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Z-ESTIMATORS AND AUXILIARY INFORMATION UNDER WEAK DEPENDENCE

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Z-Estimators and Auxiliary Information under Weak Dependence

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Abstract

In this paper we introduce a weighted Z-estimator for moment condition models in the presence of auxiliary information on the unknown distribution of the data under the assumption of weak dependence. The resulting weighted estimator is shown to be consistent and asymptotically normal. Its small sample properties are checked via Monte Carlo experiments.

Keywords: Z-estimators, M-estimators, GMM, Generalized Empirical Likelihood, blocking techniques, α-mixing.

Jel Classification: C12, C14, C22.

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1 Introduction

In this paper we introduce a weighted Z-estimator for moment condition models in the presence of auxiliary information under weak dependence[1]. The weights are estimated by means of generalized empirical likelihood (GEL) from the available auxiliary information expressed in terms of moment functions. The proposed estimator is motivated by the fact that in applied research it is often possible to retrieve some auxiliary information about the otherwise unknown distribution of the data (the population mean, or the median, for example, or other features related to the shape of the distribution) and incorporate it in the estimation algorithm. One of the most appealing features of the proposed estimator is that it is able to combine a theoretically appealing estimator (the GEL estimator) within a simple estimation setting. In theory, our estimator may be very demanding as it requires both the estimation of the weights and the optimization program associated to the Z-estimator. However, as shown by Bravo (2008, see also Zhang, 1995), it is possible to keep the two parts separated, where the first part is a maximization problem of a globally concave function, and the second part is a standard optimization problem that could be carried out by means of conventional software. In order to take into account the presence of weak dependence we use a blockwise approach (see among others Kitamura, 1997).

The contribution of this paper is twofold. First, we show that the proposed estimator is $\sqrt{n}$-consistent and asymptotically normal, then we show that the weighted Z-estimator is more efficient than its unweighted counterpart. Second, we provide Monte Carlo evidence on the performance of the weighted Z-estimator against an unweighted estimator; we also show that in finite samples the weighted estimator improves an asymptotically equivalent GMM estimator. This paper is meant to be an extension to the weak dependence case of some results of Bravo (2008) and it is related to an earlier paper of Qian and Schmidt

[1]Van der Vaart (2007, p. 41) defines a Z-estimator, for example, as the root of the first derivative of a certain criterion function (Z stands for zero). Under this definition, the estimator that derives from the first order conditions of a GMM criterion function could be thought as a Z-estimator.

(1999, QS hereafter) and to a more recent paper by Smith (2004). The latter proposes a one-step estimator where both the initial moment conditions and the auxiliary information are included into the GEL criterion function[2]. This type of approach has nice theoretical properties but it requires in general the solution of a saddle point problem, which could be computationally burdensome. The paper of QS suggests including the auxiliary information into a GMM setting. As Smith's estimator also QS's is asymptotically equivalent to ours. However, it is well known that GMM could perform very poorly in finite samples (e.g. Altonji and Segal, 1996), and it is reasonable to think that also QS's estimator inherits such finite sample features. In our simulation study we show that the efficient Z-estimator tends to improve the GMM estimator in the majority of the cases we consider.

Before concluding this section let us briefly mention some papers that are related to what we propose here. First of all, it is interesting to notice that, at least to our knowledge, most of the literature that deals with this type of problems has neglected the possibility having weakly dependent data (to our knowledge only the paper by Smith (2004) assumes strong mixing). A series of papers by Imbens and coauthors investigates the use of auxiliary information in the case of microeconometric models (see Hellerstein and Imbens, 1999, Imbens and Lancaster, 1994, Imbens, 1992). Hellerstein and Imbens (1999) for example estimate a wage regression by means of weighted least squares. The set of weights they use is based on Census data and estimated via empirical likelihood. The EL weights shift the distribution of the primary sample towards the distribution of the Census data. When the population values of the Census distribution are of greater interest such effect is desirable (Hellerstein and Imbens, 1999). Imbens and Lancaster (1994) use macro data as auxiliary information in the context of a GMM estimator. As in the case of QS our estimator may apply to rational expectation models, where the forecast error is correlated with another observable variable, which embeds the auxiliary information. In the statistical literature,

[2] In Smith (2004), the weak dependence properties of the data are taken into account by means of kernels.

similar results are for example related to the work of Kuk and Mak (1989) in the context of median estimation or to Chen and Qin (1993), who also exploit EL probabilities to carry the auxiliary information.

The rest of the paper is organized as follows. In Sections 2 and 3 we outline the estimator and the main asymptotic results. In Section 4 we describe the finite sample properties of three specifications of our Z-estimator against two competing estimators, namely an unweighted Z-estimator and an asymptotically equivalent GMM estimator. Section 5 contains some concluding remarks. Proofs and figures are relegated to the appendix.

2 Z-Estimation and Generalized Empirical Likelihood

Let $\{x_t\}$ be an $\mathbb{R}^{L_x}$-valued stationary process from an unknown distribution F, such that the following standard strong mixing conditions are satisfied

$$\alpha_x(k) \to 0, \ \ k \to \infty$$

where $\alpha_x(k) = \sup_{A,B} |\Pr(A \cap B) - \Pr(A)\Pr(B)|$, $A \in \mathcal{F}_{-\infty}^{0}$, $B \in \mathcal{F}_{k}^{\infty}$, and $\mathcal{F}_{m'}^{m''} = \sigma(x_i : m' \leq i \leq m'')$. We also assume $\sum_{k=1}^{\infty} \alpha_x(k)^{1-\frac{1}{c}} < \infty$ for some constant $c > 1$. Consider now set of differentiable functions,

$$m(\beta) = E(m(x_t, \beta))$$

such that $m : \mathbb{R}^{L_x} \times \mathbb{R}^{L_\beta} \to \mathbb{R}^{L_m}$, and $m(\beta_0) = 0$. Moreover, $\beta_0 \in int\{\mathcal{B}\}$ and $\mathcal{B} \subset \mathbb{R}^{L_\beta}$, and L_β is assumed to equal L_m. A Z-estimator for β_0, say $\hat{\beta}$, satisfies the relationship

$$\left\| \hat{m}\left(\hat{\beta}\right) \right\| = \inf_{\beta \in \mathcal{B}} \|\hat{m}(\beta)\| = 0, \tag{1}$$

where $\hat{m}(\beta) = \frac{1}{n}\sum_{t=1}^{n} m_t(\beta)$, $m_t(\beta) = m(x_t, \beta)$, and $\|\cdot\|$ is the Euclidean norm of $\cdot$. Furthermore, we take into account the presence of weak dependence by means of a blockwise approach. Let us assume that M and L are integers and $M \to \infty$ as $n \to \infty$, $M = o(\sqrt{n})$, $L = O(M)$, and $L \leq M$. The estimator we propose treats the estimation of the probabilities and of the parameter of interest separately, in order to reduce the computational complexity and exploit the desirable small sample features of the blockwise GEL (BGEL) estimator. Thus, the blockwise counterpart of (1) is

$$\left\|\hat{h}\left(\hat{\beta}\right)\right\| = \inf_{\beta \in \mathcal{B}} \left\|\hat{h}(\beta)\right\| = 0 \tag{2}$$

and $\hat{h}(\beta) = \frac{1}{b}\sum_{i=1}^{b} h_i(\beta)$, $h_i(\beta) = h(z_i, \beta)$, and $h(z_i, \beta) = \frac{1}{M}\sum_{j=1}^{M} m\left(x_{(i-1)L+j}, \beta\right)$, where $i = 1, ..., b$ and $b = \left[\frac{n-M}{L}\right] + 1$. Notice that b is the blockwise sample size, M indicates how many observations are included in a block (i.e. the blocklength), and L denotes the distance between the first observation of block i and the first observation of block $i+1$[3]. This blockwise approach is a simple method to take into account the time series properties of the data and it simply reduces to rearranging the data (or the associated moment functions) in an appropriate way[4].

Let us assume now that there exists some auxiliary information about the unknown distribution of the data, shaped into a certain function $f : \mathbb{R}^{L_x} \to \mathbb{R}^{L_f}$ that we can define in terms of a moment condition model, independent of the unknown parameter

$$E(f_t) = 0$$

[3] As Kitamura (1997) pointed out, treating the data as if they were independent would cause the estimator to be inefficient.

[4] The use of blocks does not require postulating a weighting function as in the case of kernel smoothing. In addition, Kitamura (1997) pointed out that for $L = 1$ (the fully overlapping case) the blockwise structure corresponds asymptotically to the Bartlett kernel and for other choices of L we have different kernel structures (see also Politis and Romano, 1993).

for $f_t = f(x_t)$. As for equation (2), we can define its blockwise counterpart as

$$g(z_i) = \frac{1}{M} \sum_{j=1}^{M} f\left(x_{(i-1)L+j}\right). \tag{3}$$

At this stage, our problem is to find a suitable way to incorporate the auxiliary information described in (3). In order to do that we follow Bravo (2008, see also Zhang, 1995). This is, we estimate a set of probabilities by means of GEL, using the moment functions in (3). The resulting probabilities are used to weight our initial Z-estimator (2), in order to obtain a BGEL weighted Z-estimator.

The subsequent BGEL function is

$$\hat{R}(\lambda) = \frac{1}{b} \sum_{i=1}^{b} \rho(\lambda' g_i),$$

where $g_i = g(z_i)$ and $\rho(v)$ is the so-called carrier function, concave in its domain, and normalized to be $\rho_1(0) = \rho_2(0) = -1$, given that $\rho_j(v)$, $j = 1, 2$ is the jth derivative (Newey and Smith, 2004). For $\rho(v) = \log(1 - v)$, $\rho(v) = -\exp(v)$, and $\rho(v) = -(1 + v)^2/2$ we have the empirical likelihood case, the exponential tilting case, and the Euclidean likelihood case respectively. They can be considered as special cases of the empirical Cressie-Read family of discrepancies $\rho(v) = -(1 + v)^{(1+\gamma)}/(1 + \gamma)$ where γ is a real number. Let

$$\hat{\lambda} = \arg\max_{\lambda \in \Lambda_n} \hat{R}(\lambda) \tag{4}$$

then the estimated probabilities are defined as

$$\hat{\pi}_i = \frac{\rho_1\left(\hat{\lambda}' g_i\right)}{\sum_{j=1}^{b} \rho_1\left(\hat{\lambda}' g_j\right)}.$$

The resulting BGEL-weighted estimation functions are then defined as

$$\hat{h}_\pi(\beta) = \sum_{i=1}^{b} \hat{\pi}_i h_i(\beta)$$

where $\hat{\pi}_i$ is the BGEL estimator for the probability density function as described above. Thus, the corresponding Z-estimator with auxiliary information, $\hat{\beta}_\pi$, implies

$$\left\| \hat{h}_\pi \left(\hat{\beta}_\pi \right) \right\| = \inf_{\beta \in \mathcal{B}} \left\| \hat{h}_\pi(\beta) \right\| = 0.$$

In Section 3 it will be shown that the estimator $\hat{\beta}_\pi$ is consistent and asymptotically Normal, with asymptotic variance V_β:

$$V_\beta = \left(M(\beta_0)' \right)^{-1} \left(S(\beta_0) - B(\beta_0) \Sigma^{-1} B(\beta_0)' \right) \left(M(\beta_0) \right)^{-1},$$

where $M(\beta) = E\left(\partial m_t(\beta) / \partial \beta'\right)$, $B(\beta) = E \sum_{s=-\infty}^{\infty} \left(m_{t-s}(\beta_0) f_t' \right)$, and $S(\beta_0)$ and Σ^{-1} are defined in Theorem 2 and Lemma 1. From the above expression it follows that the estimator we propose is asymptotically more efficient than an estimator that does not exploit the available auxiliary information, as its variance is $\left(M(\beta_0)' \right)^{-1} S(\beta_0) \left(M(\beta_0) \right)^{-1}$. Clearly, the efficiency of the weighted estimator depends on the relevance of the auxiliary information and, therefore, on the covariance between the original moment function m and the vector of auxiliary moments f, $B(\beta)$: thus, the larger the covariance $B(\beta)$, the smaller the resulting asymptotic variance V_β. It is also quite obvious that if the covariance is zero $\hat{\beta}$ and $\hat{\beta}_\pi$ share the same variance.

An alternative approach is due to QS, and it consists of constructing a moment vector that includes the extra moments

$$\hat{m}^f(\beta) = \frac{1}{n}\sum_{t=1}^{n}\begin{pmatrix} m_t(\beta) \\ f_t \end{pmatrix}. \tag{5}$$

The above model is overidentified, since $L_m + L_f > L_\beta$, where L_f is the length of f_t (notice that we consider $L_m = L_\beta$) and the associated parameter vector may be estimated by GMM. The resulting estimator is asymptotically equivalent to our weighted Z-estimator. The standard asymptotic variance for the GMM estimator is $\left(G(\beta_0)'\,\Omega(\beta_0)^{-1}G(\beta_0)\right)^{-1}$. In our case $G(\beta_0) = \left(M(\beta_0)', 0'\right)'$, where the presence of the zeros depends on the fact that the portion of the moment vector that carries the auxiliary information is independent of the estimand parameter vector. The matrix $\Omega(\beta_0)$ is a 2×2 block matrix, whose elements on the main diagonal are $S(\beta_0)$ and Σ, and the off diagonal entry is the covariance matrix $B(\beta_0)$. After some simple algebra the result follows, and $V_\beta = \left(G(\beta_0)'\,\Omega(\beta_0)^{-1}G(\beta_0)\right)^{-1}$.[5] A further method that is similar to ours is due to Smith (2004), and consists of estimating the parameters, given the augmented vector of moments in (5), by means of (smoothed) GEL. Such procedure consists of augmenting the GEL criterion function by the vector of auxiliary moments and simultaneously compute the an estimate of the parameters of interest.[6]

3 Asymptotic Theory

The following theorems establish consistency and asymptotic normality of the Z-estimator with auxiliary information. Proofs follow some results of Pakes and Pollard (1989), Pakes

[5] The result in Qian and Schmidt (1999) is slightly different, since the initial vector of moments, m in our notation, is overidentified.

[6] Smith (2004) assumes that the auxiliary set of moments also depends on β, while in our case it does not. The final result is different since the asymptotic variance includes extra terms that involve the first derivatives of the auxiliary moments. However, the substance is essentially the same.

and Linton (2001), Bravo (2008) and Crudu (2009, see also Bravo, 2009).

The following lemma establishes consistency and asymptotic normality of the BGEL estimator of the Lagrange multiplier in (4).

Lemma 1 *Assume 1) $\{x_t\}_{t\in\mathbb{Z}}$ is a strictly stationary strong mixing sequence, 2) $E\|f_t\|^{2(1+\eta)}$ for some small enough $\eta > 0$, $\Sigma = E(f_t f_t')$ is positive definite, 3)$R(\lambda) = E(\rho(\lambda' f_t))$ has a maximum for $\lambda = 0$ and it is unique, 4) zero is in the interior of the convex set Λ_n and $\rho(\nu)$ is concave and twice continuously differentiable about zero and its jth derivative $\rho_j(0) = -1$, $j = 1, 2$, 5) $\hat{R}(\lambda) \to_p R(\lambda)$ for all $\lambda \in \Lambda_n$, then $\hat{\lambda}$ is consistent and normally distributed*

$$\frac{\sqrt{n}}{M}\hat{\lambda} \to_d N\left(0, \Sigma^{-1}\right)$$

Theorem 1 and Theorem 2 establish consistency and asymptotic Normality for the efficient Z-estimator $\hat{\beta}_\pi$.

Theorem 1 (Consistency of $\hat{\beta}_\pi$) *Assume 1) $\mathcal{B}$ is a compact set, 2) $\forall\delta > 0$ there exists $\varepsilon(\delta)$ such that $\sup_{\|\beta-\beta_0\|>\delta}\|m(\beta)\| \geq \varepsilon(\delta) > 0$, 3) $\sup_{\beta\in\mathcal{B}}\|\hat{m}(\beta) - m(\beta)\| = o_p(1)$. Then, if also the assumptions in Lemma 1 are satisfied, $\hat{\beta}_\pi \to_p \beta_0$.*

Theorem 2 (Asymptotic Normality of $\hat{\beta}_\pi$) *Assume $\hat{\beta}_\pi$ is consistent; moreover, assume 1) $m_t(\beta)$ being continuously differentiable in a neighborhood of β_0, $\mathcal{N}(\beta_0, \delta)$, 2) $M(\beta_0) = E(\partial m_t(\beta_0)/\partial\beta)$ is continuous and nonsingular, $E\left(\|m_t(\beta_0)\|\|f_t\|^2\right) < \infty$, and $E\sup_{\beta\in\mathcal{N}(\beta_0,\delta)}(\|\partial m_t(\beta)/\partial\beta\|\|f_t\|) < \infty$, 3) $\sqrt{n}\hat{m}(\beta_0) \to_d N(0, S(\beta_0))$. Then, if assumptions in Theorem 1 are satisfied $\sqrt{n}\left(\hat{\beta}_\pi - \beta_0\right) \to N(0, V_\beta)$, where*

$$V_\beta = \left(M(\beta_0)'\right)^{-1}\left(S(\beta_0) - B(\beta_0)\Sigma^{-1}B(\beta_0)'\right)(M(\beta_0))^{-1},$$

and $B(\beta_0) = E\sum_{s=-\infty}^{\infty}(m_{t-s}(\beta_0) f_t')$.

The following corollary is a direct result of Theorem 2. It states that an estimator of the empirical distribution function based on the BGEL probabilities is more efficient than an estimator computed as $\hat{\mu}(x) = \frac{1}{n}\sum_{t=1}^{n} 1(x_t \leq x)$.

Corollary 1 *Let $\mu(x) = \Pr(x_t \leq x)$. If assumptions in Theorems 1 and 2, and assumption 1 in Lemma 1 hold, then $\hat{\mu}_\pi(z) \to_p \mu(x)$ and $\sqrt{n}(\hat{\mu}_\pi(z) - \mu(x)) \to_d N(0, \sigma^2 - a'\Sigma^{-1}a)$, where $\hat{\mu}_\pi(x)$ is the BGEL version of $\hat{\mu}(x) = \frac{1}{n}\sum_{t=1}^{n} 1(x_t \leq x)$, that is $\hat{\mu}_\pi(z) = \sum_{i=1}^{b} \hat{\pi}_i 1_M(z_i \leq z)$.*

Proofs are in the appendix.

4 Monte Carlo Experiments

In this section we study the small sample features of our weighted Z-estimator. The main objective of these experiments is to analyze the behaviour of such estimators in terms of bias and mean square error (MSE) as n and the M vary.[7] For convenience we only take into account the case $L = 1$, and we consider the effect of arbitrary values of M against an optimal M. The optimal M is computed by means of the procedure suggested by Politis and White (2004, see also Patton, Politis, and White, 2009).[8] Let us consider the estimation of a location parameter as in QS

$$y_t = \beta_0 + e_t$$

where β_0 is a scalar, and it is assumed to be equal to 1, and e_t is a zero mean disturbance. Thus, we want to find an estimate for $\beta_0 = E(y_t)$. We also assume there exists a certain random variable u_t, that is known to have zero mean and it is correlated with e_t.

[7]The figures actually present bias and MSE multiplied by their respective sample size.

[8]The optimal blocklength is the one for the circular bootstrap and it is computed using the R function `b.star` in the `np` package (Hayfield and Racine, 2008). Interestingly, the circular bootstrap is asymptotically equivalent to the block bootstrap with $L = 1$, i.e. the moving block bootstrap.

We define then the following equations:

$$\begin{aligned} y_t &= 1 + e_t \\ u_t &= \rho e_t + \sqrt{1-\rho^2}\eta_t. \end{aligned}$$

We choose two specifications for the processes e_t and η_t:

$$\begin{aligned} DGP1 &: \; e_t = \alpha e_{t-1} + \varepsilon_t^e, \eta_t = \delta \varepsilon_{t-1}^{\eta} + \varepsilon_t^{\eta} \\ DGP2 &: \; e_t = \alpha_1 e_{t-1} + \alpha_2 e_{t-2} + \varepsilon_t^e, \eta_t = \delta \eta_{t-1} + \varepsilon_t^{\eta}, \end{aligned}$$

where $\varepsilon_t^i \sim N(0,1)$, $i = e, \eta$. The parameters are $\alpha = .8$, $(\alpha_1, \alpha_2) = (.7, .2)$, $\rho = .8$ and $\delta = .4$. We compare the performance of various competing estimators for $n = 16, 32, 64, 128, 256$, and M taking values from 2 to 16.[9] The optimal Ms are computed for both y_t and u_t: M_y and M_u.[10] In Table 1 we report the average optimal blocklengths for the two DGPs.

	DGP 1		DGP 2	
n	M_y	M_u	M_y	M_u
16	2.7876	2.5156	2.7506	2.5102
32	5.0126	4.3648	5.5018	4.8820
64	7.9142	7.0892	9.5570	8.8986
128	11.5696	10.6400	15.1568	14.5900
256	16.0406	15.0726	22.3942	21.8780

Table 1: Optimal blocklengths

[9] Notice that the case $n = M = 16$ is not taken into consideration as it equivalent to $M = 1$.

[10] The blocklength and the number of resulting blocks could be different for the two series. In order to overcome this issue, the data are wrapped around a circle and extra observations are used from the beginning of the series in order to have the same number of blocks (similar procedures are suggested in Davison and Hinkley, 1997, pp. 396-397).

We compute an estimate for β_0 in five different ways. The first is a simple sample mean

$$\hat{\beta} = \bar{y} = \frac{1}{n}\sum_{t=1}^{n} y_t.$$

The second is an efficient two step GMM estimator with two moment conditions

$$\hat{\beta} = \arg\min_{\beta} \hat{g}(\beta)' \hat{\Omega}\left(\bar{\beta}\right)^{-1} \hat{g}(\beta)$$

where $\hat{g}(\beta) = \frac{1}{n}\sum_{t=1}^{n}\begin{pmatrix} y_t - \beta, & u_t \end{pmatrix}'$. The matrix of weights $\hat{\Omega}\left(\bar{\beta}\right)$ is a Newey-West matrix evaluated at a certain consistent estimator of β, $\bar{\beta}$. The remaining three estimators are weighted averages based on GEL estimators, i.e. the EL, the ET and the EU estimator,

$$\hat{\beta} = \sum_{i=1}^{b} \hat{\pi}_i^{BGEL} z_i$$

where $z_i = \sum_{j=1}^{M} y_{(i-1)L+j}$. Given the auxiliary information $w_i = \sum_{j=1}^{M} u_{(i-1)L+j}$, the three BGEL estimators for the probabilities are defined as

$$\begin{aligned}
\hat{\pi}_i^{EL} &= \frac{1}{b\left(1 + \hat{\lambda}^{EL} w_i\right)} \\
\hat{\pi}_i^{ET} &= \frac{\exp\left(\hat{\lambda}^{ET} w_i\right)}{\sum_{j=1}^{b}\exp\left(\hat{\lambda}^{ET} w_j\right)} \\
\hat{\pi}_i^{EU} &= \frac{1}{b}\left(1 - \hat{\lambda}^{EU}\left(w_i - \bar{w}\right)\right)
\end{aligned}$$

where $\bar{w} = \frac{1}{b}\sum_{i=1}^{b} w_i$. $\hat{\lambda}^{EL}$ and $\hat{\lambda}^{ET}$ are computed numerically, while it is available a close form solution for $\hat{\lambda}^{EU}$. Each weighted estimator is computed for different values of M, where M goes from 2 to 16 and for an optimal M. The calculations are carried out in R and are based on 5000 Monte Carlo repetitions.

The results of the simulations are summarized in the appendix. Figures 1 to 4 describe the behaviour of the weighted estimators as the blocklength changes compared to those estimators that are independent of M, represented by the horizontal lines. In particular, the thick horizontal lines denote the weighted estimators based on M_y and M_u and, therefore, denoted as $OptEL$, $OptET$, and $OptEU$. In several cases the sample mean is too far apart from the other estimators and it is not included in the graphs. Figures 5 to 8 describe the behaviour of all estimators as the sample size increases. Particularly when the sample size is small, the choice of M has a considerable impact on bias and MSE. For the latter (Figures 3 and 4), we see that the MSE tends to grow with M, while as n increases the slope of the curves corresponding to the EL and ET estimators becomes smaller and collapses to $OptEL$ and $OptET$. On the other hand, the EU-based estimator is upward-sloping also for $n = 256$. In Figure 5 we see that the bias does not change much for the weighted estimators and for the GMM estimator. For DGP 2 the bias varies substantially for the EL-based estimator. The bias of the ET-based estimator, however, is less sensitive to the choice of M, while the EU-based estimator has a quite persistent negative bias and tends to behave as the GMM estimator. Given the appropriate difference in scale, Figures 7 and 8 describe the same picture: the effect of an arbitrary choice of M could have a large impact on the MSE in small samples. Such an effect is more prominent for the EL case and the EU case. For the latter it persists also for larger values of n. Overall the EL estimator and the ET estimator that use an optimal blocklength have smaller MSE than GMM. Apart from small values of n, the EU estimator that uses an optimal blocklength is very similar to the GMM estimator. The MSE for the sample mean is the largest in the panel and tends to grow with the sample size.

5 Conclusion

In this paper we propose a two step procedure for Z-estimators in the presence of weakly dependent data and auxiliary information based on the estimation of BGEL probabilities. This procedure is attractive from different points of view. First of all, the computation of the BGEL probabilities is very simple, as it contemplates only the convex part of the BGEL problem (this is, the estimation of the Lagrange multiplier λ). Moreover, whenever the Z-estimator is asymptotically equivalent to a GMM estimator (QS), it does not entail the well-known small sample effects that affect GMM estimators (see for example Altonji and Segal, 1996). Our asymptotic results state that the resulting Z-estimator is consistent and Normally distributed. The resulting variance depends on the relevance of the auxiliary information. In addition, we demonstrate that the estimator of a distribution based on the BGEL weights enjoys the same favourable features of the abovementioned Z-estimator. Furthermore, by means of Monte Carlo experiments, we describe how to apply our approach to a standard time series problem. The laboratory we set is a location parameter estimation problem, similar to what is described in QS (see also Zhang, 1995). We compare three BGEL weighted estimators against a simple sample mean and an augmented GMM estimator, and we analyze their behaviour for different values of M and n. We argue that an appropriate choice of M is crucial in particular when the sample is small; because of that we advocate the use of data driven procedures for the selection of the blocklength (see Politis and White, 2004). The simulation results suggest that, in general, weighted estimators (in particular those based on ET) combined with an optimal blocklength improve over the competing estimators.

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6 Appendix: Proofs and Figures

In what follows we present the proofs of the theorems presented in Section 3 and some auxiliary results. In addition, we use the following notation: $\rightarrow_p$ and $\rightarrow_d$ denote convergence in probability and convergence in distribution; C is a generic positive constant; CS and T denote Cauchy-Schwarz inequality and triangular inequality respectively; $\|\cdot\|$ is the Euclidean norm of $\cdot$. The sums $\sum_i$ and $\sum_j$ substitute $\sum_{i=1}^{b}$ and $\sum_{j=1}^{b}$, while $\sum_t$ substitutes $\sum_{t=1}^{n}$. The CLT is meant to be a CLT for strong mixing sequences (see e.g. Ibragimov and Linnik, 1971) and CMT is the continuous mapping theorem.

Proof of Lemma 1. Consider again

$$\hat{R}(\lambda) = \frac{1}{b}\sum_i \rho(\lambda' g_i).$$

Notice that $\hat{R}(\lambda)$ is concave through $\rho(\cdot)$. Moreover, assumptions 2 to 4 match assumptions (i)-(iii) from Theorem 2.7 of Newey and McFadden (1994). Then, consistency of $\hat{\lambda}$ follows.

Consider now a mean value expansion of the first order conditions of the BGEL criterion function,

$$\begin{aligned} 0 &= \frac{\partial \hat{R}\left(\hat{\lambda}\right)}{\partial \lambda} = \frac{1}{b}\sum_i \rho_1\left(\dot{\lambda}' g_i\right) g_i \\ &= -\bar{g} + \left(\frac{M}{b}\sum_i \rho_1\left(\dot{\lambda}' g_i\right) g_i g_i'\right)\frac{\hat{\lambda}}{M} \end{aligned}$$

where $\bar{g} = \sum_i g_i/b$. Since $\hat{\lambda}$ is consistent and $\left\|\dot{\lambda}\right\| \leq \left\|\hat{\lambda}\right\|$, we have that $\rho_1\left(\dot{\lambda}' g_i\right) = -1 + o_p(1)$. Thus, multiplying by $\sqrt{n}$

$$0 = -\sqrt{n}\bar{g} - \hat{\Sigma}\sqrt{n}\frac{\hat{\lambda}}{M} - o_p(1)\,\hat{\Sigma}\sqrt{n}\frac{\hat{\lambda}}{M}$$

where $\hat{\Sigma} = M\sum_i g_i g_i'/b$. Notice that $\hat{\Sigma}\sqrt{n}\frac{\hat{\lambda}}{M} = O_p(1)$; therefore, by rearranging

$$\sqrt{n}\frac{\hat{\lambda}}{M} = -\hat{\Sigma}^{-1}\sqrt{n}\bar{g} + o_p(1). \tag{6}$$

Finally, by applying CLT to $\sqrt{n}\bar{g}$ and Slutsky theorem, the result follows. ■

Proof of Theorem 1 (Consistency of $\hat{\beta}_\pi$). Let us compute a mean value expansion of

$$\hat{\pi}_i = \frac{\rho_1\left(\hat{\lambda}' g_i\right)}{\sum_j \rho_1\left(\hat{\lambda}' g_j\right)}$$

about $\lambda = 0$, where $\hat{\lambda}$ is a consistent estimator for λ:

$$\begin{aligned}
\hat{\pi}_i &= \frac{1}{b} + \frac{1}{b}\left(\frac{\rho_2\left(\dot{\lambda}' g_i\right) g_i'}{\frac{1}{b}\sum_j \rho_1\left(\dot{\lambda}' g_j\right)} - \frac{\rho_1\left(\hat{\lambda}' g_i\right) \frac{1}{b}\sum_j \rho_2\left(\dot{\lambda}' g_j\right) g_j'}{\left(\frac{1}{b}\sum_j \rho_1\left(\dot{\lambda}' g_j\right)\right)^2} \right)\left(\hat{\lambda} - 0\right) \\
&= \frac{1}{b} + \frac{1}{b}\left(\frac{\rho_2\left(\dot{\lambda}' g_i\right) \hat{\lambda}' g_i}{\frac{1}{b}\sum_j \rho_1\left(\dot{\lambda}' g_j\right)} - \frac{\rho_1\left(\hat{\lambda}' g_i\right) \frac{1}{b}\sum_j \rho_2\left(\dot{\lambda}' g_j\right) \hat{\lambda}' g_j}{\left(\frac{1}{b}\sum_j \rho_1\left(\dot{\lambda}' g_j\right)\right)^2} \right).
\end{aligned}$$

From results of Lemma 1 we obtain

$$\hat{\pi}_i = \frac{1}{b} + \frac{1}{b}\left(\hat{\lambda}' g_i + o_p(1)\right) \tag{7}$$

and

$$\hat{\pi}_i = \frac{1}{b}\left(1 + o_p(1)\right). \tag{8}$$

From Lemma 1 in Crudu (2009) we have $\hat{h}_\pi(\beta) = \hat{m}(\beta) + O_p(M/n)$. Then, by adding and subtracting $\hat{h}_\pi\left(\hat{\beta}_\pi\right)$ and T

$$\left\| m\left(\hat{\beta}_\pi\right)\right\| \leq \left\| m\left(\hat{\beta}_\pi\right) - \hat{h}_\pi\left(\hat{\beta}_\pi\right)\right\| + \left\|\hat{h}_\pi\left(\hat{\beta}_\pi\right)\right\|$$

Moreover, by optimality of $\hat{\beta}_\pi$ and since $m(\beta_0) = 0$, and by repeated application of Lemma 1 in Crudu (2009) and T

$$\begin{aligned}
\left\| m\left(\hat{\beta}_\pi\right)\right\| &\leq \left\| m\left(\hat{\beta}_\pi\right) - \hat{m}\left(\hat{\beta}_\pi\right)\right\| + \left(1 + o_p(1)\right)\left\|\hat{m}(\beta_0) - m(\beta_0)\right\| + O_p\left(\frac{M}{n}\right) \\
&\leq \sup_{\beta\in\mathcal{B}}\left\| m(\beta) - \hat{m}(\beta)\right\| + \left(1 + o_p(1)\right)\sup_{\beta\in\mathcal{B}}\left\|\hat{m}(\beta) - m(\beta)\right\| + O_p\left(\frac{M}{n}\right).
\end{aligned}$$

By Assumption 3 $\sup_{\beta\in\mathcal{B}}\left\| m(\beta) - \hat{m}(\beta)\right\| = o_p(1)$; hence

$$\left\| m\left(\hat{\beta}_\pi\right)\right\| \leq o_p(1).$$

Since $m(\beta)$ is bounded away from zero for $\|\beta - \beta_0\| > \delta$ (assumption 2), it follows that $\hat{\beta}_\pi \in \|\beta - \beta_0\| < \delta$. As δ is arbitrary, $\hat{\beta}_\pi \to_p \beta_0$. ■

Proof of Theorem 2 (Asymptotic Normality of $\hat{\beta}_\pi$). Let us consider $\sum_i \hat{\pi}_i h_i\left(\hat{\beta}_\pi\right) = 0$, by replacing the probabilities with the expression in 7

$$0 = \frac{1}{b}\sum_i \left(1 + \hat{\lambda}' g_i + o_p(1)\right) h_i\left(\hat{\beta}_\pi\right)$$

and mean value expand $h_i\left(\hat{\beta}_\pi\right)$ about β_0, for $\hat{\beta}_\pi$ being consistent

$$\begin{aligned} 0 &= \sum_i \left(1 + \hat{\lambda}' g_i + o_p(1)\right) h_i\left(\hat{\beta}_\pi\right) \\ &= \sum_i \left(1 + \hat{\lambda}' g_i\right)\left(h_i(\beta_0) + \frac{\partial h_i\left(\dot{\beta}\right)}{\partial \beta}\left(\hat{\beta}_\pi - \beta_0\right)\right) + o_p(1)\,\hat{h}\left(\hat{\beta}_\pi\right) \end{aligned}$$

where $\left\|\dot{\beta} - \beta_0\right\| \leq \left\|\hat{\beta}_\pi - \beta_0\right\|$. Let us define $\hat{B}(\beta) = M\sum_i h_i(\beta)\, g_i'/b$. Then, by appropriate rescaling and (6)

$$\begin{aligned} 0 &= \sqrt{n}\hat{h}(\beta_0) + \hat{B}(\beta_0)\,\hat{\Sigma}^{-1}\sqrt{n}\bar{g} \\ &\quad + \left(\frac{1}{b}\sum_i \frac{\partial h_i\left(\dot{\beta}\right)}{\partial \beta} + \hat{\lambda}' \sum_i g_i \frac{\partial h_i(\beta_0)}{\partial \beta}/b\right)\sqrt{n}\left(\hat{\beta}_\pi - \beta_0\right) \\ &\quad + o_p(1)\sqrt{n}\hat{h}\left(\hat{\beta}_\pi\right) \\ &= A_1 + A_2 + A_3 \end{aligned}$$

where

$$A_1 = \sqrt{n}\hat{h}(\beta_0) + \hat{B}(\beta_0)\,\hat{\Sigma}^{-1}\sqrt{n}\bar{g},$$

$$A_2 = \left(\sum_i \partial h_i\left(\dot{\beta}\right)/\partial\beta + \hat{\lambda}' \sum_i g_i \partial h_i(\beta_0)/\partial\beta/b\right)\sqrt{n}\left(\hat{\beta}_\pi - \beta_0\right),$$

$$A_3 = o_p(1)\sqrt{n}\hat{h}\left(\hat{\beta}_\pi\right).$$

From assumption 3 and Lemma 1 in Crudu (2009) $\sqrt{n}\hat{h}(\beta_0) \rightarrow_d N(0, S(\beta_0))$ and $\sqrt{n}\bar{g} \rightarrow_d N(0,\Sigma)$. Then, after simple calculations, we get $A_1 \rightarrow_d N(0, W)$, where

$$\begin{aligned} W &= \begin{pmatrix} I, & -B(\beta_0)\Sigma^{-1} \end{pmatrix} \begin{pmatrix} S(\beta_0) & B(\beta_0) \\ B(\beta_0)' & \Sigma \end{pmatrix} \begin{pmatrix} I \\ -\Sigma^{-1}B(\beta_0)' \end{pmatrix} \\ &= S(\beta_0) - B(\beta_0)\Sigma^{-1}B(\beta_0)' \end{aligned}$$

Let us now focus attention on A_2. By Lemma 1 in Crudu (2009) and T

$$\begin{aligned} \left\| \hat{\lambda}'\frac{1}{b}\sum_i g_i \frac{\partial h_i\left(\dot{\beta}\right)}{\partial \beta'} \right\| &\leq \left\|\hat{\lambda}\right\| \left\| \frac{1}{n}\sum_t f_t \frac{\partial m_t\left(\dot{\beta}\right)}{\partial \beta'} + O_p\left(\frac{M}{n}\right) \right\| \\ &\leq \left\|\hat{\lambda}\right\| \frac{1}{n}\sum_t \sup_{\beta\in\mathcal{B}} \left\| f_t \frac{\partial m_t(\beta)}{\partial \beta'} \right\| + o_p(1). \end{aligned}$$

Thus,

$$\left\| \hat{\lambda}'\frac{1}{b}\sum_i g_i \frac{\partial h_i\left(\dot{\beta}\right)}{\partial \beta} \right\| \leq o_p(1).$$

By CMT and assumption 3 $\sqrt{n}\hat{h}\left(\hat{\beta}_\pi\right)$ is Normally distributed. Thus, its order of magnitude is $O_p(1)$ and $A_3 = o_p(1)$. Finally,

$$M(\beta_0)'\sqrt{n}\left(\hat{\beta}_\pi - \beta_0\right) = -\begin{pmatrix} I, & -\hat{B}(\beta_0)\Sigma^{-1} \end{pmatrix} \begin{pmatrix} \sqrt{n}\hat{h}(\beta_0) \\ \sqrt{n}\bar{g} \end{pmatrix} + o_p(1)$$

and

$$\sqrt{n}\left(\hat{\beta}_\pi - \beta_0\right) = -\left(M(\beta_0)'\right)^{-1} \begin{pmatrix} I, & -\hat{B}(\beta_0)\Sigma^{-1} \end{pmatrix} \begin{pmatrix} \sqrt{n}\hat{h}(\beta_0) \\ \sqrt{n}\bar{g} \end{pmatrix} + o_p(1)$$

which implies, by CLT applied to $\sqrt{n}\bar{g}$, assumption 3 and CMT,

$$\sqrt{n}\left(\hat{\beta}_\pi - \beta_0\right) \to_d N\left(0, \left(M\left(\beta_0\right)'\right)^{-1}\left(S\left(\beta_0\right) - B\left(\beta_0\right)\Sigma^{-1}B\left(\beta_0\right)'\right)\left(M\left(\beta_0\right)\right)^{-1}\right).$$

■

Proof of Corollary 1. From results in Lemma 1 and Theorem 1 we have

$$\begin{aligned} \hat{\mu}_\pi(z) &= \frac{1}{b}\sum_i 1_M\left(z_i \leq z\right)\left(1 + \hat{\lambda}' g_i + o_p(1)\right) \\ &= \hat{\mu}_b(z) - \sqrt{n}\bar{g}'\hat{\Sigma}^{-1}\frac{M}{\sqrt{n}b}\sum_i g_i 1_M\left(z_i \leq z\right) + o_p\left(\frac{1}{\sqrt{n}}\right) \end{aligned}$$

Then, by adding and subtracting $\mu(x)$ and multiplying both sides by $\sqrt{n}$, we get

$$\begin{aligned} \sqrt{n}\left(\hat{\mu}_\pi(z) - \mu(x)\right) &= \sqrt{n}\left(\hat{\mu}_b(z) - \mu(x)\right) - \sqrt{n}\bar{g}'\hat{\Sigma}^{-1}\frac{1}{b}\sum_i g_i 1_M\left(z_i \leq z\right) + o_p(1) \\ &= \begin{pmatrix} 1, & -\hat{a}'\hat{\Sigma}^{-1} \end{pmatrix}\begin{pmatrix} \sqrt{n}\left(\hat{\mu}_b(z) - \mu(x)\right) \\ \sqrt{n}\bar{g} \end{pmatrix} + o_p(1) \\ &\to {}_d N\left(0, \sigma^2 - a'\Sigma^{-1}a\right). \end{aligned}$$

The result follows by CLT applied to $\sqrt{n}\left(\hat{\mu}_b(z) - \mu(x)\right)$ and $\sqrt{n}\bar{g}$ and Slutsky theorem.

■

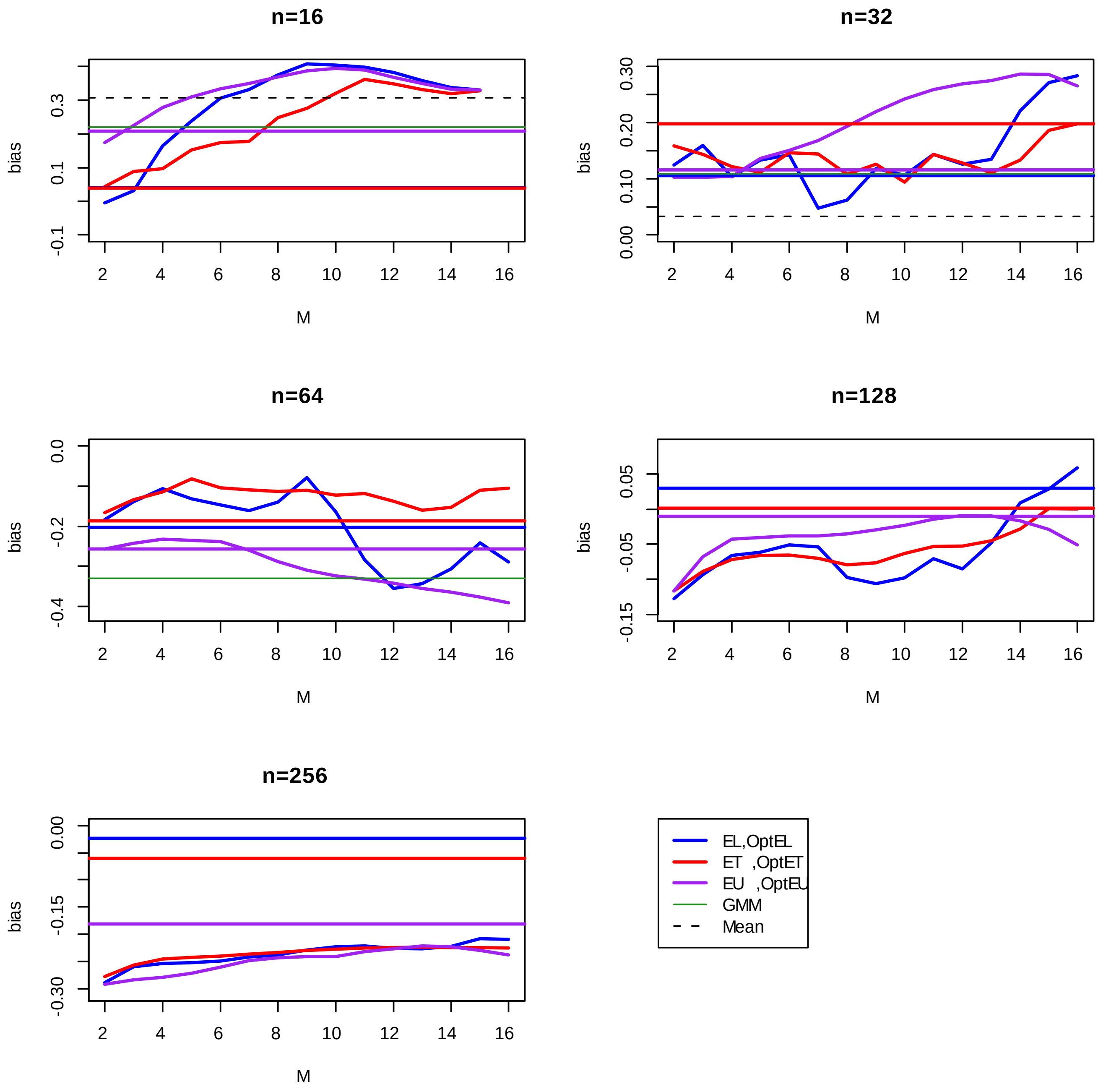

Figure 1: Bias of the Z-estimator as M varies for DGP 1

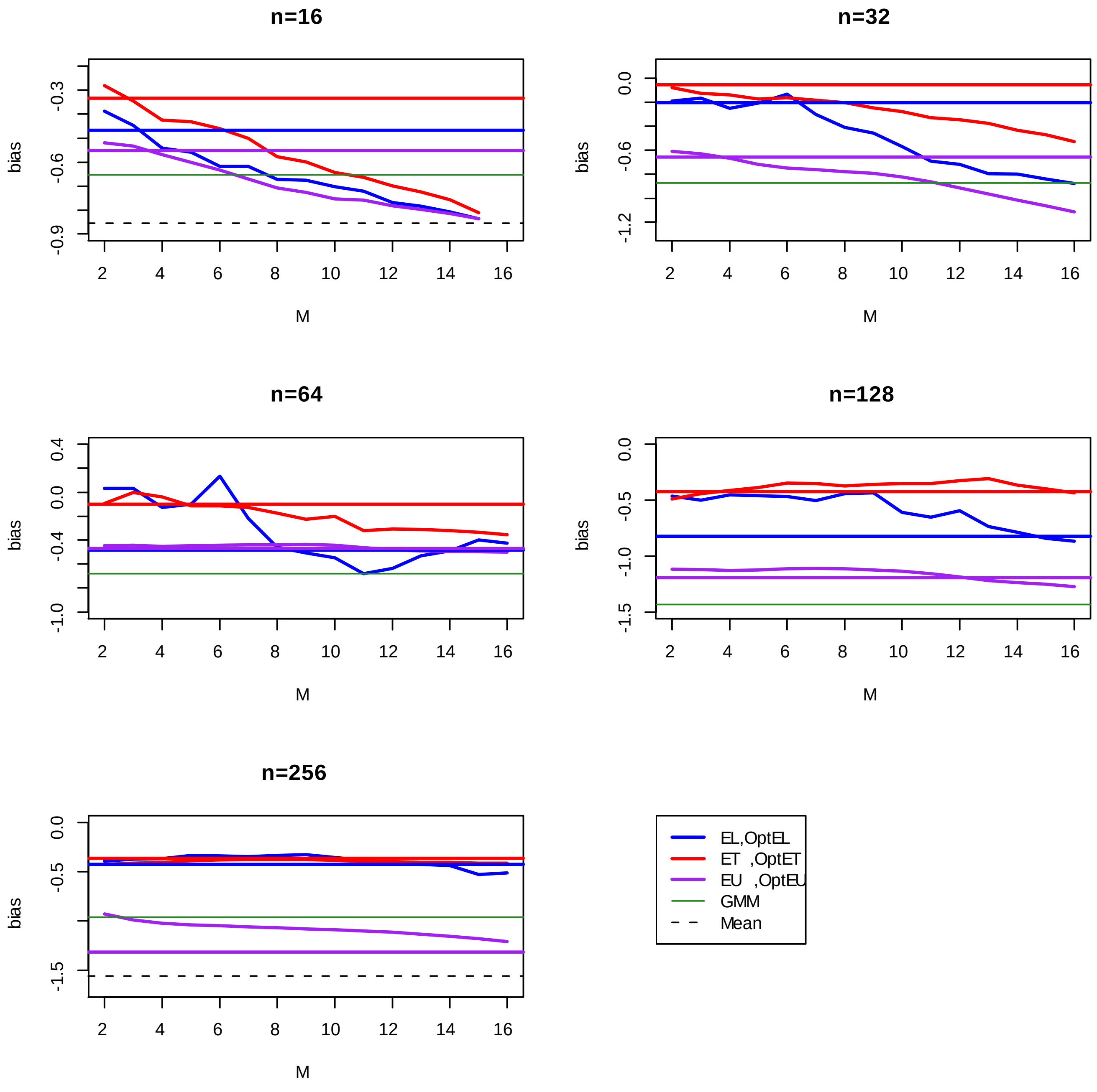

Figure 2: Bias of the Z-estimator as M varies for DGP 2

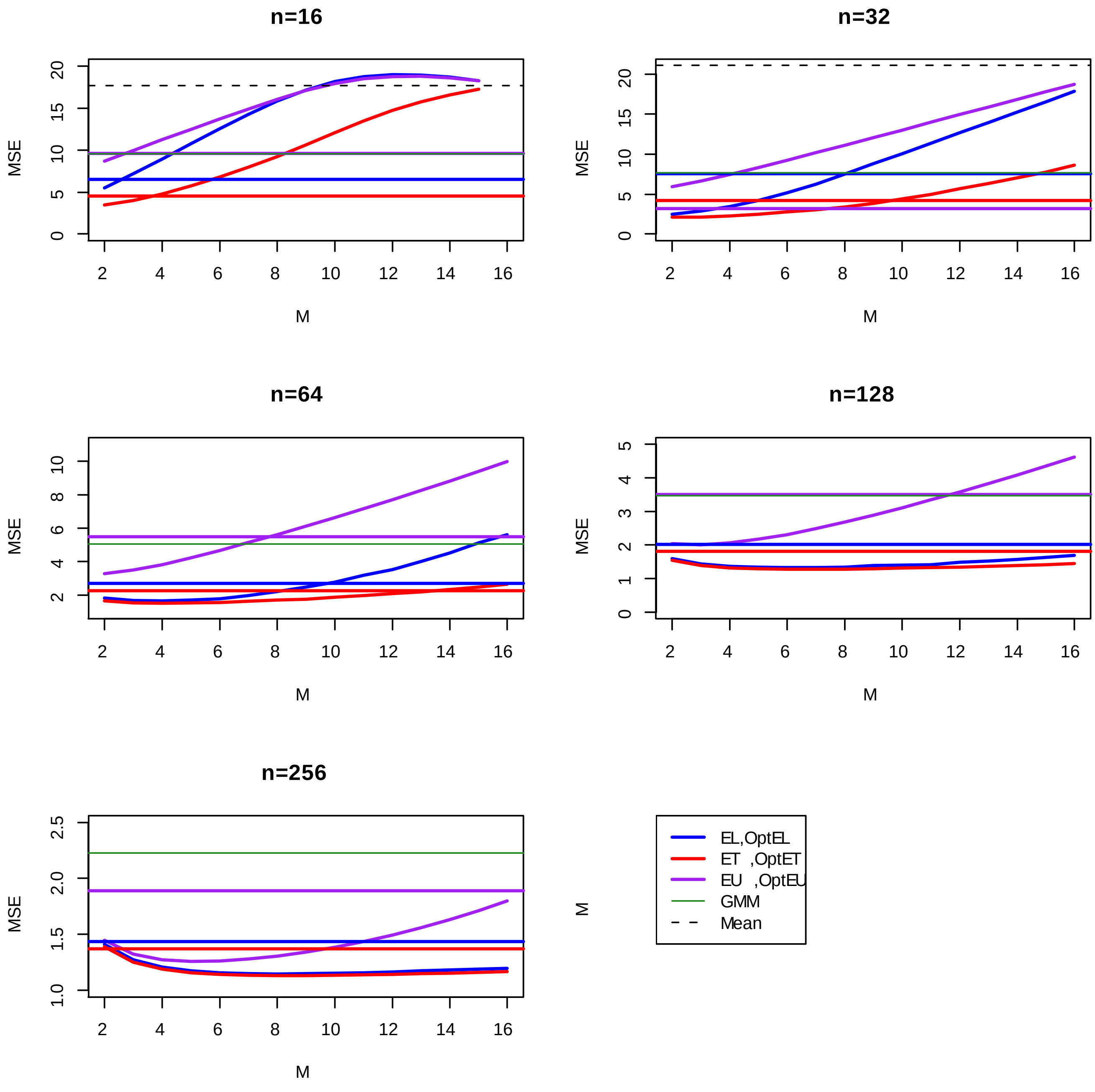

Figure 3: MSE of the Z-estimator as M varies for DGP 1

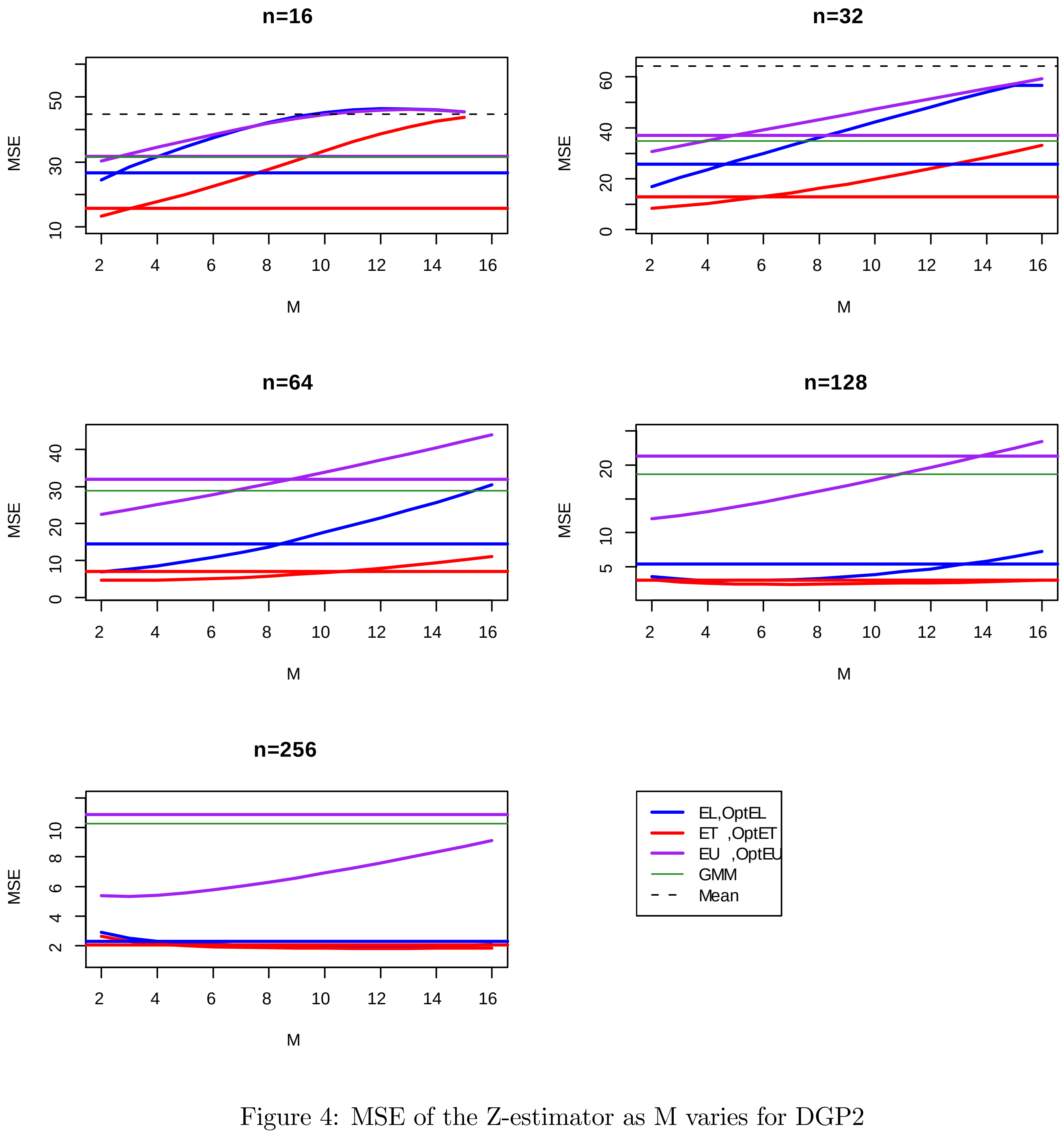

Figure 4: MSE of the Z-estimator as M varies for DGP2

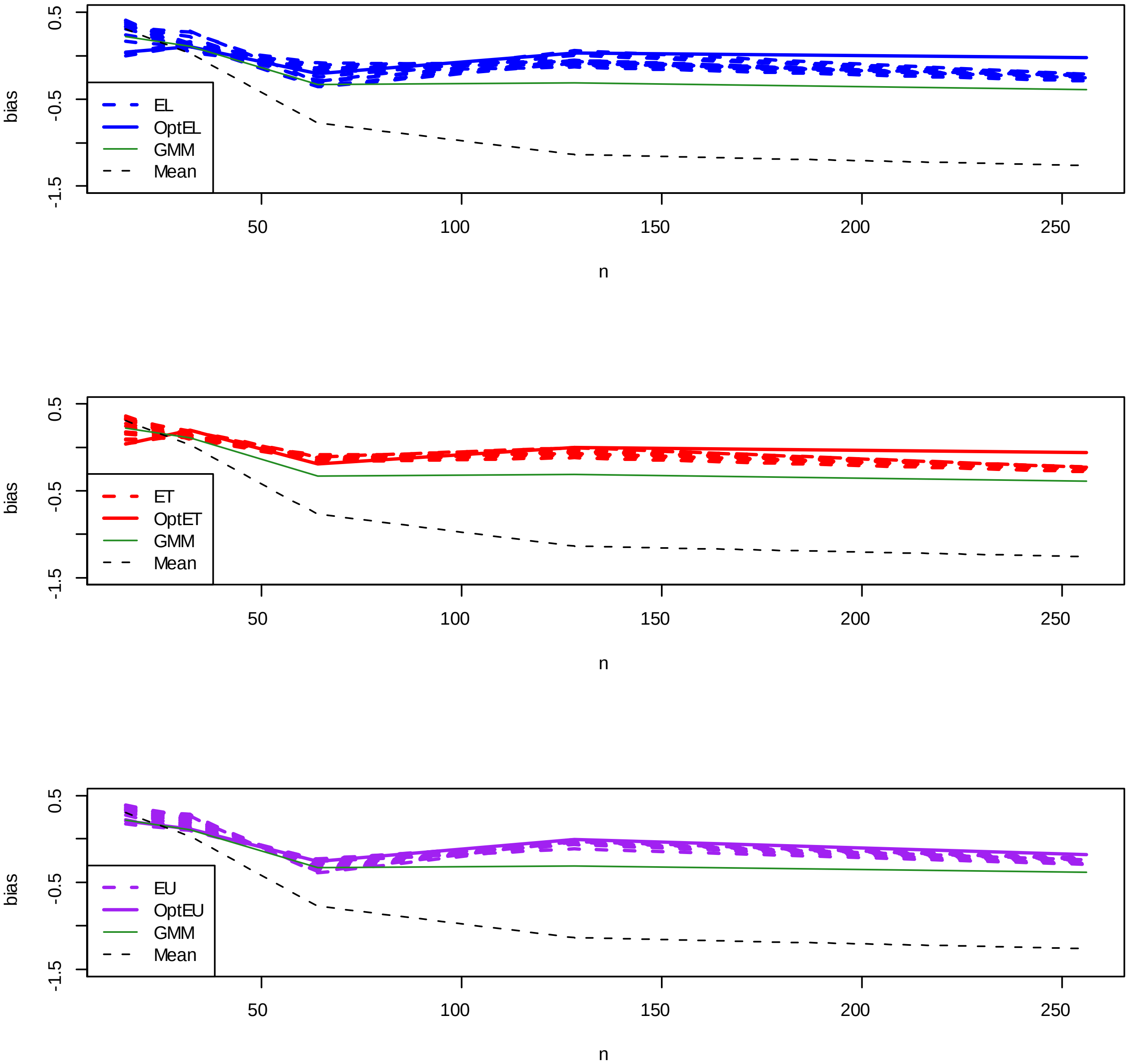

Figure 5: Bias of the Z-estimator as n varies for DGP 1

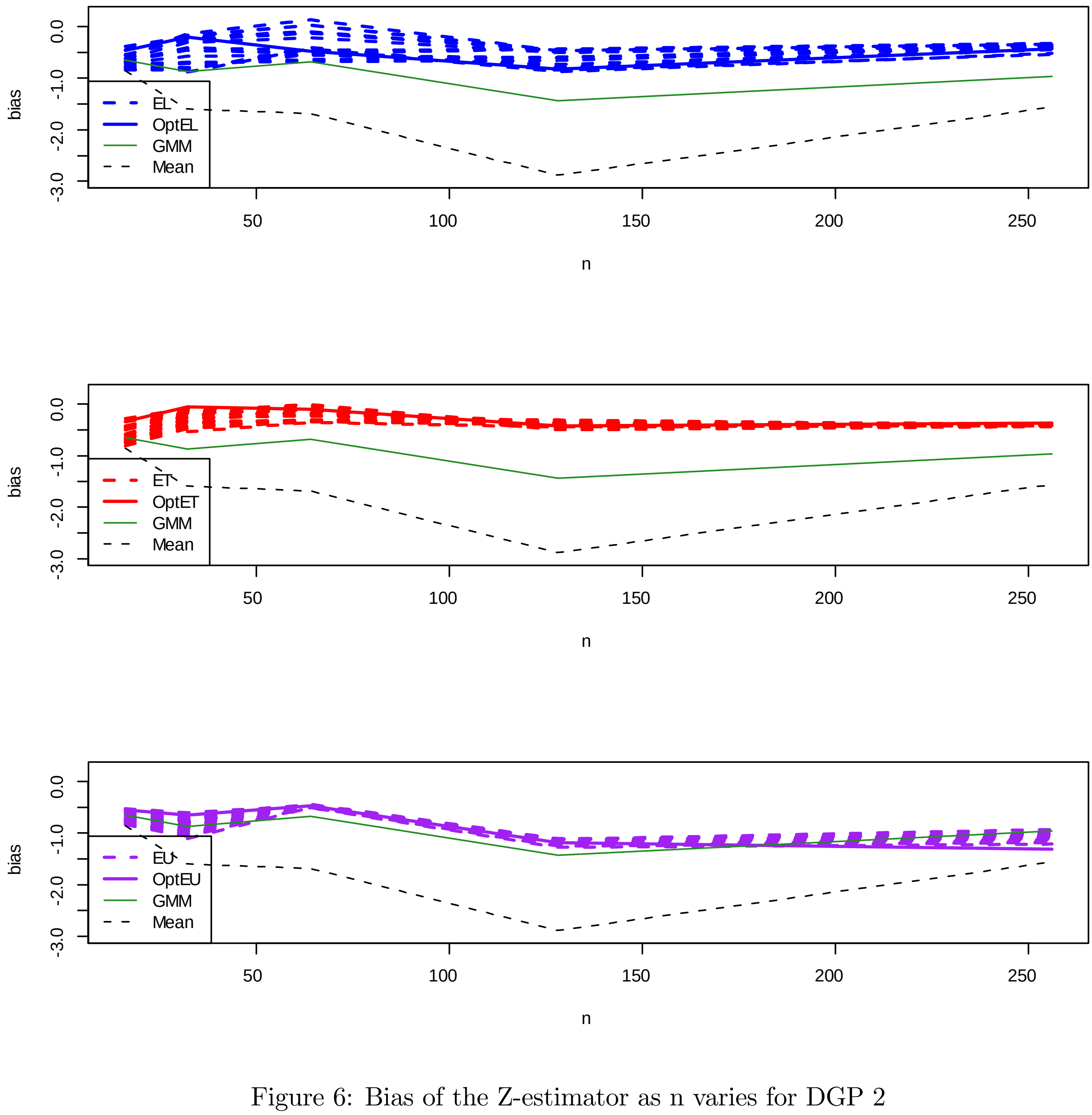

Figure 6: Bias of the Z-estimator as n varies for DGP 2

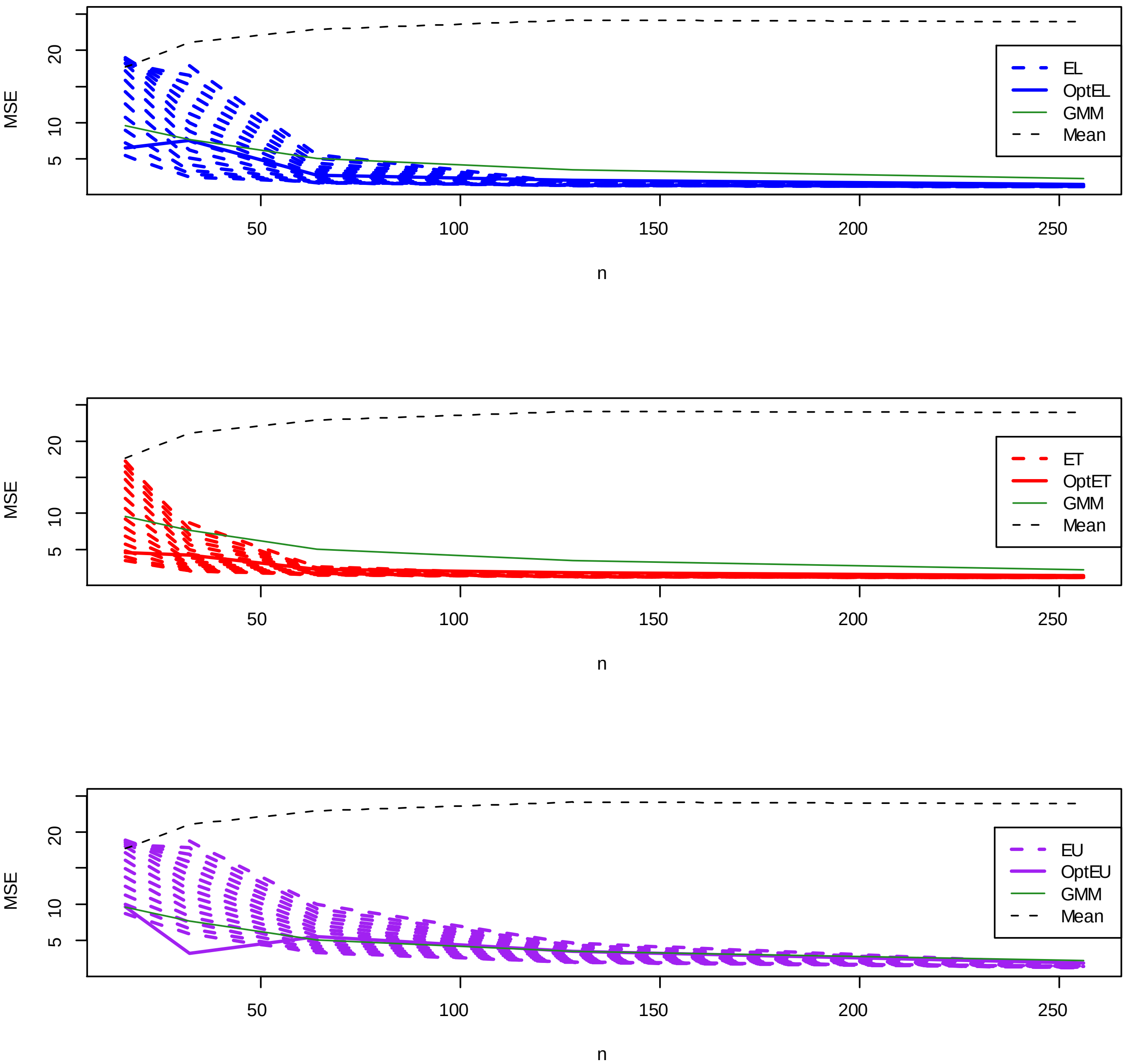

Figure 7: MSE of the Z-estimator as n varies for DGP 1

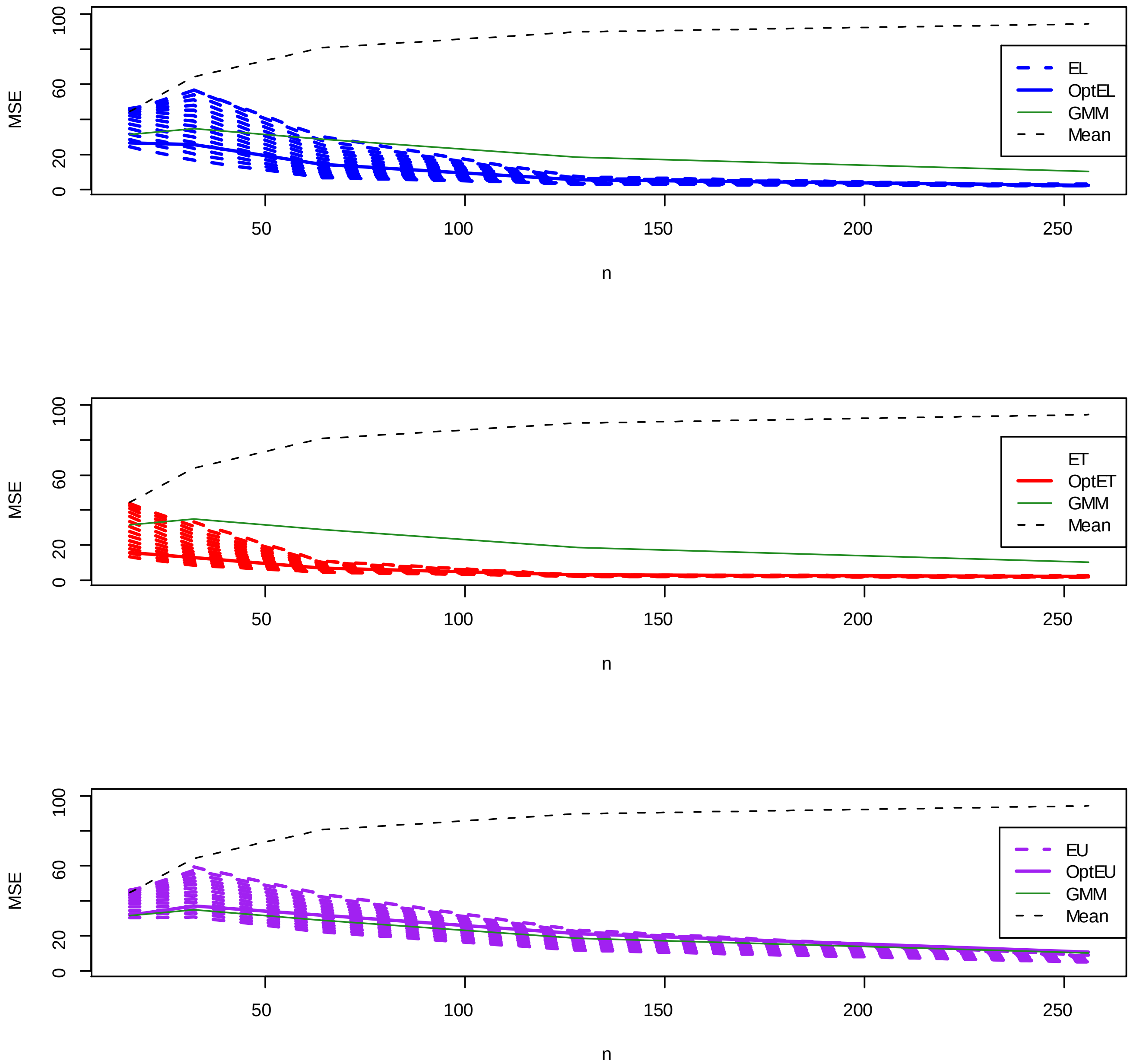

Figure 8: MSE of the Z-estimator as n varies for DGP 2

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