in any interior equilibrium there will be no incentive for a firm to move to another region. The fact that both regions' q should be equal to 1, represents the *optimal investment condition*, according to which in equilibrium firms will decide to invest up to the level at which the expected discounted value of the firm itself is equal to the replacement cost of capital. By solving this equation using (8) we find the steady-state relation between the northern market size s_E and the northern share of firms s_K:

$$s_E^N(s_K,\phi) = \frac{m^*(s_K,\phi)(s_K+(1-s_K)\phi)}{m(s_K,\phi)(\phi s_K+1-s_K)+m^*(s_K,\phi)(s_K+(1-s_K)\phi)} \quad (9)$$

[9] This feature is obtained by time-differentiating $s_K = \frac{K}{K^W}$ and then imposing a constant value for s_K

The definition of s_E when labour markets clear gives us the *permanent income condition*, which is a relation between size s_E and share of firms located in North s_K :

$$s_E^P(s_K) = \frac{L + \rho s_K}{2L + \rho} \tag{10}$$

Those two relations drive the dynamics of our economy. We can define a new implicit function whose zeros represent the interior steady-states allocations of our economy:

$$f(s_K, \phi) = s_E^N(s_K, \phi) - s_E^P(s_K) \tag{11}$$

We the define an interior steady-states allocation as any value of $s_K^* \in (0,1)$ such that $f(s_K^*, \phi) = 0$. Notice that the symmetric allocation $\left(s_K = \frac{1}{2}\right)$ is always a steady-states being $f\left(\frac{1}{2}, \phi\right) = \frac{1}{2} - \frac{1}{2} = 0$. However, and unlike most NEG models, this is not the only feasible interior steady-states as the non-linearity of $s_E^N(s_K, \phi)$ opens the door to multiple intersections with the linear function $s_E^P(s_K)$ and therefore to different values of s_K such that $f(s_K, \phi)$ is zero.

Proposition 1 *(Number of interior steady states) The system displays 1 or 3 interior steady-states allocations: the symmetric allocation* $s_K = \frac{1}{2}$ *(which always exists) and 2 non-symmetric allocations:* $s_K^*(L,\rho,\phi,\gamma)$ *and* $s_K^{**}(L,\rho,\phi,\gamma) = 1 - s_K^*(L,\rho,\phi,\gamma)$ *which emerge only for some values of the parameters. The symmetric steady-state is unique when* $f(0,\phi)\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} < 0$ *while there are 3 interior steady-states when* $f(0,\phi)\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0$.

Proof. Please refer to the downloadable mathematical appendix ■

This proposition provides a necessary and sufficient condition for the uniqueness/multiplicity of interior steady-states. However, despite its importance, it is not particularly informative as long as we do not provide an analysis concerning the way $f(0,\phi)\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ changes sign as trade costs decline. Because of the crucial linkages with the stability issues, such analysis will be performed in the next sections within the analysis of the stability map.

3.2.2 Core-periphery steady states

Interior steady-states are not the only allocations where the regional share of industrial firms is constant: s_K is constant even when the latter is equal to either 1 or 0, e.g., when the whole industrial sector is located in only one region. Since the two CP allocations are perfectly symmetric, we just focus on the first where the North gets the core. By following Baldwin and Martin (2004), we consider that for $s_K = 1$ to be an equilibrium, it must be that $q = V/F = 1$ and $q^* = V^*/F^* < 1$ for this distribution of capital ownership: continuous accumulation is profitable in the north since $V = F$, but $V^* < F$ so no southern agent would choose to setup a new firm. Defining the CP equilibrium this way, it implies that it is stable whenever it exists.

3.3 Stability analysis

3.3.1 A new agglomeration force: the Expenditure Share Effect

In this section we provide a complete stability map for the equilibria of our economy. As we will see, this analysis is intimately linked to the issue of the number of interior steady-states. At the end of this section we will be able to state, for any value of the trade costs, the existence and stability of any kind of steady-state. Following Baldwin and Martin (2004) we consider the ratio of northern and southern Tobin's q:

$$\frac{q}{q^*} = \frac{B(s_E, s_K, \phi)}{B^*(s_E, s_K, \phi)} = \frac{\left[\frac{s_E}{s_K+(1-s_K)\phi} m(s_K, \phi) + \frac{\phi(1-s_E)}{\phi s_K+(1-s_K)} m^*(s_K, \phi)\right]}{\left[\frac{s_E \phi}{s_K+(1-s_K)\phi} m(s_K, \phi) + \frac{1-s_E}{\phi s_K+(1-s_K)} m^*(s_K, \phi)\right]} = \psi\left(s_E, s_K, \phi\right) \tag{12}$$

Starting from any interior steady-state allocation where $\psi\left(s_E, s_K, \phi\right) = 1$, any increase (decrease) in $\psi\left(s_E, s_K, \phi\right)$ will make investments in the North (South) more profitable and thus will lead to a production shifting to the North (South). Hence any allocation will be stable if a production shifting, say, to the North ($\partial s_K > 0$) will reduce $\psi\left(s_E, s_K, \phi\right)$. By contrast, if $\psi\left(s_E, s_K, \phi\right)$ will augment following an increase in s_K, then the equilibrium will be unstable and agglomeration or dispersion processes might be activated. Taking the derivative of $\psi\left(s_E, s_K, \phi\right)$ with respect to s_K and then using the no-arbitrage condition (true in every interior steady-state):

$$\frac{\partial \psi\left(s_E\left(s_K\right), s_K, \phi\right)}{\partial s_K} = A\left(s_K, \phi\right) + B\left(s_K, \phi\right) + C\left(s_K, \phi\right) \tag{13}$$

where:

$$A\left(s_K, \phi\right) = \left(\frac{\partial m}{\partial s_K}/m - \frac{\partial m^*}{\partial s_K}/m^*\right)\frac{(1-\phi)}{(1+\phi)} : \text{ Expenditure Share Effect} \tag{14}$$

$$B\left(s_K, \phi\right) = -\frac{(1-\phi)^2}{\left(s_K + \left(1 - s_K\right)\phi\right)\left(\phi s_K + \left(1 - s_K\right)\right)} : \text{ Market Crowding Effect} \tag{15}$$

$$C\left(s_K, \phi\right) = \frac{(1-\phi)}{(1+\phi)} \frac{ds_E\left(s_K\right)}{ds_K} \frac{\left(m\left(\phi s_K + \left(1-s_K\right)\right) + m^*\left(s_K + \left(1-s_K\right)\phi\right)\right)^2}{mm^*\left(s_K + \left(1-s_K\right)\phi\right)\left(\phi s_K + \left(1-s_K\right)\right)} : \text{ Demand Effect} \tag{16}$$

The last two forces are the same we encounter in the standard New Economic Geography models and they are the formal representation of, respectively, the *market-crowding effect* (a dispersion force) and the *demand-linked effect* (an agglomeration force). The first force represents the novelty of our model. In the standard case, where $m^*\left(s_K, \phi\right) = m\left(s_K, \phi\right) = m$ and then $\frac{\partial m}{\partial s_K} = \frac{\partial m^*}{\partial s_K} = 0$, this force simply does not exist. We dub this force as the **expenditure share effect** in order to highlight the link between the existence of this force and the fact that the expenditure shares are endogenous. This expenditure share effect always represents a destabilizing force because $\frac{\partial m}{\partial s_K} \geq 0$ and $\frac{\partial m^*}{\partial s_K} \leq 0$. Therefore $A\left(s_K, \phi\right)$ is always positive and the new agglomeration force emerges thanks to the luxury nature of industrial good.

But what is the economic intuition behind this new agglomeration force? Imagine a firm moving from South to North ($\partial s_K > 0$). Since manufacture is a luxury good, the increase in the northern purchasing power will increase northern expenditure shares in the industrial good $\left(\frac{\partial m}{\partial s_K} \geq 0\right)$. This will amplify the demand effect in the North, thereby increasing northern profits and giving further incentive for an industrial firm to locate in the North. This new force can be so strong that a CP outcome may be reached even in case of capital mobility (see the downloadable appendix) and for whatever level of transport costs.

3.3.2 Number and stability of equilibria

An important feature of the stability analysis is the following:

$$\frac{\partial \psi\left(s_E, s_K, \phi\right)}{\partial s_K} \quad \leq \quad (>)\, 0 \Longleftrightarrow \frac{\partial f(s_K^*, \phi)}{\partial s_K} \geq (<)\, 0$$

In other words, **any interior equilibria** is stable (unstable) if the graph of $f(\cdot)$ in the plane $(s_K, f(s_K, \phi))$ crosses the horizontal axis with positive (negative) inclination. This finding links the uniqueness/multiplicity issue with the stability issue. In other words, the sign of $\frac{\partial f(\frac{1}{2},\phi)}{\partial s_K}$ is not only informative of the stability of any kind of equilibria, but it is also a determinant of the uniqueness or multiplicity regime. Moreover, the particular shape of the function $f(\cdot)$ allows us to focus only on the value of its derivative in $s_K = \frac{1}{2}$ in order to deduce the stability properties of each (interior or CP) steady-state. In fact, by proposition 1 and by continuity and symmetry of $f(\cdot)$, the sign of $\frac{\partial f(s_K^*,\phi)}{\partial s_K}$ in the symmetric equilibrium must be opposite to the sign of the same derivative in the two interior non-symmetric equilibria . As a consequence, the non-symmetric equilibria (when they exist) are unstable when the symmetric equilibrium is stable and vice versa. By applying a similar reasoning we can conclude that $s_K = 0$ and $s_K = 1$ are (local) attractors only when the non-symmetric interior steady-states exist *and* are unstable *or* when the symmetric steady-state is unique and unstable.

Before providing a complete stability map of equilibria allocation together with the associated number of interior equilibria, we need to provide some definitions. First, we call ϕ_B the *break-point* value of ϕ, i.e. a value of the freeness of trade such that the symmetric steady-state turns from stable to unstable:

$$\phi \leq (\geq)\phi_B \Longleftrightarrow \frac{\partial f(s_K^*, \phi)}{\partial s_K} \geq (<)\, 0$$

The effective value of ϕ_B will be the solution of this implicit equation:

$$\frac{L}{L+\rho} - \phi_B - \frac{(1-\mu)\,\gamma(1-\phi_B)}{\mu(L+\rho)} \left(\frac{1+\phi_B}{2}\right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} = 0 \tag{17}$$

Unfortunately, it is not possible to provide a closed-form solution for this break-point because ϕ enters with non-integer powers in (17). However, we still can obtain some qualitative results through the implicit function theorem. In particular, by writing $\phi_B = \phi_B(\rho, L, \mu, \sigma, \gamma)$ and by differentiating (17), we obtain with $\frac{\partial \phi_B}{\partial \rho} < 0$, $\frac{\partial \phi_B}{\partial L} > 0$, $\frac{\partial \phi_B}{\partial \mu} > 0$, $\frac{\partial \phi_B}{\partial \sigma} < 0$ and, more importantly, $\frac{\partial \phi_B}{\partial \gamma} < 0$ so that, since in the model with homothetic preferences $\gamma = 0$, *in our model the break-point is reached earlier than in standard models.* This should not be a surprise, given the existence of a new agglomeration force.

It is also interesting to focus on the relation between ϕ_B and ϕ_I. As already anticipated, $\frac{\partial f\left(\frac{1}{2},\phi_I\right)}{\partial s_K}$ might be negative meaning that the symmetric allocation is unstable when industrialization emerges. Meaning that, given the assumed monotonicity of $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ with respect to ϕ, we have $\phi_B < \phi_I$. By substituting the value of ϕ_I in $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ and imposing a positive value, we find the condition in terms of γ under which this is true:

$$\gamma > \frac{\mu\,(2L+\rho)}{2\,(1-\mu)} \left(\frac{2L+\rho}{2L+\rho\,(2-\sigma)}\right)^{\frac{1}{\sigma-1}} \tag{18}$$

A newly industrialized economy then experiences catastrophic agglomeration if γ is large enough.

The second value of freeness of trade we need to focus on is the value of ϕ such that $f(0, \phi)$ turns from negative to positive. This value, which we call we call $\hat{\phi}$, is important as, from proposition 1, it contributes to govern the number of interior steady-states. We assume that this value us unique and we show in the mathematical appendix that this is always the case if γ is not too large and if μ,ρ and L are not too small. We also argue that these restrictions are very mild as they are consistent with positive value for the regional expenditure shares in industrial goods. Finally, the mathematical appendix also shows that $\hat{\phi}$ *will always be lower than* ϕ_B. In other words, as trade costs gets lower, $f(0,\phi)$ becomes positive before $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ turns negative, and so before the symmetric equilibrium gets unstable.

3.3.3 Stability map

We are now ready to provide a complete characterization of the dynamics of the equilibria allocation together with the associated number of interior equilibria. Provided that $\phi_I \in (0,1)$, we discuss three cases, presented in propositions 2, 3 and 4, which differ in the relative position of ϕ_I with respect to the interval $(\hat{\phi},\phi_B)$, i.e., in the "timing" of industrialization.

Proposition 2 *In case $\phi_I < \hat{\phi} < \phi_B$ the economy starts in a pre-industrial state, and as trade costs decrease the system moves first to an early-industrialized symmetric economy then to an intermediate industrialized phase with either agglomeration or dispersion and finally towards a modern industrialized economy with agglomeration (figure (1) in the downloadable appendix).*

In this case, industrialization emerges for relatively high value of ϕ and the dynamics of the model are the richest. The case presented in the proposition encompasses four different phases:

1. $\phi \in (0,\phi_I)$: when trade costs are higher than the industrialization-triggering value, the economy is in a *rural or pre-industrial phase*: there are no industrial firm, only the agricultural (low-tech) good is produced and, for this range of trade-costs values, this represents a steady state of the economy.

2. $\phi \in (\phi_I,\hat{\phi})$: when the freeness of trade is higher than the industrialization-triggering value, industrial firms emerge because households become rich enough to afford (and demand) industrial goods. The fact that the freeness of trade is lower than $\hat{\phi}$ implies that $f(0,\phi) < 0$ which, together with the fact that $\hat{\phi} < \phi_B$ and then $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0$, implies, by proposition 1, that the symmetric allocation is the only interior steady-state. Moreover, since $\phi < \phi_B$ the symmetric allocation is also globally stable. We call this phase an *early-industrialized economy without agglomeration.*

3. $\phi \in (\hat{\phi},\phi_B)$: when the freeness of trade becomes larger than $\hat{\phi}$ but still smaller than ϕ_B, $f(0,\phi)$ switches sign from negative to positive making $f(0,\phi)\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ positive itself. By proposition 1 that means that two other interior steady-states emerge besides the symmetric one. Moreover, since $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is still positive, the two new interior equilibria are *unstable* while the symmetric one maintains its stability. That makes the two CP allocations *stable steady-states.* In other words, the emergence of two unstable interior steady-states - call them s_K^* and s_K^{**} - leads to the emergence of three connected attraction sets: $[0,s_K^*)$, $[s_K^*,s_K^{**}]$ and $(s_K^{**},1]$. In the first, industry does not have any incentive to set-up in the North so that if $s_K \in [0,s_K^*)$ than the CP allocation $s_K = 0$ is a locally stable equilibria. In particular, if the economy is in the steady-states s_K^*, then an infinitesimal relocation of a firm from North to South activates a process a catastrophic agglomeration which leads eventually to a CP pattern where the whole industrial sector is located in the South. In the second attraction set, the symmetric allocation is locally stable so that if the

economy starts from the symmetric steady-state, there is no incentive for any firm to locate elsewhere. Moreover if the economy starts from s_K^*, then an infinitesimal relocation of a firm from South to North activates a process of *catastrophic dispersion* which leads eventually to a symmetric pattern where the industrial sector is evenly distributed between North and South. Finally, the third attraction set is perfectly specular to the first one: in this case the CP allocation $s_K = 1$ is a locally stable steady-state. We call this stage an *intermediate industrialized phase with either agglomeration or dispersion.*

4. $\phi \in (\phi_B, 1)$: when economic integration is strong enough to let ϕ overcome ϕ_B, then the symmetric steady-state starts to be unstable and $f(0, \phi) \frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K}$ turns to be negative again meaning that the symmetric allocation is the unique interior steady-state. In other words, the two non-symmetric interior steady-states disappear, the two CP allocations $s_K = 0$ and $s_K = 1$ become respectively locally stable steady states in the sets $\left[0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right]$ and any shift of a firm from the symmetric allocation to the North or the South activates a process of catastrophic agglomeration respectively in the North or in the South. We call this economy a *modern industrialized economy with agglomeration.*

Proposition 3 *In case $\hat{\phi} < \phi_I < \phi_B$ the economy starts in a pre-industrial phase, and as trade costs decrease the system moves through an early state with either agglomeration or dispersion and finally towards agglomeration (figure(2) in the downloadable appendix).*

In this case, industrialization emerges for a higher level of integration, but still lower than the one which makes the symmetric steady-state unstable. There are three phases:

1. $\phi \in (0, \phi_I)$: this case is identical to case 1 related to proposition 2

2. $\phi \in (\phi_I, \phi_B)$: for this level of freeness of trade, industrialization is activated when the symmetric steady-state is still stable but $f(0, \phi)$ is positive so that $f(0, \phi) \frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K} > 0$. Therefore, at the beginning of the industrial era, the economy is stable at the symmetric allocation but two other unstable non-symmetric interior steady-states exist. That makes the two CP allocations stable equilibria so that this case behaves as phase 3 related to proposition 2. We then call this situation an *early-industrialized economy with either dispersion or agglomeration.*

3. $\phi \in (\phi_B, 1)$: this case is identical to case 4 related to proposition 2.

Proposition 4 *In case $\hat{\phi} < \phi_B < \phi_I$ the economy starts in a pre-industrial phase and as soon as industrialization occurs the symmetric equilibrium looses its stability and the system moves towards agglomeration (figure (3) in the downloadable appendix)*

So industrialization emerges when the symmetric steady-state is already unstable. That happens when condition (18) holds. In this case there are only two phases:

1. $\phi \in (0, \phi_I)$: the economy is in a pre-industrial stage of development.

2. $\phi \in (\phi_I, 1)$: this case is quite interesting because industrialization emerges when $\phi > \phi_B$ already so that the symmetric steady state is unstable and catastrophic agglomeration is activated by any firm exogenously moving from South to North or vice-versa. We are the in an it *early industrialized economy with agglomeration.*

The previous taxonomy tells us, for any value of the trade costs, the number of interior steady-state and the dynamic properties of any steady-state, interior or not. As we can appreciate, the departures from the case with homothetic preferences are quite relevant. But if the possibility of structural change leads to relevant changes in the dynamics of locational equilibria, the effects on the growth prospect is even more dramatic.

4 Growth analysis

In NEG and growth models with homothetic preferences (Baldwin and Martin 2004), growth is affected by agglomeration *only* in the presence of localized spillovers which minimize the cost of innovation when the whole manufacturing sector is located in only one region. Moreover, integration affects the growth rate only indirectly, by inducing agglomeration.

In our model, the endogeneity of expenditure shares due to non-homotheticity creates an additional and alternative channel through which the rate of growth can be affected by both integration and agglomeration. With non-homothetic preferences, integration and agglomeration affect the growth rate through their impacts on the global expenditure on industrial goods which positively affects the growth rate. Therefore, and simply enough, since integration and agglomeration respectively increase and decrease the global expenditure on industrial goods, we have the following results: *integration is good for growth and agglomeration is bad for growth.*

By the market clearing conditions in the three sectors and the fact that $E = s_E E^w$ and $E^* = (1 - s_E) E^w$ as well as the definitions of $m(s_K, \phi)$ and $m^*(s_K, \phi)$ we can write:

$$E^w = \frac{(2L - g)\sigma}{(\sigma - \mu)} - \frac{(1-\mu)}{(\sigma - \mu)}\gamma Z(s_K, \phi) \tag{19}$$

where $Z(s_K, \phi) = \left((s_K + (1 - s_K)\phi)^{\frac{1}{1-\sigma}} + (1 - s_K + s_K\phi)^{\frac{1}{1-\sigma}}\right)$. Hence world expenditure is constant in equilibrium. An interesting feature of this equation is that γ reduces E^w: there is a range of values of γ for which the manufacture good, being a luxury good, is not consumed at all. Given that world expenditure can also be expressed as $E^w = 2L + \rho$ it is possible to find an expression for the growth rate by equating the RHS of the last two expressions:

$$g(s_K, \phi) = \frac{2L\mu - \rho(\sigma - \mu) - (1-\mu)\gamma Z(s_K, \phi)}{\sigma}$$

Alike the expenditure, also g depends negatively on γ. The higher γ, the larger the range of values for which the industrial good is not consumed and the incentive to invest in new units of knowledge capital is missing. As for the effect of integration on growth we have:

$$\frac{\partial g}{\partial \phi} = -\frac{(1-\mu)\gamma}{\sigma}\frac{\partial Z}{\partial \phi} > 0$$

hence, due to the non-homotheticity of the utility function, we have the following

Proposition 5 *Integration is good for growth*

In plain words, a decrease in trade costs increases both northern and southern expenditure shares in industrial goods, thereby increasing the global expenditure in industrial goods. The latter positively affects growth so that integration is always good for growth.

By contrast the effect of agglomeration is:

$$\frac{\partial g}{\partial s_K} = -\frac{(1-\mu)\,\gamma}{\sigma}\frac{\partial Z}{\partial s_K} \geq (<) 0 \Leftrightarrow s_K \leq (>) \frac{1}{2} \tag{20}$$

This proves the following proposition

Proposition 6 *Agglomeration is bad for growth whose rate is maximized when* $s_K = \frac{1}{2}$

Why is it so? Let us see it in detail. In general, the growth rate can be written as:

$$g = \frac{m\,(s_K,\phi)\,(L+\rho s_K) + m^*\,(s_K,\phi)\,(L+\rho\,(1-s_K)) - \rho\sigma}{\sigma}$$

This expression allows us to see the growth rate as a positive function of the aggregate expenditure in industrial goods, i.e. $m\,(s_K,\phi)\,(L+\rho s_K) + m^*\,(s_K,\phi)\,(L+\rho\,(1-s_K))$. We can also appreciate the double effect of s_K on g. On the one hand, it increases growth thanks to the effect on northern industrial expenditure $m\,(s_K,\phi)\,(L+\rho s_K)$; on the other it reduces it thanks to its effect on southern industrial expenditure $m^*\,(s_K,\phi)\,(L+\rho\,(1-s_K))$:

$$\frac{\partial g}{\partial s_K} = \frac{1}{\sigma}\left(\underbrace{\frac{\partial m}{\partial s_K}\,(L+\rho s_K) + \rho m}_{\text{North-effect}} + \underbrace{\frac{\partial m^*}{\partial s_K}\,(L+\rho\,(1-s_K)) - \rho m^*}_{\text{South-effect}}\right)$$

where the North-effect is positive, while the South-effect is negative. Clearly the sign of the expression depends on the sign of the term inside brackets on the right hand side. So that:

$$s_K > (<)\,\frac{1}{2} \Longleftrightarrow \underbrace{\frac{\partial m}{\partial s_K}\,(L+\rho s_K) + \rho m}_{\text{North-effect}} + \underbrace{\frac{\partial m^*}{\partial s_K}\,(L+\rho\,(1-s_K)) - \rho m^*}_{\text{South-effect}} < (>)\,0$$

so if $s_K > \frac{1}{2}$ the negative effect induced by the decrease of the industrial expenditure in the South overcomes the positive effect of the increase in northern industrial expenditure. That happens because with non homothetic utility functions aggregate expenditure on industrial goods is maximum when industrial firms are evenly distributed in the two regions. This asymmetry leads to the fact that, following the concentration of industry in the North, the positive effect on Northern industrial expenditure would be offset by the negative effect on Southern industrial expenditure.

4.1 Agglomeration as a Function of Integration

As shown in section 3, the equilibrium value of the share of firms located in the North s_K can be thought as a function of the freeness of trade ϕ. As a matter of fact, we can think that when $\phi < \hat{\phi} < \phi_B$ the most likely steady-state (the only stable) is the symmetric one. On the opposite side, when $\hat{\phi} < \phi_B < \phi$ the most likely equilibria is one of the two CP allocations. For intermediate values of the freeness of trade, $\hat{\phi} < \phi < \phi_B$, we have three stable equilibria but if the economy locates itself in the symmetric steady-state it will stay there unless a very strong perturbation moves s_K within the attraction set of one of the two CP outcomes. Therefore, even when $\hat{\phi} < \phi < \phi_B$, the most likely steady-state is the symmetric one. And, for a given value of the freeness of trade inside this range, it is also the faster-growing equilibrium because of (20). Hence, there are good reasons to consider the symmetric and the CP equilibria as the most likely outcomes being the only steady-states that become stable depending on the value of ϕ.

How does growth look like in these equilibria? The aim of this section is to find the optimal level of economic integration with respect to growth. In other words, considering that integration affects both the allocation of industrial firms *and* the growth rate, what is the level of freeness of trade that a hypothetical central planner would choose in order to maximize growth?

For a given value of ϕ, the growth rate in these two different equilibria will be[10]:

$$g\left(\frac{1}{2},\phi\right) = \frac{2L\mu-\rho\left(\sigma-\mu\right)-\left(1-\mu\right)\gamma Z\left(\frac{1}{2},\phi\right)}{\sigma}\text{for } \phi\leq\phi_B$$

$$g\left(1,\phi\right) = \frac{2L\mu-\rho\left(\sigma-\mu\right)-\left(1-\mu\right)\gamma Z\left(1,\phi\right)}{\sigma}\text{for } \phi>\phi_B$$

It is not obvious which of the two growth rates will be higher: the first growth rate displays a lower degree of integration (which is growth-enhancing) but a lower degree of agglomeration (which is growth-detrimental) with respect to the second. It is easy to see that there is a neighborhood $(\phi_B-\epsilon,\phi_B+\epsilon)$ such that $g\left(\frac{1}{2},\phi_B-\epsilon\right) > g\left(1,\phi_B+\epsilon\right)$ so that it might be optimal to reduce the degree of integration in order to increase growth. However, if we assume that the government has total control on ϕ, a central planner would choose the level of ϕ which maximizes both growth rate for any given locational equilibria. This brings us to the last proposition

Proposition 7 *The positive effect of integration on growth more than compensates for the negative effect of agglomeration*

In order to prove this proposition we must consider that since ϕ always increases growth, the central planner would actually choose the maximum value of ϕ compatible with the two equilibria - i.e. either $\phi=\phi_B$ or $\phi=1$ according to whether growth is maximized respectively in the symmetric or in CP equilibrium. Clearly we always have:

$$g\left(\frac{1}{2},\phi_B\right)<g\left(1,1\right)$$

because $Z(1,1)=2<Z\left(\frac{1}{2},\phi_B\right)=2\left(\frac{1+\phi_B}{2}\right)^{\frac{1}{1-\sigma}}$. Hence an hypothetical central planner willing to maximize growth at the aggregate level will always choose the maximum level of freeness of trade, $\phi=1$, even if it will lead to a CP outcome.

5 Conclusions

We have presented a NEG and growth model where consumers have non-homothetic preferences and economic growth stems from new investments in R&D made by industrial firms. The main message of our paper is that non-homothetic preferences (which are widely considered as a relevant feature of real economies) uncover some theoretical mechanisms which are worth being empirically tested.

Given the appeal that NEG models has on policy-makers, we believe our results to have important policy implications as, for some plausible range of parameters' values, they are opposite to the commonly accepted models. This is particularly true concerning

[10]Without loss of generality, we assume that for $\phi=\phi_B$ the symmetric steady-state will be stable.

the way growth is affected by integration and agglomeration. The existing literature agrees in fact with the following statements: 1) integration affects growth only indirectly and only when spillovers are localized (Baldwin et al. (2001)); 2) agglomeration is never bad for growth and it boosts growth when spillovers are localized (from Martin (1999) onwards). These statements - as we have seen in details - are not true when preferences are non-homothetic and led to structural change. Hence our main normative message is that models of agglomeration and growth should not neglect the effect of trade costs and industry location on expenditure shares otherwise they can suggest misleading policy implications.

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Appendix A: Case of Capital Mobility

When capital is mobile, profits are repatriated back home so there is no connection between the set up of a new firm (ds_K) and the variation in regional expenditure (ds_E). Hence $\frac{ds_E}{\partial ds_K} = 0$ so that the demand-linked effect disappears and any equilibrium allocation is unstable if the quantity:

$$\left(\frac{\partial m^*}{\partial s_K} m - \frac{\partial m}{\partial s_K} m^*\right) (\phi s_K + 1 - s_K) (s_K + (1 - s_K) \phi) + m m^* (1 - \phi) (1 + \phi) \qquad \text{(A1)}$$

is negative. Notice that, when $\gamma = 0$, and then $\frac{\partial m^*}{\partial s_K} m = \frac{\partial m}{\partial s_K} m^*$, this can *never* be true and then every initial allocation is stable. However, with non-homothetic preferences, any steady-state allocation (in particular the symmetric one) might become unstable for sufficiently low trade costs even when capital is mobile. That can happen because when the expenditure effect is strong enough, a new industrial investment in the North might be able to generate a price reduction sufficiently large to induce an increase in northern industrial expenditure shares which in turn would able to raise northern industrial profits until more than compensating for the negative effect of the market-crowding effect.

Appendix B: proof of proposition 1

Consider $f(s_K,\phi)$, which, according to (11) and (12) can also be written as

$$f(s_K,\phi)=\frac{m^*(s_K,\phi)(s_K+(1-s_K)\phi)}{m(s_K,\phi)(\phi s_K+1-s_K)+m^*(s_K,\phi)(s_K+(1-s_K)\phi)}-\frac{L+\rho s_K}{2L+\rho} \quad \text{(A2)}$$

By substituting for the explicit form of the expenditure shares as expressed in (8) and (9), we find:

$$f(s_K,\phi)=\frac{\mu(s_K+(1-s_K)\phi)[L+\rho s_K][L+\rho(1-s_K)]-(1-\mu)\gamma(\phi s_K+(1-s_K))^{\frac{1}{1-\sigma}}(s_K+(1-s_K)\phi)[L+\rho s_K]}{\mu(1+\phi)[L+\rho s_K][L+\rho(1-s_K)]-(1-\mu)\gamma k(s_K,\phi)}-\frac{L+\rho s_K}{2L+\rho} \quad \text{(A3)}$$

where:

$$k(s_K,\phi)=(s_K+(1-s_K)\phi)(\phi s_K+(1-s_K))\left[(s_K+(1-s_K)\phi)^{\frac{\sigma}{1-\sigma}}[L+\rho(1-s_K)]+[L+\rho s_K](\phi s_K+(1-s_K))^{\frac{\sigma}{1-\sigma}}\right] \quad \text{(A4)}$$

Notice that is $f(s_K,\phi)$ symmetric with respect to the point $\left(\frac{1}{2},f\left(\frac{1}{2}\right)\right)$ meaning that $f(s_K)=-f(1-s_K)$. This symmetry is very important as it allows us to limit the analysis to the interval $s_K\in\left[0;\frac{1}{2}\right)$ and then extend it to the rest of the feasible values of $s_K\in\left[\frac{1}{2},1\right]$, by simply applying the symmetry rule.

Define now the function:

$$h(s_K,\phi)=\frac{f(s_K,\phi)}{N(s_K,\phi)}=\mu(1-2s_K)(\rho\phi-L(1-\phi))-(1-\mu)\gamma\begin{bmatrix}(\phi s_K+(1-s_K))^{\frac{1}{1-\sigma}}(s_K+(1-s_K)\phi)\\-(s_K+(1-s_K)\phi)^{\frac{1}{1-\sigma}}(\phi s_K+1-s_K)\end{bmatrix} \quad \text{(A5)}$$

where:

$$N(s_K,\phi)=\frac{(L+\rho s_K)(L+\rho(1-s_K))}{(2L+\rho)[\mu(1+\phi)[L+\rho s_K][L+\rho(1-s_K)]-(1-\mu)\gamma k(s_K,\phi)]}>0 \quad \text{(A6)}$$

Since $N(s_K,\phi)$ is positive for any $s_K\in[0,1]$ and for any $\phi>\phi_I$, we have that $f(s_K,\phi)=0 \iff h(s_K,\phi)=0$: every zero of $h(s_K,\phi)$ is also an interior steady state and vice-versa. In particular, it is easy to see that $h\left(\frac{1}{2},\phi\right)=0$.

We can then re-write $f(s_K,\phi)$ as:

$$f(s_K,\phi)=h(s_K,\phi)N(s_K,\phi) \quad \text{(A7)}$$

By differentiating with respect to s_K we find:

$$\frac{\partial f(s_K,\phi)}{\partial s_K}=\frac{\partial h(s_K,\phi)}{\partial s_K}N(s_K,\phi)+h(s_K,\phi)\frac{\partial N(s_K,\phi)}{\partial s_K} \quad \text{(A8)}$$

but, as we have seen:

$$h\left(\frac{1}{2},\phi\right)=0 \quad \text{(A9)}$$

so that:

$$\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}=\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}N\left(\frac{1}{2},\phi\right) \quad \text{(A10)}$$

thereby we can conclude that:

$$sign\left[\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}\right]=sign\left[\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}\right] \quad \text{(A11)}$$

so that, in the symmetric allocation, the sign of the slope of f is the same as the sign of the slope of h. This is also an important property which allows us to concentrate on $h\left(s_K,\phi\right)$ which is much easier to deal with from the mathematical point of view and it is of great help in the proof of the following proposition.

So now focus on the function:

$$h\left(s_K,\phi\right)=\mu\left(1-2s_K\right)\left(\rho\phi-L\left(1-\phi\right)\right)-\left(1-\mu\right)\gamma\left[\begin{array}{c}\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{1}{1-\sigma}}\left(\phi s_K+1-s_K\right)\\-\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{1}{1-\sigma}}\left(s_K+\left(1-s_K\right)\phi\right)\end{array}\right] \tag{A12}$$

An interior equilibrium is a value of $s_K\in[0,1]$ such that $h\left(s_K,\phi\right)=0$. We already know that $h\left(\frac{1}{2},\phi\right)=0$. Moreover, it's easy to see that $h\left(s_K,\phi\right)=-h\left(\left(1-s_K\right),\phi\right)$. Hence, for any interior steady state in $\left[0,\frac{1}{2}\right)$, there must be another interior steady state in $\left(\frac{1}{2},1\right]$. More formally, if $s_K^*\in\left[0,\frac{1}{2}\right)$ is such that $h\left(s_K^*,\phi\right)=0$, then there is a $s_K^{**}=\left(1-s_K^*\right)\in\left(\frac{1}{2},1\right]$ such that:

$$h\left(s_K^{**},\phi\right)=h\left(\left(1-s_K^*\right),\phi\right)=0 \tag{A13}$$

For these reasons, to prove that there are at most 3 interior steady states is sufficient to show that there can be at most 1 interior steady state in $\left[0,\frac{1}{2}\right)$. I.e., we need to show that the function $h\left(s_K,\phi\right)$ can cross the horizontal axis at most once in the interval $\left[0,\frac{1}{2}\right)$

The symmetry of f and h also implies that:

$$\frac{\partial h\left(s_K,\phi\right)}{\partial s_K}=\frac{\partial h\left(\left(1-s_K\right),\phi\right)}{\partial s_K} \tag{A14a}$$

$$\frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2}=-\frac{\partial^2 h\left(\left(1-s_K\right),\phi\right)}{\left(\partial s_K\right)^2} \tag{A14b}$$

As a consequence there is an inflexion point in $s_K=\frac{1}{2}$ as:

$$\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\left(\partial s_K\right)^2}=-\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\left(\partial s_K\right)^2}=0 \tag{A15}$$

We also have that:

$$\frac{\partial h}{\partial s_K}=-\left(\begin{array}{c}2\mu\left[\phi\rho-L\left(1-\phi\right)\right]\\+\left(1-\mu\right)\gamma\left(1-\phi\right)\left[\begin{array}{c}\frac{1}{\sigma-1}\left[\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{1}{1-\sigma}-1}\left(s_K+\left(1-s_K\right)\phi\right)+\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{1}{1-\sigma}-1}\left(\phi s_K+1-s_K\right)\right]\\+\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{1}{1-\sigma}}+\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{1}{1-\sigma}}\end{array}\right]\end{array}\right) \tag{A16a}$$

$$\frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2}=-\frac{\left(1-\mu\right)\gamma\left(1-\phi\right)^2}{\sigma-1}\left[\begin{array}{c}\frac{\sigma}{1-\sigma}\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(\phi s_K+1-s_K\right)}{\left(s_K+\left(1-s_K\right)\phi\right)}-2\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\\-\frac{\sigma}{1-\sigma}\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(s_K+\left(1-s_K\right)\phi\right)}{\left(\phi s_K+\left(1-s_K\right)\right)}+2\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\end{array}\right] \tag{A16b}$$

A necessary and sufficient condition for the single-crossing properties of h is the monotonicity of $\frac{\partial h(s_K,\phi)}{\partial s_K}$ in $\left[0,\frac{1}{2}\right)$ or, equivalently, the fact that $\frac{\partial^2 h(s_K,\phi)}{\left(\partial s_K\right)^2}$ does not change sign in $\left[0,\frac{1}{2}\right)$. We prove that this is the case.

Notice that:

$$sign\frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2}=-sign\left[\begin{array}{c}\frac{\sigma}{1-\sigma}\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(\phi s_K+1-s_K\right)}{\left(s_K+\left(1-s_K\right)\phi\right)}-2\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\\-\frac{\sigma}{1-\sigma}\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(s_K+\left(1-s_K\right)\phi\right)}{\left(\phi s_K+\left(1-s_K\right)\right)}+2\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\end{array}\right] \tag{A17}$$

re-write the member on the RHS to obtain:

$$\left(\phi s_K+\left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{\left(s_K+\left(1-s_K\right)\phi\right)}{\left(\phi s_K+\left(1-s_K\right)\right)}+2\right)-\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{\left(\phi s_K+1-s_K\right)}{\left(s_K+\left(1-s_K\right)\phi\right)}+2\right) \tag{A18}$$

Notice that:

$$s_K < \frac{1}{2} \Leftrightarrow \frac{(s_K + (1 - s_K)\,\phi)}{(\phi s_K + (1 - s_K))} < 1 < \frac{(\phi s_K + 1 - s_K)}{(s_K + (1 - s_K)\,\phi)} \tag{A19}$$

so that:

$$s_K < \frac{1}{2} \Leftrightarrow \left(\frac{\sigma}{\sigma - 1}\frac{(s_K + (1 - s_K)\,\phi)}{(\phi s_K + (1 - s_K))} + 2\right) < \left(\frac{\sigma}{\sigma - 1}\frac{(\phi s_K + 1 - s_K)}{(s_K + (1 - s_K)\,\phi)} + 2\right) \tag{A20}$$

Moreover, since $\sigma > 1$:

$$s_K < \frac{1}{2} \Leftrightarrow (\phi s_K + (1 - s_K))^{\frac{\sigma}{1-\sigma}} < (s_K + (1 - s_K)\,\phi)^{\frac{\sigma}{1-\sigma}} \tag{A21}$$

hence it should be that when $s_K < \frac{1}{2}$ we have that

$$(\phi s_K + (1 - s_K))^{\frac{\sigma}{1-\sigma}} \left(\frac{\sigma}{\sigma - 1}\frac{(s_K + (1 - s_K)\,\phi)}{(\phi s_K + (1 - s_K))} + 2\right) < (s_K + (1 - s_K)\,\phi)^{\frac{\sigma}{1-\sigma}} \left(\frac{\sigma}{\sigma - 1}\frac{(\phi s_K + 1 - s_K)}{(s_K + (1 - s_K)\,\phi)} + 2\right) \tag{A22}$$

so that:

$$s_K < \frac{1}{2} \Leftrightarrow \frac{\partial^2 h\,(s_K, \phi)}{(\partial s_K)^2} > 0 s_K > \frac{1}{2} \Leftrightarrow \frac{\partial^2 h\,(s_K, \phi)}{(\partial s_K)^2} < 0 \tag{A23}$$

Hence $\frac{\partial h(s_K,\phi)}{\partial s_K}$ is monotone (and increasing) in $\left[0, \frac{1}{2}\right)$. Therefore h can have at most 1 zero in $\left[0, \frac{1}{2}\right)$ and, by the symmetry rule and since $h\left(\frac{1}{2}, \phi\right) = 0$, h can have at most 3 zeros in $[0, 1]$. And since $h\,(s_K^*, \phi) = 0 \Leftrightarrow f\,(s_K^*, \phi) = 0$, we have shown that the interior steady state allocations (i.e. the values of $s_K \in (0, 1)$ such that $f\,(s_K, \phi) = 0$) can be *1 or at most* **3**.

Once we have limited the number of equilibria, we are almost ready to provide the necessary and sufficient condition for uniqueness and multiplicity. A necessary and sufficient condition for the existence of three distinct interior steady states is the following:

$$h\,(0, \phi)\,\frac{\partial h\left(\frac{1}{2}, \phi\right)}{\partial s_K} > 0$$

We first show that the condition is sufficient. If $h\,(0, \phi)\,\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0$ then $h\,(0, \phi) > 0\,(< 0)$ when $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0\,(< 0)$. Since $h\left(\frac{1}{2}, \phi\right) = 0$ and h is continuous, h must cross the horizontal axis at least once and thus there must be *at least* one $s_K^* \in \left[0, \frac{1}{2}\right)$ such that $h\,(s_K^*, \phi) = 0$. As we have already shown such value is unique. Hence, by the symmetry rule, $h\,(0, \phi)\,\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0$ is a sufficient condition for the existence of **three** interior steady state allocations in the whole interval $[0, 1]$. As for necessity, assume that there are three points $s_K^* \in \left[0, \frac{1}{2}\right)$, $\bar{s}_K = \frac{1}{2}$ and $s_K^{**} = 1 - s_K^*$ such that $h\,(s_K^*, \phi) = h\left(\frac{1}{2}, \phi\right) = h\,(1 - s_K^*, \phi) = 0$. When $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0\ (< 0)$, since $h\left(\frac{1}{2}, \phi\right) = 0$ and $h\,(s_K, \phi)$ crosses the horizontal axis only once in $\left[0, \frac{1}{2}\right)$, then it must be $h\,(0, \phi) > 0\,(< 0)$. By contrast, when $h\,(0, \phi)\,\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} \leq 0$, the *interior steady state allocation is unique and equal to the symmetric allocation* $s_K = \frac{1}{2}$. As for $h\,(0, \phi)\,\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} < 0$, it is sufficient to notice that, since the necessary condition for 3 interior steady states does not apply and since there cannot be more than 3 interior steady states, hence there is only one interior steady state, the symmetric allocation $\bar{s}_K = \frac{1}{2}$. As for the knife-edge case $h\left(0,\phi\right)\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} = 0$, again the interior steady-state is unique because of the following reasoning. We have three possible cases:

1. $h\left(0,\phi\right) = 0$ and $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} \neq 0$. In this case, since we already know that there is only one $s_K^* \in \left[0,\frac{1}{2}\right)$ such that $h\left(s_K^*,\phi\right) = 0$, then it should be $s_K^* = 0$ which does not satisfy the definition of interior steady state [11].

2. $h\left(0,\phi\right) \neq 0$ and $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} = 0$. Since $\frac{\partial h(s_K,\phi)}{\partial s_K}$ is monotone in $s_K \in \left[0,\frac{1}{2}\right]$ there cannot be other $s_K \in \left(0,\frac{1}{2}\right)$ such that $\frac{\partial h(s_K,\phi)}{\partial s_K} = 0$. Hence h cannot cross the horizontal axis in $\left(0,\frac{1}{2}\right)$ and the symmetric equilibrium is unique in $s_K \in [0,1]$.

3. $h\left(0,\phi\right) = 0$ and $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} = 0$: this case is ruled out by the strict monotonicity of $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}$.

Appendix C: Unique ϕ_B and $\hat{\phi}$

Along the text we have assumed that ϕ_B (the value of the freeness of trade such that $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ switches from negative to positive) and $\hat{\phi}$ (the value of the freeness of trade such that $f(0,\phi)$ turns from positive to negative) are both unique. We now express these conditions in terms of the value of the model's parameters.

First focus on ϕ_B. From Section A we already know that we can write:

$$\frac{\partial f(\frac{1}{2},\phi)}{\partial s_K} = \frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} N\left(\frac{1}{2},\phi\right) \tag{A24}$$

where $N\left(\frac{1}{2},\phi\right)$ is always positive. Hence, to study the sign of $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ we can focus on the sign of $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}$. The latter, by differentiating $h(s_K,\phi)$ with respect to s_K and then setting $s_K = 1/2$, can be written as

$$\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} = 2\mu\left(L+\rho\right)\left[\frac{L}{L+\rho} - \phi - \frac{(1-\mu)\,\gamma(1-\phi)}{\mu(L+\rho)}\left(\frac{1+\phi}{2}\right)^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\right] \tag{A25}$$

We know that the symmetric allocation (which is always a steady state) is unstable if $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} < 0$. We aim at studying how the sign of this partial derivative changes along the feasible interval of ϕ that is $(\phi_I,1)$. Let's check the sign at the extreme of the interval. When trade costs are nil we have

$$\frac{\partial h\left(\frac{1}{2},1\right)}{\partial s_K} = -2\mu\rho < 0 \tag{A26}$$

meaning that for very low trade costs the symmetric equilibrium is *always unstable*. This result is in line with the existing literature.

[11] Actually in this case $s_K^* = 0$ is a core-periphery equilibrium which also satisfies the interior equilibrium property, i.e., it is such that $f\left(s_K^*\right) = 0$. By contrast, the core-periphery outcome needs not to satisfy this condition.

At the industrialization-triggering value of trade costs, that is, ϕ_I, we have:

$$\frac{\partial h\left(\frac{1}{2},\phi_I\right)}{\partial s_K}=2\mu\left(L+\rho\right)\left[\frac{L}{L+\rho}-\phi_I-\frac{1-\mu}{\mu}\frac{\gamma}{L+\rho}(1-\phi_I)\left(\frac{1+\phi_I}{2}\right)^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\right] \tag{A27}$$

which can be either positive or negative. As a consequence, for high trade costs (yet not too high, because otherwise industrialization would not be feasible), the symmetric equilibrium might be either stable or unstable. Hence, in order to have at most one break-point value, it is sufficient that $\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K\partial\phi}<0$. By computation we obtain:

$$\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K\partial\phi}=2\mu\left(L+\rho\right)\left[-1+\frac{\sigma}{\sigma-1}\frac{(1-\mu)\gamma}{\mu(L+\rho)}\left(\frac{1+\phi}{2}\right)^{\frac{1}{1-\sigma}}\left[1+\frac{1}{\sigma-1}\frac{1-\phi}{1+\phi}\right]\right] \tag{A28}$$

so that:

$$\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K\partial\phi}<0\Longleftrightarrow\frac{(1-\mu)\gamma}{\mu(L+\rho)}<\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\left[\frac{(\sigma-1)\left(1+\phi\right)}{(\sigma-1)\left(1+\phi\right)+1-\phi}\right] \tag{A29}$$

This condition tells us that, in order for the break-point to be unique, the quantity $\frac{(1-\mu)\gamma}{\mu(L+\rho)}$ should not be too high.

Let's now focus on $\hat{\phi}$. We observe that

$$h\left(0,1\right)=\mu\rho>\ 0 \tag{A30}$$

while $h\left(0,\phi_I\right)$, might be positive or negative. Similarly to what we have done for ϕ_B, in order to guarantee that there is at most one value of ϕ - call it $\hat{\phi}$ - such that $h(0,\phi)=0$, it is sufficient (but not necessary) to assume that the derivative of $h(0,\phi)$ with respect to ϕ is positive for any $\phi\in(\phi_I,1)$. When this is the case, $h\left(0,\phi\right)$ always increases with ϕ.

If we compute $\frac{\partial h(0,\phi)}{\partial\phi}$ we find:

$$\frac{\partial h\left(0,\phi\right)}{\partial\phi}=\mu\left(L+\rho\right)-\left(1-\mu\right)\gamma\left[\frac{(\sigma-1)\,\phi^{\frac{\sigma}{\sigma-1}}+1}{(\sigma-1)\,\phi^{\frac{\sigma}{\sigma-1}}}\right] \tag{A31}$$

so that:

$$\frac{\partial h\left(0,\phi\right)}{\partial\phi}>0\Longleftrightarrow\frac{\left(1-\mu\right)\gamma}{\mu\left(L+\rho\right)}<\frac{(\sigma-1)\,\phi^{\frac{\sigma}{\sigma-1}}}{(\sigma-1)\,\phi^{\frac{\sigma}{\sigma-1}}+1} \tag{A32}$$

Hence both conditions can be expressed in terms of $\frac{(1-\mu)\gamma}{\mu(\rho+L)}$ and in both cases the RHS of the condition is a quantity which should be smaller than 1 in order to ensure uniqueness. Unfortunately, we cannot say which of the two RHS is larger but we can anyway express the requirements for the uniqueness of ϕ_B and $\hat{\phi}$ with a single condition. Our analysis then shows that $\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K\partial\phi}<0$ and $\frac{\partial h(0,\phi)}{\partial\phi}>0$ for any $\phi\in(\phi_I,1)$ if:

$$\frac{\left(1-\mu\right)\gamma}{\mu\left(\rho+L\right)}<min\left(\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\left[\frac{(\sigma-1)\left(1+\phi\right)}{(\sigma-1)\left(1+\phi\right)+1-\phi}\right];\frac{(\sigma-1)\,\phi^{\frac{\sigma}{\sigma-1}}}{(\sigma-1)\,\phi^{\frac{\sigma}{\sigma-1}}+1}\right) \tag{A33}$$

This last relation tells us that a single break-point ϕ_B and a single $\hat{\phi}$ is guaranteed if γ is not too large, μ,ρ and L are not too small. To see that this condition is not particularly strong notice that the positivity of the industrial expenditure shares requires (in the North) that:

$$m\left(s_K,\phi\right)>0\Longleftrightarrow\frac{\left(1-\mu\right)\gamma}{\mu\left(L+\rho s_K\right)}<\left(s_K+\left(1-s_K\right)\phi\right)^{\frac{1}{\sigma-1}} \tag{A34}$$

In the symmetric equilibrium this condition becomes:

$$m\left(\frac{1}{2},\phi\right)>0 \Longleftrightarrow \frac{(1-\mu)\gamma}{\mu(L+\rho)}<\frac{2L+\rho}{2(L+\rho)}\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}<1 \tag{A35}$$

This condition, which should always be true for the industrial sector to exist, can be expressed as well in terms of $\frac{(1-\mu)\gamma}{\mu(L+\rho)}$ and tells us that the industrial sector exists if the latter quantity is smaller than a quantity which is itself smaller than 1. In other words, even the same existence of the industrial sector is guaranteed if γ is not too large, μ,ρ and L are not too small. Hence, there is a wide range of parameters' values such that the condition:

$$\frac{2L+\rho}{2(L+\rho)}\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}<min\left(\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\left[\frac{(\sigma-1)(1+\phi)}{(\sigma-1)(1+\phi)+1-\phi}\right];\frac{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}}{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}+1}\right) \tag{A36}$$

is true for any $\phi\in(\phi_I,1)$. If this is the case, the same emergence of the industrial sector guarantees that $\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K\partial\phi}<0$ and $\frac{\partial h(0,\phi)}{\partial\phi}>0$ and therefore ϕ_B and $\hat{\phi}$ are always unique.

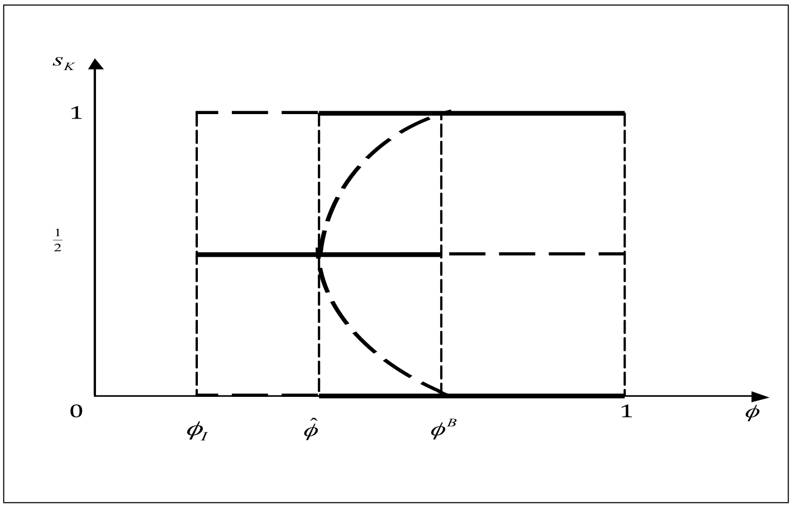
We have also assumed that $\frac{\partial h(0,\phi)}{\partial\phi}>0$ which is a sufficient (but not necessary) condition in order for the existence of a single ϕ (we have called it $\hat{\phi}$) which changes the sign of $h(0,\phi)$ from negative to positive.

Appendix D: $\hat{\phi}$ is always lower than ϕ_B

We provide the proof for the interval $s_K\in\left[0,\frac{1}{2}\right)$ as, by symmetry, the same argument applies for $s_K\in\left[\frac{1}{2},1\right]$. We know that, by definition, $f\left(0,\widehat{\phi}\right)=h\left(0,\widehat{\phi}\right)=0$. We also know that $f\left(\frac{1}{2},\phi\right)=h\left(\frac{1}{2},\phi\right)=0$. As the latter relationship is true for any ϕ, it also holds when $\phi=\hat{\phi}$. Hence, $h\left(0,\widehat{\phi}\right)=h\left(\frac{1}{2},\hat{\phi}\right)=0$. By proposition 1, we also know that, since $\frac{\partial^2 h(s_K,\phi)}{\partial s_K^2}$ is positive for $s_K\in\left[0,\frac{1}{2}\right)$, there cannot be other value of $s_K\in\left[0,\frac{1}{2}\right)$ such that $h\left(s_K,\widehat{\phi}\right)=0$. Now $\frac{\partial^2 h(s_K,\phi)}{\partial s_K^2}>0$ for $s_K\in\left[0,\frac{1}{2}\right)$ implies that $\frac{\partial h\left(\frac{1}{2},\widehat{\phi}\right)}{\partial s_K}>\frac{\partial h\left(0,\widehat{\phi}\right)}{\partial s_K}$. Moreover, since $h\left(0,\widehat{\phi}\right)=h\left(\frac{1}{2},\hat{\phi}\right)=0$, by continuity of h and $\frac{\partial h(s_K,\phi)}{\partial s_K}$, there should be a unique $s'_K\in\left(0,\frac{1}{2}\right)$ where $\frac{\partial h\left(s'_K,\phi\right)}{\partial s_K}=0$ and $\frac{\partial h(s_K,\phi)}{\partial s_K}$, being $\frac{\partial^2 h(s_K,\phi)}{\partial s_K^2}$ positive in the whole interval, switches sign from negative to positive. As a consequence, we have $\frac{\partial h\left(\frac{1}{2},\widehat{\phi}\right)}{\partial s_K}>0>\frac{\partial h\left(0,\widehat{\phi}\right)}{\partial s_K}$. Since $\frac{\partial h\left(\frac{1}{2},\phi_B\right)}{\partial s_K}=0$ and $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is decreasing in ϕ (provided that γ is small enough) then it must be $\phi_B>\widehat{\phi}$.

Appendix E: Stability Maps

Figure 1: Stability map when $\phi_I < \hat{\phi} < \phi_B$



The dotted and solid lines represent respectively the unstable and the stable equilibria

Figure 2: Stability map when $\hat{\phi} < \phi_I < \phi_B$

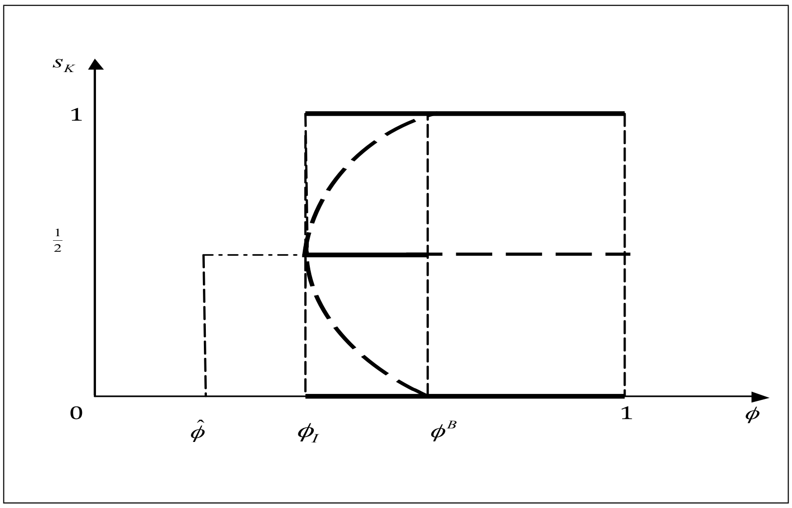
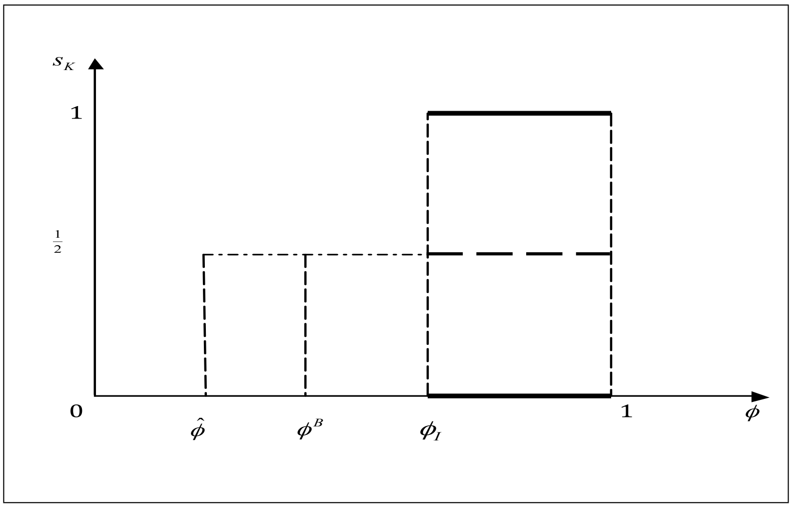


Figure 3: Stability map when $\hat{\phi} < \phi_B < \phi_I$



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Appendix A: proof of proposition 1

Consider the function:

$$h(s_K,\phi) \quad = \quad \mu(1-2s_K)(\rho\phi - L(1-\phi)) - (1-\mu)\gamma \begin{bmatrix} (s_K+(1-s_K)\phi)^{\frac{1}{1-\sigma}}(\phi s_K+1-s_K) \\ -(\phi s_K+(1-s_K))^{\frac{1}{1-\sigma}}(s_K+(1-s_K)\phi) \end{bmatrix}$$

An interior equilibrium is a value of $s_K \in [0,1]$ such that $h(s_K,\phi)=0$. We surely have $h\left(\frac{1}{2},\phi\right)=0$. Moreover, $h(s_K,\phi) = -h((1-s_K),\phi)$. Hence, for any interior steady state in $\left[0,\frac{1}{2}\right)$, there is also an interior steady state in $\left(\frac{1}{2},1\right]$. More formally, if $s_K^* \in \left[0,\frac{1}{2}\right)$ is such that $h(s_K^*,\phi)=0$, then there is a $s_K^{**}=(1-s_K^*) \in \left(\frac{1}{2},1\right]$ such that:

$$h(s_K^{**},\phi) = h((1-s_K^*),\phi) = 0$$

For these reasons, to prove that there are at most 3 interior steady states is sufficient to show that there can be at most 1 interior steady state in $\left[0,\frac{1}{2}\right)$. I.e., we need to show that the function $h(s_K,\phi)$ can cross the horizontal axis only once in the interval $\left[0,\frac{1}{2}\right)$

The symmetry of f also implies that:

$$\begin{aligned} \frac{\partial h\left(s_K,\phi\right)}{\partial s_K} &= \frac{\partial h\left(\left(1-s_K\right),\phi\right)}{\partial s_K} \\ \frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2} &= -\frac{\partial^2 h\left(\left(1-s_K\right),\phi\right)}{\left(\partial s_K\right)^2} \end{aligned}$$

As a consequence there is an inflexion point in $s_K = \frac{1}{2}$ as:

$$\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\left(\partial s_K\right)^2} = -\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\left(\partial s_K\right)^2} = 0$$

We also have that:

$$\frac{\partial h}{\partial s_K} = -\left(\begin{array}{c} 2\mu\left[\phi\rho - L\left(1-\phi\right)\right] \\ +\left(1-\mu\right)\gamma\left(1-\phi\right)\left[\begin{array}{c} \frac{1}{\sigma-1}\left[\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{1}{1-\sigma}-1}\left(s_K + \left(1-s_K\right)\phi\right) + \left(s_K + \left(1-s_K\right)\phi\right)^{\frac{1}{1-\sigma}-1}\left(\phi s_K + 1 - s_K\right)\right] \\ +\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{1}{1-\sigma}} + \left(s_K + \left(1-s_K\right)\phi\right)^{\frac{1}{1-\sigma}} \end{array} \right] \end{array} \right)$$

$$\frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2} = -\frac{\left(1-\mu\right)\gamma\left(1-\phi\right)^2}{\sigma-1}\left[\begin{array}{c} \frac{\sigma}{1-\sigma}\left(s_K + \left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(\phi s_K + 1 - s_K\right)}{\left(s_K + \left(1-s_K\right)\phi\right)} - 2\left(s_K + \left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}} \\ -\frac{\sigma}{1-\sigma}\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(s_K + \left(1-s_K\right)\phi\right)}{\left(\phi s_K + \left(1-s_K\right)\right)} + 2\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}} \end{array} \right]$$

A necessary and sufficient condition for the single-crossing properties of h is the monotonicity of $\frac{\partial h(s_K,\phi)}{\partial s_K}$ in $\left[0,\frac{1}{2}\right)$ or, equivalently, the fact that $\frac{\partial^2 h(s_K,\phi)}{(\partial s_K)^2}$ does not change sign in $\left[0,\frac{1}{2}\right)$. We prove that this is the case.

Notice that:

$$sign\frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2} = -sign\left[\begin{array}{c} \frac{\sigma}{1-\sigma}\left(s_K + \left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(\phi s_K + 1 - s_K\right)}{\left(s_K + \left(1-s_K\right)\phi\right)} - 2\left(s_K + \left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}} \\ -\frac{\sigma}{1-\sigma}\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\frac{\left(s_K + \left(1-s_K\right)\phi\right)}{\left(\phi s_K + \left(1-s_K\right)\right)} + 2\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}} \end{array} \right]$$

re-write the member on the RHS to obtain:

$$\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{\left(s_K + \left(1-s_K\right)\phi\right)}{\left(\phi s_K + \left(1-s_K\right)\right)} + 2\right) - \left(s_K + \left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{\left(\phi s_K + 1 - s_K\right)}{\left(s_K + \left(1-s_K\right)\phi\right)} + 2\right)$$

Notice that:

$$s_K < \frac{1}{2} \Leftrightarrow \frac{\left(s_K + \left(1-s_K\right)\phi\right)}{\left(\phi s_K + \left(1-s_K\right)\right)} < 1 < \frac{\left(\phi s_K + 1 - s_K\right)}{\left(s_K + \left(1-s_K\right)\phi\right)}$$

so that:

$$s_K < \frac{1}{2} \Leftrightarrow \left(\frac{\sigma}{\sigma-1}\frac{\left(s_K + \left(1-s_K\right)\phi\right)}{\left(\phi s_K + \left(1-s_K\right)\right)} + 2\right) < \left(\frac{\sigma}{\sigma-1}\frac{\left(\phi s_K + 1 - s_K\right)}{\left(s_K + \left(1-s_K\right)\phi\right)} + 2\right)$$

Moreover, since $\sigma > 1$:

$$s_K < \frac{1}{2} \Leftrightarrow \left(\phi s_K + \left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}} < \left(s_K + \left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}$$

combining 5 and 5 leads to:

$$\left(\phi s_K + \left(1-s_K\right)\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{\left(s_K + \left(1-s_K\right)\phi\right)}{\left(\phi s_K + \left(1-s_K\right)\right)} + 2\right) < \left(s_K + \left(1-s_K\right)\phi\right)^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{\left(\phi s_K + 1 - s_K\right)}{\left(s_K + \left(1-s_K\right)\phi\right)} + 2\right)$$

so that:

$$\begin{aligned} s_K &< \frac{1}{2} \Leftrightarrow \frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2} > 0 \\ s_K &> \frac{1}{2} \Leftrightarrow \frac{\partial^2 h\left(s_K,\phi\right)}{\left(\partial s_K\right)^2} < 0 \end{aligned}$$

Hence $\frac{\partial h(s_K,\phi)}{\partial s_K}$ is monotonic (and increasing) in $\left[0,\frac{1}{2}\right)$ so that $h\left(s_K,\phi\right)$ can cross the horizontal axis at most once in the same interval. And since $signh\left(s_k,\phi\right) = signf\left(s_K,\phi\right)$, then, there can be 1 (the symmetric allocation) or at most 3 interior steady-states.

Appendix B: Unique ϕ_B and $\hat{\phi}$

Along the text we have assumed that $\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K \partial \phi} < 0$ which is a sufficient (but not necessary) condition for the existence of a single break-point ϕ_B. We have also assumed that $\frac{\partial h(0,\phi)}{\partial \phi} > 0$ which is a sufficient (but not necessary) condition in order for the existence of a single ϕ (we have called it $\hat{\phi}$) which changes the sign of $h(0,\phi)$ from negative to positive. In the following, we provide the condition in terms of the models' parameters for this to happen.

By computation we obtain:

$$\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K \partial \phi} = 2\mu\left(L+\rho\right)\left[-1+\frac{\sigma}{\sigma-1}\frac{(1-\mu)\gamma}{\mu(L+\rho)}\left(\frac{1+\phi}{2}\right)^{\frac{1}{1-\sigma}}\left[1+\frac{1}{\sigma-1}\frac{1-\phi}{1+\phi}\right]\right]$$

so that:

$$\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K \partial \phi} < 0 \Longleftrightarrow \frac{(1-\mu)\gamma}{\mu(L+\rho)} < \left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\left[\frac{(\sigma-1)(1+\phi)}{(\sigma-1)(1+\phi)+1-\phi}\right]$$

As for $\frac{\partial h(0,\phi)}{\partial \phi}$ we have:

$$\frac{\partial h(0,\phi)}{\partial \phi} = \mu\left(L+\rho\right)-\left(1-\mu\right)\gamma\left[\frac{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}+1}{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}}\right]$$

so that:

$$\frac{\partial h(0,\phi)}{\partial \phi} > 0 \Longleftrightarrow \frac{(1-\mu)\gamma}{\mu(L+\rho)} < \frac{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}}{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}+1}$$

Notice that both condition can be expressed in terms of $\frac{(1-\mu)\gamma}{\mu(\rho+L)}$ and in both cases the RHS of the condition is a quantity which is smaller than 1. Unfortunately, we cannot say which of the two RHS is larger but we can anyway express the requirements for the uniqueness of ϕ_B and $\hat{\phi}$ with a single condition. Our analysis then shows that $\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K \partial \phi} < 0$ and $\frac{\partial h(0,\phi)}{\partial \phi} > 0$ for any $\phi \in (\phi_I, 1)$ if:

$$\frac{(1-\mu)\gamma}{\mu(\rho+L)} < min\left(\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\left[\frac{(\sigma-1)(1+\phi)}{(\sigma-1)(1+\phi)+1-\phi}\right];\frac{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}}{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}+1}\right)$$

This last relation tells us that a single break-point ϕ_B and a single $\hat{\phi}$ is guaranteed if γ is not too large, μ, ρ and L are not too small. To see that this condition is not particularly strong notice that the positivity of the industrial expenditure shares requires (in the North) that:

$$m(s_K,\phi) > 0 \Longleftrightarrow \frac{(1-\mu)\gamma}{\mu(L+\rho s_K)} < \left(s_K+(1-s_K)\phi\right)^{\frac{1}{\sigma-1}}$$

In the symmetric equilibrium this condition becomes:

$$m\left(\frac{1}{2},\phi\right) > 0 \Longleftrightarrow \frac{(1-\mu)\gamma}{\mu(L+\rho)} < \frac{2L+\rho}{2(L+\rho)}\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}} < 1$$

This condition, which should always be true for the industrial sector to exist, can be expressed as well in terms of $\frac{(1-\mu)\gamma}{\mu(L+\rho)}$ and tells us that the industrial sector exists if the latter quantity is smaller than a quantity which is itself smaller than 1. In other words, even the same existence of the industrial sector is guaranteed if γ is not too large, μ, ρ and L are not too small. Hence, there is a wide range of parameters' values such that the condition:

$$\frac{2L+\rho}{2(L+\rho)}\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}} < min\left(\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\left[\frac{(\sigma-1)(1+\phi)}{(\sigma-1)(1+\phi)+1-\phi}\right];\frac{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}}{(\sigma-1)\phi^{\frac{\sigma}{\sigma-1}}+1}\right)$$

is true for any $\phi \in (\phi_I, 1)$. If this is the case, the same emergence of the industrial sector guarantees that $\frac{\partial^2 h\left(\frac{1}{2},\phi\right)}{\partial s_K \partial \phi} < 0$ and $\frac{\partial h(0,\phi)}{\partial \phi} > 0$ and therefore ϕ_B and $\hat{\phi}$ are always unique.

Appendix C: $\hat{\phi}$ is always lower than ϕ_B

We provide the proof for the interval $s_K \in \left[0, \frac{1}{2}\right)$ as, by symmetry, the same argument applies for $s_K \in \left[\frac{1}{2}, 1\right]$. We know that, by definition, $f\left(0, \widehat{\phi}\right) = h\left(0, \widehat{\phi}\right) = 0$. We also know that $f\left(\frac{1}{2}, \phi\right) = h\left(\frac{1}{2}, \phi\right) = 0$. As the latter relationship is true for any ϕ, it also holds when $\phi = \hat{\phi}$. Hence, $h\left(0, \widehat{\phi}\right) = h\left(\frac{1}{2}, \hat{\phi}\right) = 0$. By proposition 1, we also know that, since $\frac{\partial^2 h(s_K, \phi)}{\partial s_K^2}$ is positive for $s_K \in \left[0, \frac{1}{2}\right)$, there cannot be other value of $s_K \in \left[0, \frac{1}{2}\right)$ such that $h\left(s_K, \widehat{\phi}\right) = 0$. Now $\frac{\partial^2 h(s_K, \phi)}{\partial s_K^2} > 0$ for $s_K \in \left[0, \frac{1}{2}\right)$ implies that $\frac{\partial h\left(\frac{1}{2}, \widehat{\phi}\right)}{\partial s_K} > \frac{\partial h\left(0, \widehat{\phi}\right)}{\partial s_K}$. Moreover, since $h\left(0, \widehat{\phi}\right) = h\left(\frac{1}{2}, \hat{\phi}\right) = 0$, by continuity of h and $\frac{\partial h(s_K, \phi)}{\partial s_K}$, there should be a unique $s'_K \in \left(0, \frac{1}{2}\right)$ where $\frac{\partial h\left(s'_K, \phi\right)}{\partial s_K} = 0$ and $\frac{\partial h(s_K, \phi)}{\partial s_K}$, being $\frac{\partial^2 h(s_K, \phi)}{\partial s_K^2}$ positive in the whole interval, switches sign from negative to positive. As a consequence, we have $\frac{\partial h\left(\frac{1}{2}, \widehat{\phi}\right)}{\partial s_K} > 0 > \frac{\partial h\left(0, \widehat{\phi}\right)}{\partial s_K}$. Since $\frac{\partial h\left(\frac{1}{2}, \phi_B\right)}{\partial s_K} = 0$ and $\frac{\partial h\left(\frac{1}{2}, \phi\right)}{\partial s_K}$ is decreasing in ϕ (provided that γ is small enough) then it must be $\phi_B > \widehat{\phi}$.

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