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COMPETITION AND THE SIGNALING ROLE OF PRICES

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Competition and the signaling role of prices[*]

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Abstract

In a market where sellers are heterogeneous with respect of the quality of their good and are more informed than buyers, high quality sellers' chances to trade might depend on their ability to inform buyers about the quality of the goods they offer. We study how the strength of competition among sellers affects the ability of sellers of high quality goods to achieve communication by means of appropriate pricing decisions in the context of a market populated by a large number of strategic price setting sellers and a large number of buyers. When competition among sellers is weak high quality sellers are able to use prices as a signaling device and this enables them to trade. By contrast, strong competition among sellers inhibits the role of prices as signals of high quality, and high quality sellers are driven out of the market.

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1 Introduction

A buyer interested in a specific digital camera could find out the list of retailers'
price quotes at shopper.com in just a few seconds. For any model, such a list would
invariably contain substantially dispersed prices.[1] Why would these price quotes not
obey the law of one price? After all, if consumers have access to market prices at a
negligible cost, one would expect all cameras of the same model and brand to trade
approximately at the same price.

The Internet allows consumers to observe prices of any specific camera at almost no
cost. Whether the Internet is as informative about other relevant characteristics of the
product they are interested in, is far less clear. For example, Lin and Scholten (2005)
document that not all the firms selling electronic products are explicit on whether
they sell brand new or refurbished or open box products. Information about delivery,
assistance, and customer care in general is also much less available than price quotes,
and is rather opaque anyway. Since the availability of hard information is typically
limited in online markets, sellers may attempt to signal quality through the choice of
price.

When sellers are more informed than buyers, the ability of sellers of high quality
goods to inform buyers about the quality of their goods might indeed be crucial in keep-
ing these sellers from being wiped out by price competition. However, is this ability
independent of the competitive pressure faced by the sellers? Does price competition
alter the effectiveness of price as a signaling device?

The signaling role of sellers' strategic price decisions in the presence of asymmetric
information has been the subject of extensive research and a summary of the related
literature is clearly beyond the scope of this paper. Within this literature, important
contributions including Milgrom and Roberts (1986), Laffont and Maskin (1987), Bag-
well and Riordan (1991) Bagwell (1991), Overgard (1987) and Ellingsen (1997) have
focused on the case of monopoly.[2] Bagwell (1991) finds that, with a downward sloping

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[1]Various studies document the presence of persistent significant price dispersion in internet markets
for final goods. See, for instance, Clay, Krishnan and Wolff (2001) for the case of electronic bookstores
and Baye, Morgan and Scholten (2004) for the case of consumer electronic products.

[2]Representative contributions focusing on other market structures include Laffont and Maskin
(1989) for the case of oligopoly and Cooper and Ross (1982) for the case of free entry, Wolinski

demand, the only equilibrium which satisfies the Intuitive Criterion (Cho and Kreps 1987) is a separating equilibrium in which the high quality is traded at a higher price but sells less than the low quality. Ellingsen (1997), in a model with one seller and one buyer with inelastic demand, finds that there is a unique equilibrium surviving D1 (Cho and Kreps 1987). The equilibrium is separating: the seller sells with probability one at the low price and with probability less than one (but positive) at the high price. Hence, the general consensus is that, in the absence of competition, a high quality seller is able to signal quality by distorting his price upwards and reducing the volume of trade relative to the first best.

The main contribution of the present paper is to understand to what extent (and under what conditions) this conclusion applies when sellers might face competition. We identify two regimes: *weak competition* (buyers outnumber sellers of low quality goods), and *strong competition* (buyers are outnumbered by low quality sellers). When competition is weak, different qualities trade at different prices and in different amounts. This matches the standard result obtained with monopoly. By converse, when competition is strong, only the low quality is traded in any robust equilibrium. In this case, prices do not serve as signals of quality and sellers of high quality goods are driven out of the market. We thus establish a causal link between the competitive pressure faced by sellers and the information conveyed through prices, as well as with the associated volume of trade and its quality. This is, to our knowledge, a novel result within the literature on the signaling role of prices.

The result is established in a model where there are two qualities of the same good, and therefore two types of sellers. However, as shown in the appendix, the result holds in the more general case of an arbitrary number of qualities. The stronger the competitive pressure faced by the sellers, the lower the number of qualities that will be traded and the associated degree of price heterogeneity observed in the market.

Prices serve as signals only when different types of sellers have different incentives to announce a particular price. How could this depend on the competitive pressure faced by sellers? Consider the strong competition regime. Low quality sellers undercut each other until, in equilibrium, they announce the zero profit price. So long as sellers' costs

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(1983) on search costs, and Jansen and Roy (2002) on durable goods.

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are increasing in the quality of the good, sellers of high quality goods cannot afford to sell at the price that gives zero profits to low quality sellers. Yet, in an equilibrium in which low quality sellers make zero profits, no trade can occur at higher prices. High quality sellers are thus unable to trade. Since both types of sellers make zero profits, they have identical incentives to deviate to any price at which high quality sellers would be willing to trade. Off-equilibrium prices cannot be used to signal the quality of the good. As a result, high quality sellers are driven out of the market.

Consider now the weak competition regime. Buyers compete and, therefore, low quality sellers make positive profits in equilibrium. Prices thus become an effective communication device. Consider, for instance, a candidate equilibrium in which only the low quality is traded. High quality sellers, who in equilibrium are out of the market, have an incentive to announce any off-equilibrium price greater than their production cost, whenever there is a positive chance to sell at those prices. Low quality sellers would announce such prices only if the chances to sell were sufficiently good, since they would make strictly positive profits by announcing the equilibrium price. Therefore, sellers of high quality goods seem more likely to benefit from announcing such prices than sellers of low quality goods. High quality sellers can thus use these prices to signal the quality of their goods. As a result, high quality sellers are never driven out of the market when competition is weak. In this case, the robust equilibrium is one in which both qualities are traded and the higher quality trades at a higher price.

Interestingly, incentive compatibility for the low quality sellers requires that the probability to sell at the higher price should be lower than one. Independently of the strength of demand, some high quality sellers will always be unable to sell, even when the price of high quality goods exceeds the production cost. The price should fall to equate demand and supply, but imperfect information inhibits such a market-clearing role of prices. Thus, sellers of high quality could be rationed in a sense similar to Stiglitz and Weiss (1981).[3] On the other hand, the upward pressure on prices induced by signaling reduces the downward pressure on prices resulting from competition among sellers. This is consistent with Daughety and Reinganum (2005)

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[3]See also Hellmann and Stiglitz (2000) on how rationing can emerge in the presence of inter-markets competition.

and (2007) who find that duopolists might benefit from the upward distortion on prices of high quality goods due to signaling. Intuitively, a side effect of the upward distortion is that prices become closer to cartel prices.

Following the seminal work by Akerlof (1970), the case of a market with competition between price setting sellers more informed than buyers has been considered by Wilson (1979 and 1980). He documents how price dispersion can emerge as the result of a separating equilibrium. However, "the absence of restrictions on the expectations of agents outside the set of [equilibrium] prices actually announced" [Wilson, 1980, page 126] implies a huge degree of indeterminacy. Many types of equilibria could actually exist, each associated with a particular degree of price dispersion.[4] Subsequent works have have exploited the predictive power of forward induction refinements to address this issue. Consistent with the rest of the literature, we use D1 to restrict beliefs out of the equilibrium path.[5] The result is a set of robust equilibria which share all the same unique outcome in terms of prices, quantity and quality of trade.

Although the issue of equilibrium selection is a delicate one, the obvious advantage of using a strong solution concept is that the predictive power of the theory is greatly enhanced – whether high quality sellers are driven out of the market or not only depends on the competitive pressure faced by sellers. Since different qualities always trade at different prices in robust equilibria, price dispersion is also uniquely determined by the strength of competition.[6]

Competition also determines the magnitude of price distortions due to the use of prices as signals. In the strong competition regime, the upward distortion is extreme: High quality sellers can only reveal their type by announcing prices so high that no trade can take place. By contrast, when competition is weak, high quality sellers are able to separate themselves with little or no distortion on prices.

Recent contributions use the mechanism design methodology to study the maximum level of welfare achievable in an economy characterized by asymmetric infor-

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[4]As already mentioned, Laffont and Maskin (1989) study an oligopolistic market where sellers can signal quality through their prices. They conclude that, even in the case of just two firms, the problem of characterizing all perfect Bayesian equilibria seems intractable.

[5]See Banks and Sobel (1987) and Cho and Kreps (1987).

[6]Villeneuve (2003) considers privately informed insurers competing to insure a single consumer. Differently from our framework, the type – which is determined by the consumer's risk profile – is the same for all insurers.

mation. Gul and Postlewaite (1992) study the conditions under which an economy characterized by asymmetric information can achieve efficiency as it becomes large. Muthoo and Mutuswami (2005) characterize the second best solution in markets with quality uncertainty where sellers are more informed than buyers.

Our approach is complementary to theirs to the extent that we study the prevailing equilibrium associated with a specific price convention. This enables us to find a relationship between competition and observable features of the market such as price dispersion. Moreover, we can assess how the use of prices as a communication device could affect the level of welfare and the distribution of surplus between buyers and sellers. In particular, while such a role of price could sometimes help high quality sellers to trade, it does not always lead to a welfare improvement. On the contrary, there are instances where welfare would be higher if sellers of goods of different qualities could be forced to pool at the same price.

In many cases, policy makers have argued that regulations restricting competition serve the purpose of maintaining the quality of products and services and protecting customers from malpractice. This is the case, for instance, of professional services defined as services that require practitioners to display a high level of technical knowledge which consumers might not have. Our results suggest that while limiting competition does raise the average quality traded in the market, it also reduces the surplus available to the buyers. Accordingly, when there are positive gains from trading low quality goods, buyers (customers) are unambiguously better off in the strong competition regime, in spite of the adverse effect on the quality of goods.

Similarly, we show that, when high quality sellers are driven out of the market by competition, imposing a price floor might help to restore trade of the high quality. This is broadly consistent with recent evidence by Huck et al. (2007) (although their environment is slightly different from the one considered here). Intuitively, a price floor would prevent the profits of low quality sellers from falling to zero when competition is strong. This would allow high quality sellers to separate themselves by charging prices above the price floor. In particular cases, price controls might thus increase efficiency. On the other hand, price controls have relevant redistributive effects. Any gain in overall efficiency necessarily comes at the cost of a lower consumer surplus.

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The paper is organized as follows. In section two we present the model. In Section three we describe the equilibrium concept and its refinements, and we characterize the set of robust equilibria. Section four describes the features of the equilibrium outcome and discusses potential policy interventions. Section five discusses issues of robustness. A final section concludes the paper.

2   The Model

We consider a large market populated by B buyers, and S sellers. The set $\mathcal{S}$ of sellers is indexed by $s = 1, ..., S$; $s \in \mathcal{S}$. Each seller can produce either one unit of an indivisible good or nothing. Goods come in two different qualities, $q \in \{h, l\}$, where l (h) stands for *low* (*high*). The general case of a finite number of qualities is analyzed in the Appendix. We refer to sellers producing a quality q good as sellers of quality (or type) q. The quality of each seller is decided by nature: Each seller has a probability λ to be of quality h and probability $1 - \lambda$ to be of quality l. The distribution of qualities is common knowledge. However, buyers cannot observe individual qualities. Moreover, quality is not verifiable ex post. The cost of producing one unit of the good is denoted with $c(q)$ and increases with the seller's quality: $c(h) > c(l)$.

The set $\mathcal{B}$ of buyers is indexed by $b = 1, ..., B$; $b \in \mathcal{B}$. Each buyer consumes either one good or nothing. Buyers share identical preferences defined by the monetary utility function $u(q) > 0$, with $u(h) > u(l)$. We impose $u(q) > c(q)$ for all $q \in \{h, l\}$, which implies that under full information there are always gains from trade to be realized. For expositional purposes, we also impose $u(l) < c(h)$: Buyers are never willing to buy a low quality good at any price that is profitable for a high quality seller.

We are mainly interested in characterizing the behavior of agents in a large market. Accordingly, we consider the case in which both the number of buyers and the number of sellers go to infinity and their ratio, B/S, converges to some value $\theta \in \mathbb{R}^+$.[7]

The market functions as follows.

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[7]As it is well known (see Judd 1985), the use of the law of large numbers with a continuum of agents may be inappropriate.

Pricing

At stage zero, each seller s observes his quality. Endowed with this piece of information, sellers move first, by simultaneously choosing their action, while buyers do nothing. The action p_s played by seller s consists in announcing a price $p \in [0, \overline{p}]$, where $\overline{p}$ is finite and strictly greater than $u(h)$, so that in equilibrium trade never occurs at $\overline{p}$. For simplicity, we adopt the convention that sellers who choose not to trade always announce $\overline{p}$. A strategy for seller s is a map from $\{l, h\}$ into the set $\mathbb{A}$ of probability distributions over $[0, \overline{p}]$. An action profile for the sellers is a collection $\mathbf{p} \equiv \{p_1, ..., p_S\}$, with $p_s \in [0, \overline{p}]$.

Beliefs

At stage 1, buyers observe the prices announced by sellers at stage zero and choose whether to buy and at what prices. Buyers' prior beliefs assign a probability λ to the event that an individual seller s is of type h. Upon observing the price p announced by a seller s, and given the sellers' action profile $\mathbf{p}$, buyers' (posterior) beliefs that the seller announcing p is of type q are denoted with the conditional probability function $\sigma(q|p, \mathbf{p})$.

Demand

We do not model explicitly strategic interaction between buyers since this would require the choice of a mechanism matching buyers and sellers. We choose to simplify the analysis by specifying only the minimum requirements that the demand side must satisfy for the results. As will be made clear in the next section, this reduces to assuming that the essential properties of a market à la Bertrand are preserved.

We therefore assume that, after observing $\mathbf{p}$, buyers play a (continuation) game in which each buyer chooses an action (or a set of actions) which may or may not result in a purchase. For instance, buyers can compare prices, inquire about availability, and then choose whether to buy at some price. The outcome of this game is a function $J(p; \mathbf{p}, \sigma)$, $J : [0, \overline{p}] \to [0, 1]$ which, for any $\mathbf{p}$ and σ, specifies the probability to sell at some price p. Given that all sellers are ex-ante identical and they move simultaneously, the function $J(p; \mathbf{p}, \sigma)$ will be the same to all sellers.

Finally, if a seller is able to sell to a buyer, he receives the price he announced and incurs the production cost $c(q)$, while the buyer obtains utility $u(q)$ from consumption and pays the price announced by the seller. We denote with K the share of buyers who are able to obtain a good. The values of $J(p; \mathbf{p}, \sigma)$ and K are connected by the restriction that the number of goods sold must equal the number of goods bought. Denote with $s(p)$ the fraction of sellers announcing p. Using the law of large numbers, this restriction can be stated as

$$\sum_p s(p) J(p; \mathbf{p}, \sigma) = K\theta, \tag{1}$$

where both sides are scaled by the total number of sellers in the economy (S).

3 Equilibrium

We base our equilibrium analysis on the concept of perfect Bayesian equilibrium (PBE). We now discuss the restrictions that sellers' strategies, beliefs σ, and buyers' behavior (summarized by the function J) must satisfy.

Sellers' strategies

The expected payoff of a seller of type $q \in \{l, h\}$ announcing price p when prices announced by all sellers are summarized by $\mathbf{p}$ is $J(p; \mathbf{p}, \sigma)[p - c(q)]$. In equilibrium, the sellers' strategy profile must satisfy the following two restrictions

R1 All sellers play best replies given other sellers' strategies, beliefs σ, and the function J;

R2 Strategies are symmetric: all sellers of the same quality announce the same prices with the same probabilities.

Condition R1 is entirely standard and requires no explanation. Condition R2 is commonly invoked when dealing with many agents. Symmetric strategies are usually imposed to simplify the equilibrium analysis. With price-setting sellers, this particular condition is further motivated by the fact that buyers' beliefs are derived from sellers' strategies. If strategies were not symmetric, buyers could assign different probabilities

to be of a given type to sellers announcing the same price. This is at odds with the conventional idea of a large market in which trade is not affected by the identity of individuals. Moreover, since we allow for mixed strategies, symmetric strategies do not rule out asymmetric actions as an equilibrium outcome. Thus, imposing symmetric strategies does not imply a great loss of generality in terms of agents' behavior.

We also restrict our attention to equilibria in which sellers randomize over prices by using distributions with finite support. This restriction, together with the assumption that all sellers adopt the same strategies, permits to exploit the law of large numbers (see below).

Beliefs

R3 All buyers share the same beliefs and these are derived from sellers' strategies using Bayes rule where possible;

R4 Buyers' beliefs about a seller announcing a given price are not affected by the price announced by another seller, even in the presence of deviations.[8]

Condition R3 is again standard. Symmetry, together with the assumption that buyers' beliefs obey Bayes' rule, imply that buyers should assign the same probability to be high quality to any pair of sellers announcing the same price. Condition R4 implies that beliefs about seller s are independent of other sellers' actions, even in the presence of deviations (Fudenberg and Tirole, 1991, p. 332). This condition is consistent with the fact that sellers different from s have no information about s's type that is not also available to the buyers.

Buyers' behavior

As already mentioned, rather than assuming a specific market structure, we consider a broad range of possible interactions. We require however that the function $J(p, \mathbf{p}, \sigma)$ satisfy some familiar properties of competitive behavior,

R5 For all $\mathbf{p}$, and beliefs σ, the function $J(p; \mathbf{p}, \sigma)$ satisfies:

    i) $J(p; \mathbf{p}, \sigma) = 0$ for all p such that $\sigma(h|p, \mathbf{p})u(h) + [1 - \sigma(h|p, \mathbf{p})]u(l) < p$;

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[8]Formally, this is equivalent to saying that, for all p, $\hat{p}$, $\mathbf{p}$, $\hat{\mathbf{p}}$, and q, $\sigma(q|p, \mathbf{p}) = \sigma(q|\hat{p}, \hat{\mathbf{p}})$ if $p = \hat{p}$.

ii) If, at some p, $J(p; \mathbf{p}, \sigma) > 0$ and $\mathbf{p}$ contains some p' such that

$$\sigma(h|p', \mathbf{p})u(h) + [1 - \sigma(h|p', \mathbf{p})]u(l) - p' >$$
$$\sigma(h|p, \mathbf{p})u(h) + [1 - \sigma(h|p, \mathbf{p})]u(l) - p, \tag{2}$$

then $J(p'; \mathbf{p}, \sigma) = 1$;

iii) If $K < 1$, then $J(p, \mathbf{p}, \sigma) = 1$ for all p such that

$$\sigma(h|p, \mathbf{p})u(h) + [1 - \sigma(h|p, \mathbf{p})]u(l) > p. \tag{3}$$

Condition (i) is the result of a standard participation constraint. If, given beliefs, buyers expect to make a loss at p, no buyer would buy at p. Hence, the probability to sell must be zero. Condition (ii) says that if at p the probability to sell is positive and, given beliefs, there is another price p' at which buyers make higher surplus, then the probability to sell at p' must be one. Intuitively, it is possible to sell at p only if at all prices that guarantee a better deal to the buyer there is no excess supply. This is akin to assuming Bertrand competition when supply is inelastic. Condition (iii) says that if there are buyers who are unable to obtain a good, then it must be possible to sell with probability one at all prices that, given beliefs, leave the buyer with a positive surplus. Again, the intuition is obvious. Notice also that conditions (i-iii) do not impose any restriction on how beliefs should vary according to the observed price. As it usually happens in the presence of adverse selection, the prices that leave the highest expected surplus to the buyer are not necessarily the lowest.

Conditions (i-iii) are natural when search costs are not particularly high. As observed by Bester (1993), this is exactly the case when we should expect posted prices as opposed to bargaining. Our results thus apply to any specific market setting where sellers post prices and buyers interact in a way that is compatible with R5.

For an immediate example, consider a situation in which buyers arrive sequentially and choose at which price to buy and then select at random among the sellers who have announced that price and have not sold to previous buyers. This is reminiscent of customers visiting a price comparison site listing sellers' quotes. Otherwise, if buyers simultaneously choose the price at which to buy, the purchase (sale) may be allocated through a lottery whenever there is excess demand (supply). This is the approach

taken in Wilson (1980). A third example is a situation in which, after observing the prices posted by the sellers, buyers simultaneously submit a ranking of the prices at which they accept to buy. A seller is then matched with a buyer only when all sellers offering prices that are better placed in the buyers' rankings have been able to sell their goods.

As already mentioned, we are interested in the behavior of a market characterized by a large number of agents. The law of large numbers works on two levels here. First, the individual realizations of Nature's draws are irrelevant: the fraction of type h (resp. l) sellers in the market is always equal to λ (resp. $1-\lambda$). Second, given symmetry and the assumption that agents randomize over distributions with finite support, individual realizations of agents' randomization are also irrelevant. In equilibrium, the fraction of sellers announcing a given price is certain. Given R1-R5, in the remainder of the paper, we will denote simply with $\sigma(q|p)$ the posterior probability that a seller announcing price p is of type q (or, equivalently, the fraction of type q sellers among sellers announcing p). Also, when there is no risk of confusion, action profiles and beliefs will be omitted when referring to the probability to make a sale. This will be denoted simply as $J(p)$.

Equilibria can take different forms:

a. *Separating equilibria*, in which, by definition, different seller-types take different actions;

b. *Pooling equilibria*, in which all seller-types take the same action;

c. *Partially separating* or *Hybrid equilibria*, in which heterogenous poolings of sellers take different actions.

Associated with this variety of equilibria is a great deal of indeterminacy with respect to the market's outcome in terms of prices, traded quantities and qualities, as well as with respect to the associated expected payoffs of market's participants. For this very reason it is important to investigate how a robustness analysis helps restricting the set of possible equilibria.

3.1 Restrictions on out of equilibrium beliefs

The high degree of indeterminacy is due to a typical "unsent message" problem: if a seller deviates to a price p that is announced with probability zero in equilibrium, Bayes' rule cannot determine the posterior beliefs of the buyers. Thus, upon observing a profile $\mathbf{p}$ containing the deviation p, buyers could hold arbitrary beliefs about the quality of the seller who is announcing p. Therefore, we impose that buyers' out of equilibrium beliefs be consistent with a standard equilibrium refinement.

R6 Buyers' out of equilibrium beliefs satisfy D1 (Cho and Kreps 1987).

In this section we discuss how this restriction can be used to eliminate equilibria in the case of many buyers and sellers.[9] If seller s deviates and announces a new price p, his probability to sell at p depends on whether p is more or less appealing to buyers than the prices announced by other sellers. However, what beliefs should buyers hold about sellers who did not deviate? If a seller sticks to his equilibrium strategy, equilibrium requires that beliefs about him should not be affected by the deviation of another seller. This follows from restriction R4. As a result, buyers' beliefs about sellers who did not deviate are the same as in the candidate equilibrium. This implies that we can just focus on whether the beliefs about the seller *who actually deviated* are consistent or not with the D1 criterion.

Section A.1 in the Appendix contains a formal definition of D1. We now describe how the refinement works in practice. Fix a price p that is never announced in equilibrium and let $J(p)$ denote the probability to sell at p. Then, if for all $J(p)$ such that a type q seller weakly benefits from the deviation (relative to his equilibrium payoff) a type $q' \neq q$ seller would strictly benefit, then buyers' posterior beliefs assign probability zero to type q. That is, $\sigma(q|p) = 0$, or equivalently $\sigma(q'|p) = 1$.

In our framework, it will be often more practical to focus on D1-robust equilibrium *outcomes* – the collection of prices, quantities and qualities exchanged – rather than

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[9]Mailath et al. (1993) point out that the D1-robust outcome may not converge to the full information outcome as buyers' information becomes "almost" perfect (e.g. when λ approaches one or to zero in our setup). In a monopolistic framework, Adriani and Deidda (2009) show that this may be problematic. As will become clear, the discontinuity affects the D1-robust outcome in our game when λ is close to one. However, allowing for "continuous" equilibria when λ is close to one would change some quantitative results, but not the main qualitative results (i.e. what qualities are traded).

on robust equilibria. The reason is that the same outcome can be typically induced by many distinct equilibria. An outcome is said to be robust to D1 if there exist D1-robust equilibria that induce the outcome. We now provide a simple condition that allows to test whether a given equilibrium outcome is robust to D1. Let $\pi^*(q)$ denote the expected equilibrium payoff of a type q seller.

Lemma 1. *An outcome is robust to D1 if and only if there exist equilibria that induce the outcome which specify $\sigma(h|p) = 1$ for all out of equilibrium prices $p > c(h) + \pi^*(h)$ such that*

$$\frac{\pi^*(l)}{p - c(l)} > \frac{\pi^*(h)}{p - c(h)}. \tag{4}$$

Proof. See Appendix.

Since the condition above is a direct implication of D1, it is useful to gather intuition about the implications of D1 in our framework. Inequality (4) is intuitively appealing: If the opportunity cost $(\pi^*(q))$ of a deviation p relative to the potential gain $(p - c(q))$ is larger for a low quality seller than for a high quality one, then buyers should infer that the deviation comes from a high quality seller. In what follows, we will thus restrict attention to equilibria such that $\sigma(h|p) = 1$ for all p such that (4) holds. The reason why Lemma 1 only considers deviations such that $p - c(h) > \pi^*(h)$ is that type h should be able to benefit from the deviation conditional on the probability to sell at p being sufficiently large. These are the deviations where D1 bites.[10]

For the purposes of the robustness analysis we distinguish the candidate equilibria into two broad categories on the basis of how many qualities are traded.

Definition 1. *A type I equilibrium is a PBE where both qualities are traded. A Type II equilibrium is a PBE where only the low quality is traded.*

It is worth noting that, in general, there would be a third category, which includes those equilibria in which no quality is traded. However, as it turns out, the low quality is always traded given our assumptions.[11]

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[10]Notice also that, as shown in the proof of Lemma 1, not all equilibria that satisfy $\sigma(h|p) = 1$ for all p such that (4) holds are robust to D1. Moreover, there are equilibria that do not satisfy that condition that are robust. However, the firsts always have an outcome-equivalent equilibrium that is robust and the seconds always have an outcome-equivalent equilibrium that satisfies the condition. Since we are focusing on outcomes, $\sigma(h|p) = 1$ for all p such that (4) holds is thus both necessary and sufficient for robustness.

[11]Consider an equilibrium where all sellers announce prices at which trade does not occur. Given $K < 1$, $J(p) = 1$ for all $p < u(l)$ independently of out of equilibrium beliefs. Then, deviating and announcing $p \in (c(l), u(l))$ is always profitable for a seller of type l.

3.2 Type I equilibria: both qualities are traded

Type I equilibria can take two forms: a. Separating equilibria (SE); b. Pooling equilibria (PE) and Hybrid equilibria (HE).

In any PE, by definition, there is a single equilibrium price p^* at which both high and low qualities are traded. In HE sellers of the same type may announce different prices and there is at least one price that is announced with positive probability by both types. It is well known that D1 tends to select SE (see Cho and Sobel, 1990). The next Lemma shows that also in the present model, within the set of type I equilibria, D1 discards PE and HE.

Lemma 2. *No pooling/hybrid equilibrium of type I survives D1.*

Proof. See Appendix.

In any PE or HE where both qualities are traded, sellers of type h make lower profits than sellers of type l since they face higher production costs ($c(h) > c(l)$). Sellers of type h have therefore a lower opportunity cost of deviating. Buyers typically infer that any deviation to a price above the price at which both qualities are pooled must come from a high quality seller. Sellers of type h are thus able to "stand out of the crowd" by announcing a price that is slightly higher than the equilibrium price at which both qualities are traded. Accordingly, no PE or HE of type I is ever robust: Robust equilibria of type I are always SE.

In a separating equilibrium, low quality sellers announce a price p_l, while high quality sellers announce a different price $p_h \neq p_l$. Prices constitute a perfect signal of quality: $\sigma(h|p_h) = 1$, and $\sigma(h|p_l) = 0$. In any SE, p_h and p_l satisfy

$$[p_l - c(l)]J(p_l) \geq [p_h - c(l)]J(p_h), \tag{5}$$

$$[p_h - c(h)]J(p_h) \geq [p_l - c(h)]J(p_l). \tag{6}$$

These two inequalities represent the Incentive Compatibility Constraints (ICC) for low and high quality sellers, respectively. Any SE in which quality $q \in \{l, h\}$ is traded must also: 1) satisfy the participation constraint of sellers ($p_q \geq c(q)$) and buyers ($p_q \leq u(q)$); 2) ensure that if buyers obtain higher surplus from buying at p_q than at

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$p_{q'}$, then $J(p_q) = 1$. These conditions imply that any SE is characterized by $p_h > p_l$ and $J(p_h) < J(p_l)$.[12]

Whenever the equilibrium is separating, the robustness condition (4) can be rewritten as

$$\frac{[p_l - c(l)] \, J(p_l)}{p - c(l)} > \frac{[p_h - c(h)] \, J(p_h)}{p - c(h)}. \tag{9}$$

If (9) holds for some p such that type h may benefit, then $\sigma(h|p) = 1$. The next Lemma illustrates how D1 helps to restrict the set of separating equilibria.

Lemma 3. *In any D1-robust SE of type I, the ICC of low quality sellers is satisfied with equality unless $p_h = c(h)$.*

Proof. See Appendix.

In any SE of type I the probability to sell at p_h must be less than 1 to prevent mimicking from type l. Consider a deviation to a price p slightly lower than p_h. A high quality seller would be willing to trade off a small reduction in price for (a possibly small) increase in the probability to sell. If the ICC of type l does not hold with equality, a low quality seller strictly prefers p_l to p_h. This implies that type l are reluctant to deviate unless the chances to sell at p become relatively high. According to D1, buyers should infer that the deviation p must come from a high quality seller, which in turn gives sellers the incentive to deviate. By contrast, when type l ICC holds with equality, low quality sellers are indifferent between p_l and p_h. They would thus profit from the deviation whenever the type h would profit. Hence, beliefs that the deviation emanates from a type l seller with sufficiently high probability are consistent with D1.

We are interested in market conditions under which a SE of type I is robust to D1. A key parameter in our discussion is the ratio, θ, between potential demand, given by the number of buyers, and potential supply, given by the number of sellers. This

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[12]If $J(p_h) \geq J(p_l)$, then, from type l ICC, $p_h < p_l$. Type l ICC can be rewritten as

$$\frac{p_l - c(l)}{p_h - c(l)} \geq \frac{J(p_h)}{J(p_l)}. \tag{7}$$

But then, given $p_h < p_l$ and $c(h) > c(l)$,

$$\frac{p_l - c(h)}{p_h - c(h)} > \frac{p_l - c(l)}{p_h - c(l)} \geq \frac{J(p_h)}{J(p_l)}, \tag{8}$$

so that type h ICC is violated. By converse, if $J(p_h) < J(p_l)$, then $p_h > p_l$ follows from R5(ii).

is a measure of the competitive pressure faced by buyers and sellers. Before stating the conditions under which robust type I equilibria emerge, we also need to define the following quantities

$$GFT_q \equiv u(q) - c(q), \ \Delta GFT \equiv GFT_l - GFT_h, \tag{10}$$

$$\delta \equiv \frac{GFT_l}{u(h) - c(l)}, \ \gamma \equiv \frac{\Delta GFT}{c(h) - c(l)}, \tag{11}$$

$$\hat{\theta} \equiv 1 - \lambda + \delta\lambda, \ \theta_\gamma \equiv 1 - \lambda + \gamma\lambda I_{\{\gamma>0\}}, \tag{12}$$

where $I_{\{\gamma>0\}} : \mathbb{R} \to \{0, 1\}$ is an indicator function that takes value 1 if $\gamma > 0$ and zero otherwise. Notice that since $\gamma < \delta$, $\hat{\theta}$ is always strictly greater than θ_γ. The next result gives necessary and sufficient conditions for robust type I equilibria and characterizes the robust equilibrium outcome.

Proposition 1. *D1-robust equilibria of type I emerge if and only if $1 - \lambda < \theta$. In all these equilibria: i) $J(p_l) = 1$, and $J(p_h) = \min \left[\frac{\theta - (1-\lambda)}{\lambda}, \delta \right]$, ii) p_l and p_h are uniquely determined:*

   i. $p_h = u(h), \ p_l = u(l), \ if \ \theta \in [\hat{\theta}, \infty);$

   ii. $p_h = c(l) + \frac{\lambda[u(h)-u(l)]}{1-\theta}, \ p_l = c(l) + \frac{\theta - (1-\lambda)[u(h)-u(l)]}{1-\theta} \ if \ \theta \in (\theta_\gamma, \hat{\theta});$

   iii. $p_h = c(h), \ p_l = u(l) - [u(h) - c(h)] \ if \ \theta \in (1 - \lambda, \theta_\gamma].$

Proof. See Appendix.

To gather intuition, we start by discussing why $1 - \lambda < \theta$ is necessary and sufficient for robust equilibria of type I. Consider a SE of type I so that sellers of type l announce p_l and sellers of type h announce p_h. If $1 - \lambda$ exceeds θ, low quality sellers are relatively more numerous than buyers (i.e. they are the long side of the market). This implies that sellers compete to sell, while buyers face no competitive pressure. Accordingly, competition among low quality sellers ensures that type l make zero profits: $p_l = c(l)$.[13] However, if $p_l = c(l)$, then any price $p_h \geq c(h)$ with a positive probability to sell would violate incentive compatibility of low quality sellers. As a result, the probability to trade at any price higher than (or equal to) $c(h)$ must be zero, so that high quality sellers are driven out of the market.

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[13]Price competition works here because the worst belief that buyers can assign to a seller announcing a price lower than p_l is that he is of type l with probability 1, and buyers are buying quality l at p_l.

Consider now the case $1 - \lambda < \theta$. Low quality sellers are on the short side of the market. Buyers, on the other hand, face competitive pressure. If $1 - \lambda < \theta$, low quality sellers are not enough to satisfy the whole demand. Therefore, they will raise their prices until some of the buyers will be willing to buy (from high quality sellers) at a price $p_h \geq c(h)$. Notice however that the probability to sell at p_h must be sufficiently low to ensure that low quality sellers do not have incentive to announce p_h.

As for the characterization, Proposition 1 identifies a unique robust equilibrium outcome in terms of prices and traded quantities and qualities. Again, the prices, qualities, and quantities crucially depend on the buyers to sellers ratio, θ. The relationship between θ and the equilibrium prices is illustrated in Figure 1. If θ is very large, i.e. greater than $\hat{\theta}$, trade (of both qualities) occurs at buyers' reservation prices, $u(h)$ and $u(l)$ (case i). If θ is only moderately large, i.e. greater than $1 - \lambda$ but lower than $\hat{\theta}$, trade (of both qualities) occurs at prices that guarantee a positive surplus to the buyers (cases ii and iii). Notice that, provided that $\gamma \leq 0$, p_h will exceed type h sellers' reservation production cost in any robust SE of type I. This occurs even though the probability to sell at p_h, $J(p_h)$, is always less than one, which would suggest that high quality sellers' profits should be competed away. This is a standard effect of asymmetric information. Price competition among type h is impaired by buyers' fear that type l may deviate and announce p_h if the probability to sell at p_h becomes too large. Hence, price competition among sellers of type h comes to a halt when the demand at p_h is such that low quality sellers are indifferent between announcing p_h and announcing p_l. Limited price competition causes high quality sellers' profits to remain positive, even if $J(p_h) < 1$.

Things change if the gains from trading the low quality exceed those from trading the high quality ($\gamma > 0$) and $\theta \leq \theta_\gamma$ (if $\theta > \theta_\gamma$, the previous discussion applies). In this case, the ICC of low quality sellers does not hold with equality and the price announced by high quality sellers drops to $c(h)$. In other words, high quality sellers must forgo their profits in order to trade. Low quality sellers will then announce the highest possible price at which buyers (weakly) prefer to buy the low quality, given the option to buy the high quality at $c(h)$.

3.3 Type II equilibria: Only the low quality is traded

In this section we turn our attention to the typical lemon-market situation in which the high quality is driven out of the market (type II equilibria). We will characterize the (unique) robust outcome and show that type II equilibria emerge if and only if there is no robust equilibrium of type I.

Proposition 2. *D1-robust equilibria of type II emerge if and only if $1 - \lambda \geq \theta$. In all these equilibria, all buyers purchase a low quality good. All trade occurs at a unique price p^*, which is equal to $c(l)$ if $1 - \lambda > \theta$.*[14]

Proof. See Appendix.

In order to gather intuitions on Proposition 2, notice that $1 - \lambda < \theta$ implies that sellers of low quality are on the short side and buyers face competitive pressure. If only the low quality were traded, low quality sellers would announce $p^* = u(l)$. At this price, they would extract all the surplus from the buyers and make strictly positive profits. It is easy to see why these equilibria fail D1. Whenever sellers of low quality make strictly positive profits, their opportunity cost of deviating, measured by $\pi^*(l)$, is larger than that of high quality sellers – which is zero since they are not trading. Accordingly, while high quality sellers are never worse off when deviating, low quality sellers might be hurt. Upon observing a deviation $p > c(h)$, buyers should accordingly infer that the seller deviating is of high quality.

By contrast, when $1 - \lambda > \theta$, (low quality) sellers' profits are driven to zero by competition ($p^* = c(l)$). Hence, the opportunity cost of deviating is the same for low and high quality sellers alike, and both types have nothing to lose from deviating. Thus, deviating to prices at which no trade occurs in equilibrium is as cheap a way to signal quality for the low type as it is for the high type. Unsurprisingly, deviations to higher prices thus fail to convey information about quality.

Finally, notice that robust type II equilibria can be either SE or hybrid equilibria (HE). In SE all low quality sellers announce p^* while high quality sellers announce $\bar{p}$. In HE, all sellers of quality h and a fraction smaller than $1 - \lambda - \theta$ of low quality sellers announce $\bar{p}$ (and do not trade) while the rest announce p^*.[15]

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[14]The analysis of the equilibrium price for the special case $1 - \lambda = \theta$ is presented in the proof. There, it is shown that a discontinuity arises when $\gamma > 0$.

[15]One might wonder whether D1 is necessary for the result that the high quality is traded whenever

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4 Properties of the robust equilibrium outcome

We now discuss the properties of the equilibrium outcome and characterize the amounts of low and high quality goods traded and the distribution of surplus.

Cho and Sobel (1990) show that in signaling games that satisfy specific monotonicity and sorting conditions, D1 selects a unique equilibrium, which is a SE. In the model we analyze, given any two prices p and $p' < p$ and associated probabilities to sell $J(p)$ and $J(p')$ their sorting condition would be

$$J(p)[p - c(l)] \geq J(p')[p' - c(l)] \Rightarrow J(p)[p - c(h)] > J(p')[p' - c(h)]. \tag{13}$$

Whenever low quality sellers benefit from announcing a higher price, high quality sellers would strictly benefit from doing the same. Condition (13) is of course satisfied for all prices at which trade occurs, i.e. provided that $J(p) > 0$ and $J(p') > 0$. However, at prices at which the probability to sell is zero the net payoff is independent of the announced price and seller's type. Hence, at such prices, no sorting is possible and (13) is not satisfied. These observations help explaining why in the model we analyze the set of $D1$ robust equilibria does not include only separating equilibria and generally contains more than one equilibrium. Nevertheless, as it directly follows from the combination of Propositions 1-2 and Lemma 2, $D1$ guarantees separation at prices at which trade occurs. Pooling can only occur at prices at which trade does not occur.[16] As a result, the equilibrium outcome is essentially unique in terms of quantities and prices at which trade occurs.

Proposition 3. *Given the values of λ, $\theta \neq 1 - \lambda$, $u(l)$, $c(l)$, $u(h)$, $c(h)$, all the resulting D1-robust equilibria yield the same unique outcome in terms of prices of traded goods, quality and quantity of trade. In particular: i. The fraction of quality l goods being traded (over the total supply of quality l) is $f(l) = \min[\theta/(1 - \lambda), 1]$; ii. The fraction of quality h goods being traded (over the total supply of quality h) is $f(h) = \max \left[0, \min \left[\frac{\theta - (1 - \lambda)}{\lambda}, \delta \right] \right]$.*

$1 - \lambda < \theta$. We considered robustness of type II equilibria to less powerful refinements such as Divinity (Banks and Sobel, 1987) and Sequential Perfection (Grossman and Perry, 1986). Both criteria give the same results. There is only a significant difference between these criteria and D1: when $1 - \lambda < \theta$, type II equilibria may pass Divinity and Sequential Perfection (but not D1) if the proportion of high quality sellers is sufficiently small.

[16]In particular, pooling survives $D1$ in equilibria of type II where only a fraction of low quality sellers announce $p^* = c(l)$ at which trade occurs, while high quality sellers and the rest of low quality sellers who decide not to trade announce $\bar{p}$. However, such HE yield the same equilibrium outcome in terms of quantities and qualities traded and agents' interim payoffs as the robust SE of type II in which all sellers of type l announce p^*.

Proof. See Appendix.

Because of this uniqueness property, the model implies a very precise relationship between the market conditions (as measured by θ) and:

a. Quantity and quality of trade;

b. Distribution of trade surplus;

c. Price dispersion and distortions;

d. Signaling role of prices.

a. Quantity and quality of trade.

If low quality sellers are relatively more numerous than buyers, the only D1-robust equilibrium is one in which only the low quality is traded; all buyers are able to buy. By converse, if buyers are relatively more numerous than low quality sellers, in the robust equilibrium both low quality and high quality sellers are able to sell their goods with positive probability. All low quality sellers are able to sell while only a fraction of sellers of high quality is able to find a buyer.

The fraction of high quality goods traded is a nondecreasing function of θ, as illustrated in Figure 2. For values of $\theta \leq 1 - \lambda$, the fraction of high quality traded, $f(h)$, is equal to zero, while the fraction of low quality, $f(l)$, increases in θ. If $\theta > 1 - \lambda$, $f(h)$ linearly increases in θ until it reaches the value $\delta < 1$ where θ equals the critical value $\hat{\theta}$; $f(l)$ stays constant. Once θ has reached $\hat{\theta}$, further increases in θ do not affect $f(h)$ any longer. Notice that, while all buyers are able to buy one good if $\theta \leq \hat{\theta}$, a fraction $\lambda \delta$ of buyers do not obtain any good when the reverse (strict) inequality holds. This happens in spite of the fact that high quality sellers sell with probability $\delta < 1$. Although trade would be mutually beneficial, there might be buyers and sellers who are unable to trade. On the other hand, D1 selects the equilibrium where the amount of trade is maximized among all SE. Hence, the prevailing SE is the one in which the potential inefficiency related to the quantity of trade is minimized.

It is important to note that δ is decreasing in $u(h)$. When demand is sufficiently high, the higher is $u(h)$ the higher must be the price of high quality goods. However, the probability to sell a high quality good is bounded above by δ. As a result, the

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more buyers value high quality goods, the lower must be the maximum fraction of high quality sellers able to sell their good. The model thus displays a sort of curse on high quality sellers.

b. Distribution of trade surplus

The strength of competition among sellers has important redistributive implications. On the one hand, when competition is weak, the average quality traded is higher. On the other hand, sellers appropriate a larger share of the surplus. Our results show that, provided that there are potential gains from trading the low quality $(c(l) < u(l))$, buyers unambiguously benefit from strong competition. To see this, notice that if $1 - \lambda$ exceeds θ the low quality is traded at $p = c(l)$. On the other hand, if θ exceeds $1 - \lambda$, the low quality is always traded at a price higher than $c(l)$. In equilibrium, the surplus obtained from buying either quality is the same. Hence, buyers' surplus is always lower under weak competition than under strong competition. In other words, all the potential benefits from trading the high quality accrue to the sellers.

c. Price dispersion and distortions

Prices are non decreasing functions of θ as described in Figure 1. When $1 - \lambda > \theta$, all trade occurs at price p_l (no price dispersion). Under reversed market conditions $(1 - \lambda < \theta)$, trade occurs at two different prices, p_l and p_h (price dispersion).

Wilson (1980) first argued that, in a market for lemons with price setting sellers, trade may occur at a distribution of prices rather than at a unique price. The results of our analysis have precise implications regarding the conditions under which a distribution of prices should arise in a lemon market. According to our model, the stronger the competition, the less price dispersion and variety of trade we should observe, and viceversa. This implication holds not just in the case of a market with two types of sellers and two qualities of goods, but also in the general case of any finite number N of qualities, which is analyzed in the Appendix.

As in other models with adverse selection, signaling through prices might result in price distortions. Notably, the magnitude of such distortionary effects varies with market conditions. In particular, price distortion vanishes for sufficiently weak competition $(\theta > \hat{\theta})$, while it is extreme in the case of strong competition, where the high

quality price is so high that no trade is possible.

d. Signaling role of prices

Off the equilibrium path, the information content of a deviation to a price higher than the equilibrium price changes according to whether sellers face weak competition or strong competition. If competition among sellers is weak, starting from an equilibrium in which only the low quality is traded, a deviation to a higher price allows high quality sellers to reveal themselves. As a consequence, the initial equilibrium unravels. By converse, in case of strong competition among sellers, both high and low quality sellers make zero profits and therefore have identical incentives to deviate. As a result, the deviation does not serve as a signal for high quality sellers and the initial equilibrium holds.

In equilibria that are robust to D1, the effectiveness of the price system at reflecting information along the equilibrium path also depends on the strength of competition among sellers. As we have shown, when competition is weak, both qualities are traded and each quality is traded at a different price. That is, prices are fully informative and enable high quality sellers to trade. However, when competition is strong, only low quality sellers are able to trade. The price system associated with this equilibrium is still fully informative, but the only credible way for high quality sellers to reveal their type is to decide not to trade (i.e. to announce a price at which trade does not take place). In other words, when competition is strong, there is no equilibrium price system that is both informative and would allow the high quality goods to be traded.

These observations lead us to the the following conclusion. In lemon markets, whenever competition among sellers is strong, the traditional role of prices as a device for competing against rival sellers impairs the effectiveness of prices as a device for conveying information to the buyers.

4.1   Policy implications: Price controls

We have shown that, when $1 - \lambda > \theta$, strong competition among sellers inhibits the signaling role of prices and drives high quality sellers out of the market. The resulting equilibrium outcome is inefficient whenever gains from trading quality h exceed those from trading quality l, i.e. $GFT_h > GFT_l$. We now show that the introduction of a

price-floor – which forces sellers to set prices above a minimum price – could help to restore trade of the high quality in these circumstances. Intuitively, the introduction of a price floor leaves positive profits to low quality sellers, thus making separation possible by reducing the incentive to mimic.

Before moving to the analysis, a note of caution is in order. The results we are going to present depend in different measures on the equilibrium selection criterion we used in the previous sections. One should be cautious in invoking equilibrium selection arguments when discussing policy implications. Moreover, if compensating transfers are unfeasible, price controls have clear redistributive consequences. For these reasons, one should not interpret what follows as a generic endorsement of price controls, but rather as an *example* where price controls are not necessarily a misguided policy.

Let $\underline{p}$ denote the price-floor. It is immediate to verify that Lemma 2 still applies, so that pooling/hybrid equilibria in which trade occurs at $p \geq \underline{p}$ are not robust to D1.[17] Therefore, we restrict attention to SE.

Assume that the price-floor $\underline{p}$ is set to satisfy $c(l) < \underline{p} < u(l)$. In a SE, low quality sellers are able to trade only if $p_l \in [\underline{p}, u(l)]$. Given $1 - \lambda > \theta$, there is always excess supply at p_l. Hence, price competition will drive the price of the low quality down to the price floor: $p_l = \underline{p}$. In contrast with the previous analysis, however, low quality sellers now make positive expected profits at $p_l = \underline{p}$. According to D1, if the high quality is not traded, any deviation to a price above $c(h)$ should be considered as emanating from a type h seller. It is then clear that, so long as $GFT_h > GFT_l$, the high quality must always be traded with positive probability in equilibrium. Otherwise, sellers could deviate to prices slightly above $c(h)$ and be able to attract buyers. Moreover, $GFT_h > GFT_l$ implies that p_h must exceed $c(h)$ in any separating equilibrium. Otherwise, buyers would not buy at p_l. This implies – see Lemma 3 – that the ICC of type l must be satisfied with equality in any D1-robust equilibrium

$$J(\underline{p}) \left[\underline{p} - c(l) \right] = J(p_h) \left[p_h - c(l) \right]. \tag{14}$$

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[17] In these equilibria, type l sellers make higher expected profits than type h sellers. Accordingly, sellers of high quality are more likely to benefit from deviating to a higher price, which undermines the equilibrium.

Since both qualities are necessarily traded with probability less than one, buyers must be indifferent between buying the low quality at $\underline{p}$ and buying the high quality at p_h, i.e.

$$u(h) - p_h = u(l) - \underline{p}. \tag{15}$$

Finally, the requirement that the number of goods bought equals the number of good sold (i.e. equation 1) implies

$$\lambda J(p_h) + (1 - \lambda)J(p_l) = \theta, \tag{16}$$

where $K = 1$ follows from $1 - \lambda > \theta$. Solving for p_h, $J(p_h)$, and $J(\underline{p})$ the system formed by equations (14), (15), and (16), yields the following equilibrium prices and the associated probabilities to trade

$$p_l = \underline{p}; \ p_h = u(h) - u(l) + \underline{p}, \tag{17}$$

$$J(\underline{p}) = \theta \frac{\underline{p} - c(l) + u(h) - u(l)}{\underline{p} - c(l) + (1 - \lambda)\left[u(h) - u(l)\right]}, \tag{18}$$

$$J(p_h) = \theta \frac{\underline{p} - c(l)}{\underline{p} - c(l) + (1 - \lambda)\left[u(h) - u(l)\right]}. \tag{19}$$

From (18) and (19), an increase in the price floor $\underline{p}$ within the interval $[c(l), u(l)]$ increases the probability to trade for type h and reduces the probability to trade for type l. When $GFT_h > GFT_l$, an increase in the price floor thus increases overall efficiency. However, notice that (15) implies that any increase in the price floor always reduces the surplus available to the buyers. As a result, any efficiency gain associated with a price floor comes at the cost of a lower buyers' surplus.

The introduction of price controls in the form of a price-floor can thus help to restore trade of the high quality under strong competition. That said, the price-floor only works if it is neither too low (i.e. lower than $c(l)$) nor too high (i.e. higher than $u(l)$). In the first case, it would simply be ineffective and undercutting by low quality sellers would drive the high quality out of the market. In the second case, complete market breakdown would be the only D1-robust equilibrium outcome. The argument for the second result is similar to the one developed by Adriani and Deidda (2009).

If $\underline{p} > u(l)$, there is no separating equilibrium with trade. Since all pooling equilibria with trade fail D1, the unique D1-robust outcome must involve no trade.[18]

Finally, we note that the presence of a price-floor does not eliminate price-distortions. As a result, the probability of trading the high quality could be higher if the price control were to take the form of a fixed price. Forcing both qualities to stick to the same price would eliminate any scope for price signaling, thus implementing the outcome of a pooling equilibrium. A pooling price that is compatible with trade exists if[19]

$$\lambda[u(h) - c(h)] + (1 - \lambda)[u(l) - c(h)] \geq 0. \tag{20}$$

If trade occurs at the fixed price, the probability to sell is equal to θ for both types. Hence, type h sell with a probability greater than $J(p_h)$, as given by equation (19). By converse, type l sell with a probability lower than $J(\underline{p})$ (see equation 18). Hence, if the fraction of high quality is sufficiently large or the gains from trading quality h are particularly large – so that (20) is satisfied – efficiency may be increased by forcing both qualities to trade at a fixed price. Otherwise, a price floor, which allows for separation, would be more effective.

Taken altogether, our results suggest that limiting the sellers' ability to choose their prices might be beneficial. This is broadly consistent with recent evidence by Huck et al. (2007). Although the problem that subjects face in their experiment is slightly different from the one considered here, the main effect at work is essentially the same.[20] Limiting the scope for Bertrand competition boosts trade of high quality goods.

An interesting question is why posted prices are so widespread, even though this price convention does not always attain the second best. While we believe that this issue deserves further scrutiny, Bester (1993) suggests a possible answer. A posted price convention tends to endogenously emerge in markets with asymmetric information, as opposed to bargaining for instance, when the search costs faced by the buyers become

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[18]If $\underline{p} > u(l)$ there is always a D1-robust equilibrium with no trade. Intuitively, since both types make zero profits in equilibrium, D1 does not restrict buyers' beliefs in the presence of a deviation to any price above $\underline{p}$. As a result, beliefs that assign a sufficiently low probability to type h when observing an out of equilibrium price do not violate D1 and sustain the no trade equilibrium.

[19]A pooling price at which trade occurs must be at most equal to the expected quality $\lambda u(h) + (1 - \lambda)u(l)$ and greater than or equal to the cost of type h, $c(h)$.

[20]In Huck et al. (2007), sellers choose ex-post the quality to be delivered.

small.

5   Robustness and extensions

In order to better understand the relevance of the model to specific markets, it seems important to assess the robustness of the model's results with respect to its main features, and in particular: i. The information available to buyers and sellers; ii. The assumption of indivisible goods; and iii. The property of the cost function according to which $c(h) > c(l)$.

Regarding the information available to buyers , let us assume that –rather than gathering information only through prices – buyers also observe a noisy signal about the quality. With minor changes to the analysis carried forward in the previous sections, it is possible to show that this would not change the nature of the problem. Suppose that each buyer $b = 1, ..., B$ observes a private noisy signal x_b^s about seller s's quality after observing the seller's price but before choosing whether to buy.[21] Conditional on a given seller s's quality, the signals about seller s $(x_1^s, ..., x_B^s)$ are independently drawn from the same distribution. To simplify the discussion, we assume that x_b^s can only take two values; $x_b^s \in \{L, H\}$. [22] We denote with ρ_q the probability to observe a high signal $(x_b^s = H)$ when the seller is of type $q \in \{h, l\}$. We also assume $1 > \rho_h > \rho_l > 0$, so that a buyer is more likely to observe H when the seller is of type h than when the seller is of type l. Since we are adding a further stage to the game – the stage in which buyers observe their signals – we need to distinguish between buyer's beliefs after observing the price (but not the signal) and beliefs after observing both price and signal. We refer to the first as "price-induced beliefs" and to the second simply as posterior beliefs.[23]

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[21]To keep things simple, we also assume that a buyer's decision to buy or not is not observable by other buyers.

[22]This is however not necessary for the point we want to make. The crucial assumption is that the conditional distribution of x has the same support independently of whether the seller's type is h or l – so that there is no realization of x_b^s that perfectly reveals the seller's quality.

[23]Formally, denoting the price-induced beliefs with $\sigma(q|p)$ as in the previous sections, the posterior beliefs are now

$$\Pr(q|p, x = H) = \frac{\sigma(q|p)\rho_q}{\sigma(q|p)\rho_q + \sigma(q'|p)\rho_{q'}}, \ \Pr(q|p, x = L) = \frac{\sigma(q|p)(1 - \rho_q)}{\sigma(q|p)(1 - \rho_q) + \sigma(q'|p)(1 - \rho_{q'})}, \quad (21)$$

for $q, q' \in \{l, h\}$, $q \neq q'$.

The crucial difference with the previous analysis is that, if the signal is informative, the probability to sell at a given price could now depend on the seller's type. Intuitively, given $\rho_h > \rho_l$, each buyer is more likely to observe H when the seller is of type h than when he is of type l. As a result, when deviating to an out of equilibrium price p, the probability to sell at p for a type h cannot be lower (and is possibly higher) than the same probability for a type l, independently of the beliefs induced by the deviation p.[24] In other words, denoting with $J(p,q)$ the probability to sell at price p for a type q, in the previous sections we had $J(p,h) = J(p,l)$ for all p. We can now only impose the weaker restriction $J(p,h) \geq J(p,l)$ for all p. The problem analyzed in the previous sections is thus a special case of the problem we analyze here.

Nevertheless, it is still easy to establish that, when $1 - \lambda > \theta$, there exists a D1-robust equilibrium outcome where only the low quality is traded and both types make zero profits. This outcome is sustained by out of equilibrium beliefs that assign a sufficiently high probability to type l whenever observing a deviation to a price greater than $c(h)$. These out of equilibrium beliefs are clearly robust to D1 since both types make zero profits in equilibrium, so that both types would always deviate whenever the probability to sell is positive.

By the same token, we argue that the high quality is traded with positive probability in any D1-robust equilibrium if $1 - \lambda < \theta$. Suppose that the high quality is not traded. In this case, the equilibrium necessarily involves type l making strictly positive profits, thus leaving no surplus to the buyers. A type h would strictly benefit from deviating to $p > c(h)$ whenever the probability to sell at p is positive. By converse, a type l would deviate to p only if the probability to sell were sufficiently high. Given $J(p,h) \geq J(p,l)$, a type h would strictly benefit whenever a type l weakly benefits from the deviation. Hence, all equilibria where the high quality is driven out of the market fail D1.

To summarize, when competition is strong, there is still a D1-robust equilibrium in which only the low quality is traded. When competition is weak, the high quality

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[24]More precisely, for any price-induced beliefs, the expected quality for a buyer observing H is (weakly) higher than the expected quality for a buyer observing L. Hence, a buyer who finds it optimal to buy at a price p when observing L must necessarily find it optimal to buy when observing H. Since the probability that a buyer observes H is larger when the seller is of type h, the probability to sell at any price for a type h seller cannot be larger than the same probability for a type l.

is necessarily traded in any D1-robust equilibrium. This illustrates that the effect of competition we described in the previous sections is unaffected by the introduction of noisy signals. However, if we want to conclude that competition *necessarily* drives the high quality out of the market, we need to rule out equilibria where the high quality is traded when competition in strong. This basically reduces to ruling out pooling and hybrid equilibria where both qualities are traded. Unfortunately, this would require adding more structure to the model, i.e. explicitly modelling how buyers and sellers are matched. Rather than following this route and assume an explicit market structure, we turn the problem on its head and show that there are intuitive restrictions on the probability to sell that are sufficient to rule out pooling equilibria under D1. These boil down to a weak monotonicity condition for the ratio $J(p, h)/J(p, l)$. The following Lemma extends Lemma 2.

Lemma 4. *If $J(p, h)/J(p, l)$ is non-decreasing in p, then no pooling/hybrid equilibrium of type I survives D1.*

Proof. See Appendix.

Intuitively, the requirement that $J(p, h)/J(p, l)$ is non-decreasing is equivalent to saying that buyers' information matters (weakly) more when the price is high than when the price is low.

To summarize, under a monotonicity condition for the ratio $J(p, h)/J(p, l)$, strong competition always drives the high quality out of the market. The previous analysis can be extended without further complications to the case in which the signal is public, so that, for any seller s, all buyers observe the same (noisy) realization of x^s. This may be relevant for e-commerce applications, since many web sites list measures of the seller's quality (e.g. ratings) in addition to the sellers' prices.

Let us now turn to the issue of the information available to sellers. Hertzendorf and Overgaard (1999, 2001, 2002) consider duopoly models where sellers know not only their own type, but also the type of their competitor. By contrast, we have considered the case of sellers who have no private information about their competitors. This assumption allows us to impose that the behavior of a seller should not affect buyers' beliefs about other sellers (R4). While such a restriction may result crucial in environments with a limited number of competitors, it is natural in a world with

many *a priori* identical competitors whose identity is irrelevant and, more importantly, with no aggregate uncertainty. Independently of the behavior of sellers, it is common knowledge that there are a fraction λ $(1-\lambda)$ of type h (l) sellers. A natural extension of the model would allow for aggregate shocks observed only by sellers, so that the behavior of one seller may convey information about other sellers.

Finally, we focus on the assumption of indivisibility of goods and the property of the cost function such that $c(h) > c(l)$. Suppose then that sellers could choose a pair (p, x) where $p \in [0, \overline{p}]$ denotes the unit price and $x \in [0, 1]$ the quantity on offer. Assume for simplicity that the marginal cost $c(q)$ is constant in x. Given two price-quantity combinations (p, x) and (p', x') with $p' > p$ and $x' < x$, Cho's and Sobel's (1990) sorting condition (equation 13) becomes

$$J(p')x'[p' - c(l)] \geq J(p)x[p - c(l)] \Rightarrow J(p')x'[p' - c(h)] > J(p)x[p - c(h)], \qquad (22)$$

i.e. whenever a type l weakly prefers the high-price-low-quantity combination to the low-price-high-quantity one, a type h strictly prefers the first. Given $c(h) > c(l)$, (22) is satisfied for all positive x and x', whenever the probabilities to sell are positive. The result of Cho and Sobel (1990) thus suggests that any D1-robust equilibrium exhibits separation at prices where trade occurs. But this implies that, whenever competitive pressures drive the profits of type l to zero, the high quality cannot be traded in equilibrium. Otherwise, the low quality would profit from mimicking.

This clarifies that the crucial assumption of our model is $c(h) > c(l)$. If this inequality does not hold, the sorting condition is never satisfied. Without sorting, D1 does not yield sharp predictions about the relationship between the extent of competition among sellers and the equilibrium outcome in terms of quantities and qualities traded and related prices.

6   Conclusions

This paper tackled the issue of strategic pricing in a competitive market for lemons where the potential gains from trade are always positive. Sellers' pricing decisions are affected by two types of considerations. On the one hand, sellers want to maximize

the chance to find a buyer. On the other hand, they may want to use prices to conceal/reveal their true quality. Thus, in markets for lemons, pricing decisions retain a double function. Sellers may lower prices to undercut competitors or increase them to signal high quality. We argue that these two roles of prices may be at odds with each other. When competition among sellers is strong, the use of prices to compete with other sellers prevails. Announcing a price that is higher than the price at which trade occurs in the market does not help to be recognized as a high quality seller by the buyers. The reason is that profits of all sellers are driven to zero by competition. Hence, all sellers, irrespectively of their quality, have no opportunity cost of deviating and announcing a higher price.

By contrast, when competition is weak, announcing a price higher than the lowest price at which trade occurs conveys relevant information to the buyers. The rationale is that in this case at least some of the sellers make positive profits. Thus, announcing a higher price (which harms the seller by reducing the likelihood of making a sale) is relatively more costly for low quality sellers.

The model generates various predictions, some of which are empirically relevant. First, the degree of price dispersion is inversely related to the degree of competition among sellers, as measured by the sellers to buyers ratio. We should observe concentration of trade around few low prices when competition is strong, whereas trade should spread upon a distribution of relatively dispersed prices when competition is weak. Second, the average quality traded in the market should increase (although in a nonlinear fashion) as competition decreases. This is because weak competition allows high quality sellers (who would be driven out of the market in the presence of fierce competition) to sell their goods.

Finally, the paper suggests that, in some circumstances, imposing some degree of price control might increase efficiency by restoring a positive amount of trade of the high quality. However, any efficiency gain would come at the expenses of lower buyers' surplus.

A Appendix

A.1 Proof of Lemma 1

Definitions and Preliminaries

Let p denote an out of equilibrium price and $\pi^*(q)$ the expected payoff of a type $q \in \{l, h\}$ seller in the candidate equilibrium. With a slight abuse of notation, let also $\mathcal{J}(p, \sigma(q|p))$ denote the set of functions $J : [0, \bar{p}] \to [0, 1]$ that do not violate R5 given $\sigma(q|p)$ and[25]

$$\hat{\mathcal{J}}(p) \equiv \bigcup_{\{\sigma(q|p):\sum_{q\in\{h,l\}} \sigma(q|p)=1\}} \mathcal{J}(p, \sigma(q|p)). \tag{A.1}$$

Finally, let

$$D(q|p) \equiv \{J \in \hat{\mathcal{J}}(p)|\ J(p)[p - c(q)] > \pi^*(q)\}, \tag{A.2}$$

and

$$D^0(q|p) \equiv \{J \in \hat{\mathcal{J}}(p)|\ J(p)[p - c(q)] = \pi^*(q)\}. \tag{A.3}$$

Then, D1 requires that hat $\sigma(q|p) = 0$ (and $\sigma(q'|p) = 1$) whenever $D(q|p) \bigcup D^0(q|p) \subset D(q'|p)$. We say that R5 *has no bite* for a deviation p if, for any $x \in [0, 1]$, there is a function $J \in \hat{\mathcal{J}}(p)$ with $J(p) = x$. The proof of Lemma 1 is structured as follows. For all deviations $p \leq c(h) + \pi^*(h)$ and p such that (4) does not hold, we show that any equilibrium either satisfies the D1 restriction or has an outcome-equivalent equilibrium that satisfies it. For deviations $p > c(h) + \pi^*(h)$ such that (4) holds, equilibria with $\sigma(h|p) = 1$ satisfy the D1 restriction; equilibria with $\sigma(h|p) < 1$ either fail D1 (when R5 has no bite) or (when R5 bites) have an outcome equivalent equilibrium with $\sigma(h|p) = 1$ that satisfies D1.

Proof

We start by establishing three facts. First, if a seller has no incentive to deviate to p when $\sigma(h|p) > 0$, then he has no incentive to deviate when $\sigma(h|p) = 0$. Hence, any equilibrium with beliefs such that $\sigma(h|p) > 0$ at an out of equilibrium price p has an outcome-equivalent equilibrium sustained by beliefs such that $\sigma(h|p) = 0$. Second, since $D(q|p) = \varnothing$ for all $p \leq c(q) + \pi^*(q)$, beliefs such that $\sigma(h|p) = 0$ for all $p \leq c(h) + \pi^*(h)$ never violate D1. Third, $D(h|p) \subseteq D(l|p)$ for all p such that (4) does not hold, implying that $\sigma(h|p) = 0$ is always consistent with D1. Combining the last two facts with the first shows that, for deviations $p \leq c(h) + \pi^*(h)$ and for deviations $p > c(h) + \pi^*(h)$ such that (4) does not hold, any equilibrium either satisfies D1 or has an outcome-equivalent equilibrium that satisfies D1.

We can accordingly restrict attention to $p > c(h) + \pi^*(h)$ such that (4) holds. Consider first deviations p such that R5 has no bite. Notice that any $J \in \hat{\mathcal{J}}(p)$ such

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[25]Notice that, since buyers face many competing sellers, $\mathcal{J}$ depends through R5(ii) on the prices announced by other sellers – and on the beliefs at those prices. However, since there are a large number of sellers announcing a finite set of prices in any equilibrium, the fraction of sellers announcing each of these prices cannot be affected by the deviation of a single seller. Buyers' beliefs at these prices are also unaffected by deviations. Hence, knowledge of the candidate equilibrium is sufficient to perfectly predict the set of prices that will appear in **p**, the supply at each price, and associated beliefs (although a seller chooses whether to deviate before observing **p**). Since $\mathcal{J}$ depends on the prices announced by others and the associated beliefs only through the candidate equilibrium – which is fixed when checking robustness – we omit these variables.

that $J(p) > \pi^*(q)/(p - c(q))$ belongs to $D(q|p)$ (or to $D^0(q|p)$ if the strict equality holds). Consider x such that $\pi^*(l)/(p-c(l)) > x > \pi^*(h)/(p-c(h))$. Since there exists $J \in \hat{\mathcal{J}}(p)$ with $J(p) = x$, then $D(l|p) \bigcup D^0(l|p) \subset D(h|p)$. As a result, an equilibrium satisfies D1 if and only if it specifies $\sigma(h|p) = 1$.

Consider now what happens when R5 bites. It is straightforward (but tedious) to verify that, for deviations $p > u(l)$, it is always possible to find beliefs $\sigma(h|p)$ such that, for any $x \in [0,1]$, $J(p) = x$ does not violate R5 (i-iii), except in the following two cases

1. $p > u(h)$ so that R5(i) implies $J(p) = 0$ for all values of $\sigma(h|p)$.

2. a price p' such that $\sigma(h|p')u(h) + [1 - \sigma(h|p')]u(l) - p' > u(h) - p$ is announced in the candidate equilibrium, so that, if $J(p') < 1$, R5(ii) requires $J(p) = 0$ for all values of $\sigma(h|p)$.

Hence, if R5 bites, $J(p) = 0$ *for all values of* $\sigma(h|p)$. But this implies that any equilibrium with $\sigma(h|p) < 1$ has an outcome-equivalent equilibrium with $\sigma(h|p) = 1$. Moreover, when $\hat{\mathcal{J}}(p)$ contains only J with $J(p) = 0$, $D(q|p) = \varnothing$, so that D1 imposes no restriction. Hence, for all out of equilibrium prices p such that R5 bites, any equilibrium with $\sigma(h|p) < 1$ has an outcome-equivalent equilibrium with $\sigma(h|p) = 1$ that satisfies D1.

$\square$

We conclude this section with a technical remark. The original definition of D1 restricts attention to actions by the receiver that are best replies for some beliefs (see Cho and Kreps, 1987). Since our model does not specify the action of the buyers, we have stated the D1 criterion in terms of probability to sell. This is analogous to most of the literature on price signaling which typically specifies a demand function rather than modelling the buyer's problem (see for instance the formulation of the Intuitive Criterion in Bagwell, 1991, or Milgrom and Roberts, 1986). In our definition we have replaced the requirement that there exist beliefs such that the receivers' responses are best replies with the requirement that there exist beliefs such that J satisfies R5 – which describes buyers' strategic behavior in our model. As the proof suggests, however, nothing would change if we were to concentrate on a definition of D1 that allows for any $J(p) \in [0,1]$.

Proof of Lemma 2

We first consider pooling equilibria (PE), then Hybrid Equilibria (HE). In any PE of type I there is a single equilibrium price p^* at which both high and low qualities are traded. Hence, all sellers have the same probability to sell, $J(p^*)$. Equilibrium profits are $\pi^*(q) = J(p^*)[p^* - c(q)]$, with $q \in \{l, h\}$. The payoff of a buyer obtaining the good at p^* is $\lambda[u(h) - p^*] + (1 - \lambda)[u(l) - p^*]$. Since buyers' payoff must be non-negative a necessary condition for a PE is $p^* \leq \lambda u(h) + (1 - \lambda)u(l)$.

Consider then robustness. Condition (4) can be rewritten as

$$\frac{p^* - c(l)}{p - c(l)} > \frac{p^* - c(h)}{p - c(h)} \Rightarrow c(h)(p - p^*) > c(l)(p - p^*). \tag{A.4}$$

Condition (A.4) holds for all $p > p^*$ so that, for deviations above the pooling equilibrium price, $\sigma(h|p)$ must be one. From R5 (ii), if

$$u(h) - p > \lambda[u(h) - p^*] + (1 - \lambda)[u(l) - p^*], \tag{A.5}$$

then $J(p) = 1$. The above inequality is satisfied for all p lower than $p^* + \eta$, where

$$\eta \equiv (1 - \lambda)[u(h) - u(l)] > 0. \tag{A.6}$$

Hence, a seller deviating to $p \in (p^*, p^* + \eta)$ would be able to sell with probability one at a higher price. The candidate equilibrium thus fails D1.

The argument for hybrid equilibria (HE) is essentially the same. In any HE, there is always a type $q \in \{l, h\}$ who announces at least two different prices with positive probability and there is always a price p^* announced with positive probability by both types. In a HE of type I, $J(p^*) > 0$, since quality h can be traded only if type l sellers make strictly positive profits. Since $\pi^*(q)$ is the same at all prices announced by type q, in order to asses the robustness, one can just focus on the incentives to deviate from p^*. Hence, robustness again requires $\sigma(h|p) = 1$ whenever (A.4) holds. From R5 (ii), if

$$u(h) - p > \sigma(h|p^*)[u(h) - p^*] + (1 - \sigma(h|p^*))[u(l) - p^*], \tag{A.7}$$

then $J(p) = 1$. Since pooling occurs at p^*, $\sigma(h|p^*)$ must be strictly lower than one. But then, any deviation $p \in (p^*, p^* + \tilde{\eta})$, with

$$\tilde{\eta} \equiv (1 - \sigma(h|p^*))[u(h) - u(l)] > 0, \tag{A.8}$$

would cause the equilibrium to unravel.

$\square$

Proof of Lemma 3

Assume $p_h > c(h)$ and consider a deviation to a price $p < p_h$, but such that $p - c(h) > J(p_h)[p_h - c(h)]$. In a more compact form, $p \in (p_h - \Delta, p_h)$, where $\Delta = (1 - J(p_h))[p_h - c(h)] > 0$ (notice that Δ is positive since in any SE of type I, $0 < J(p_h) < J(p_l) \leq 1$). We want to show that

1. If condition (9) holds at p, then the equilibrium fails D1.

2. For condition (9) not to hold at p, type l ICC must hold with equality whenever $p_h > c(h)$.

If condition (9) holds at p, then $\sigma(h|p) = 1$. Given $p < p_h$, condition R5 (ii) then ensures that $J(p) = 1$. Hence, given $p > p_h - \Delta$, it is optimal for type h sellers to deviate to p. For the equilibrium to be robust to D1, it is thus necessary that condition (9) does not hold for all $p \in (p_h - \Delta, p_h)$, so that $\sigma(h|p) < 1$ does not violate D1. We now prove point 2. For $\xi \geq 0$, rewrite the ICC of type l as

$$J(p_l)[p_l - c(l)] = J(p_h)[p_h - c(l)] + \xi, \tag{A.9}$$

so that the ICC is satisfied as strict equality when $\xi = 0$. Use (A.9) to substitute for $[p_l - c(l)] J(p_l)$ in (9) to obtain

$$(p_h - p)J(p_h)[c(h) - c(l)] + \xi(c(h) - p) \geq 0. \tag{A.10}$$

By inspecting (A.10) it is immediate to verify that, given $p_h > c(h)$, whenever $\xi > 0$ there exist values of $p \in (p_h - \Delta, p_h)$ so that (9) holds. Hence, $p_h > c(h)$ necessarily implies that the ICC of low quality sellers holds with equality.

$\square$

Proof of Proposition 1

Preliminary results

From Lemma 2, any robust equilibrium of type I is separating. A D1-robust separating equilibrium outcome must satisfy: 1) Participation constraints and incentive compatibility constraints for both types, 2) Condition R5, 3) $\sigma(h|p) = 1$ for all $p > c(h)+\pi^*(h)$ such that condition (9) holds (robustness to D1).

The next Lemma establishes that $1 - \lambda < \theta$ is necessary for a type I equilibrium and provides some intermediate necessary conditions which are useful for the characterization of the robust equilibrium outcome when $1 - \lambda < \theta$.

Lemma 5. *If $1 - \lambda \geq \theta$ there is no SE of type I. If $1 - \lambda < \theta$, then any robust SE satisfies the following*

I)

$$J(p_l) = 1; \tag{A.11}$$

II)

$$K < 1 \Rightarrow p_l = u(l) \Leftrightarrow p_h = u(h); \tag{A.12}$$

III)

$$J(p_h) = \frac{\theta K - (1 - \lambda)}{\lambda} > 0 \tag{A.13}$$

(i.e. the equilibrium is of type I.);

IV)

$$p_l = p_h - [u(h) - u(l)]. \tag{A.14}$$

Proof. Assume $1 - \lambda \geq \theta$ and consider a SE of type I, i.e. both $J(p_h)$ and $J(p_l)$ are greater than zero. If $1 - \lambda \geq \theta$ there is at least a low quality seller for every buyer. Hence, $J(p_h) > 0$ implies $J(p_l) < 1$ (see equation 1). Notice that, from condition R5 (ii), $p_l > c(l) \Rightarrow J(p_l) = 1$. Otherwise a low quality seller would profit from announcing a price slightly below p_l and sell with probability one. Hence, $p_l = c(l)$. The ICC of type l sellers then implies $J(p_h) = 0$, i.e. the high quality is never traded in equilibrium, so there is no SE of type I. This proves the first statement of the Lemma.

From now on, restrict attention to the case $1 - \lambda < \theta$. We start from point (I): $J(p_l) = 1$. As argued above, $p_l > c(l) \Rightarrow J(p_l) = 1$. If $p_l = c(l)$, then type l ICC implies $J(p_h) = 0$. But then, (1) reduces to $J(p_l) = K\theta/(1 - \lambda)$. Given $1 - \lambda < \theta$ and $J(p_l) \leq 1$, $K < 1$ follows. However, given $K < 1$, $J(p_l) = 1$ from R5 (iii). (*This proves I.*)

Consider now point (II). If $K < 1$ and $p_l < u(l)$, then, from R5 (iii), a type l seller could profit from slightly raising his price. Given $p_l = u(l)$, R5 (ii) implies that $p_h = u(h)$ (otherwise $J(p_h)$ should be equal to one so that the ICC of type l

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would be violated). (*This proves $K < 1 \Rightarrow p_l = u(l) \Rightarrow p_h = u(h)$, see below for* $p_h = u(h) \Rightarrow p_l = u(l)$.)

We now move to point (III). Given $J(p_l) = 1$, equation (1) implies

$$J(p_h) = \frac{\theta K - (1 - \lambda)}{\lambda}. \tag{A.15}$$

We still have not proved that $J(p_h) > 0$. Suppose by contradiction that $J(p_h) = 0$. Then, since buyers outnumber low quality sellers, $K < 1$ follows from (A.15) given $1 - \lambda < \theta$. Moreover, if $J(p_h) = 0$, then (9) holds for all $p \in (c(h), u(h))$ unless $p_l = c(l)$. Hence, robustness implies that either $p_l = c(l)$ or $\sigma(h|p) = 1$ for all $p \in (c(h), u(h))$. Both cases are inconsistent with $K < 1$, given R5(iii). In the first case, type l can profitably deviate to prices above $c(l)$. In the second case, $J(p) = 1$ for all $p \in (c(h), u(h))$, which implies that type h can profitably deviate to p. Hence, $J(p_h) > 0$. (*This proves III.*)

Finally, consider point (IV). If $u(l) - p_l < u(h) - p_h$, then $J(p_h)$ would be one – see R5 (ii) – and the ICC of type l would be violated. If $u(l) - p_l > u(h) - p_h$, then again R5 (ii) implies that type l sellers could raise their price and still be able to sell with probability one, since $J(p_h) > 0$. Hence, $u(l) - p_l = u(h) - p_h$ so that (A.14) must hold. (*This proves IV and also shows that $p_h = u(h) \Rightarrow p_l = u(l)$, which proves II.*)

$\square$

To sum up, we have shown that $1 - \lambda < \theta$ is necessary for a type I equilibrium and that the attribute of being of type I is necessary for a robust equilibrium when $1 - \lambda < \theta$ (see point III). This implies that we can impose that the ICC of type l holds with equality whenever $p_h > c(h)$ (Lemma 3). We now (i) show that when $1 - \lambda < \theta$, there exists a unique outcome that satisfies the necessary conditions of Lemmata 3 and 5; and (ii) provide a full characterization of the outcome in question.

Characterization and uniqueness

Lemma 5 establishes that $J(p_l) = 1$. Replacing p_l from (A.14) of Lemma 5 into the ICC of type l, Lemma 3 can be restated as

$$p_h = c(h) \Leftrightarrow J(p_h) \leq \gamma, \tag{A.16}$$

$$p_h > c(h) \Leftrightarrow J(p_h) = \frac{u(l) - c(l) - (u(h) - p_h)}{p_h - c(l)} > \gamma. \tag{A.17}$$

$J(p_h)$, K, p_l and p_h must thus necessarily satisfy (A.16-A.17) and (A.12-A.14) of Lemma 5.

We now show that there are unique values of $J(p_h)$, K, p_h, and p_l that satisfy the (necessary) conditions of Lemma 5 and are compatible with Lemma 3. We divide the argument into three cases according to the value of θ.

Case 1: $\theta \geq \hat{\theta}$.
It follows from (A.13) that $\theta \geq \hat{\theta}$ implies either $K < 1$ or $J(p_h) \geq \delta$, or both. From (A.16-A.17), the maximum value that $J(p_h)$ can take is δ when $p_h = u(h)$. Hence, either $K < 1$ or $J(p_h) = \delta$ (and $p_h = u(h)$), or both. In both cases, $p_l = u(l)$ and $p_h = u(h)$ from (A.12). From (A.17), it then follows that $J(p_h) = \delta$. Finally, $K = \hat{\theta}/\theta$ follows from (A.13).

Case 2: $\theta_\gamma < \theta \le \hat{\theta}$.

From (A.13), $\theta < \hat{\theta} \Rightarrow J(p_h) < \delta$. In turn, $J(p_h) < \delta \Rightarrow K = 1$, since $K < 1$ would imply $p_l = u(l)$ and $p_h = u(h)$, which, given (A.17), would imply $J(p_h) = \delta$, which is not possible.

Also from (A.13), $\theta > \theta_\gamma \Rightarrow J(p_h) > \gamma$. Using (A.17) and (A.14) one obtains

$$p_h = \frac{u(h) - [u(l) - c(l) + c(l)J(p_h)]}{1 - J(p_h)}, \tag{A.18}$$

$$p_l = \frac{c(l) + J(p_h)[u(h) - c(l) - u(l)]}{1 - J(p_h)}. \tag{A.19}$$

Values for p_h and p_l can then be found by replacing $J(p_h)$ with

$$J(p_h) = \frac{\theta - (1 - \lambda)}{\lambda}. \tag{A.20}$$

Case 3: $\theta \le \theta_\gamma$.

As above, given $\theta_\gamma < \hat{\theta}$, $\theta < \hat{\theta} \Rightarrow J(p_h) < \delta \Rightarrow K = 1$. However, now $\theta \le \theta_\gamma \Rightarrow J(p_h) \le \min\{\gamma, 0\} \Rightarrow J(p_h) \le \gamma \Rightarrow p_h = c(h)$ (the last statement comes from A.16). Then, $p_h = c(h)$ implies $p_l = u(l) - [u(h) - u(l)]$. Finally, $J(p_h)$ is given by (A.20).

Existence

For each of the three possible cases, the necessary conditions identify unique values for $J(p_l)$, $J(p_h)$, p_h, p_l, and K. All that is left to prove is that there exists a robust equilibrium that induces this outcome. It is immediate to check that, by construction, incentive compatibility constraints, participation constraints, and R5 are satisfied *along the equilibrium path.* All we need to show is that, for all out of equilibrium prices $p \notin \{p_l, p_h\}$, there exists $J(p)$ and out of equilibrium beliefs σ such that

1. sellers are not willing to deviate to p

2. $J(p)$ is consistent with R5 given σ

3. $\sigma(h|p) = 1$ for all $p > c(h) + \pi^*(h)$ such that (9) holds (D1)

Consider the following candidates

$$J(p) = \begin{cases} 1 & p < p_l \\ 0 & p > p_l \end{cases}, \sigma(h|p) = \begin{cases} 1 & p > p_h \\ 0 & p < p_h \end{cases}, \tag{A.21}$$

for $p \notin \{p_l, p_h\}$. Given $J(p_l) = 1$ in the candidate outcome, no seller has any incentive to deviate to $p < p_l$. Since $J(p) = 0$ for $p > p_l$, no seller has incentive to deviate to $p > p_l$. As for the second requirement, it is immediate to check that (A.21) satisfies R5(i) since $p_l \le u(l)$ in the candidate outcome. Hence, $J(p) = 0$ at all p such that buyers would not participate. Condition R5(iii) is also satisfied since the candidate outcome prescribes $p_l = u(l)$ and $p_h = u(h)$ whenever $K < 1$. Hence, when $K < 1$, (A.21) implies $J(p) = 1$ at all p that leave positive expected surplus to the buyers. Moreover, $\sigma(h|p) = 0$ for all $p < p_h$ ensures that $\sigma(h|p)u(h) + \sigma(l|p)u(l) - p \le u(h) - p_h = u(l) - p_l$ for all $p > p_l$. Hence, given σ, $J(p)$ satisfies R5 (ii). We must thus check that these

beliefs are robust to D1. When $p_h > c(h)$ then the the ICC of type l holds with equality in the candidate outcome, so that condition (9) reduces to

$$\frac{p_h - c(l)}{p - c(l)} > \frac{p_h - c(h)}{p - c(h)}, \tag{A.22}$$

which is satisfied for all $p > p_h$ and is not satisfied for all p such that $c(h) < p < p_h$. When $p_h = c(h)$, then again condition (9) holds for all $p > p_h$. Hence, (A.21) prescribes $\sigma(h|p) = 1$ whenever the robustness condition requires so. This shows that σ is consistent with D1.

Proof of Proposition 2

We start by showing that all trade occurs at a unique price p^* in all type II equilibria. Then, we characterize the equilibrium outcomes of all type II equilibria in terms of the price at which trade occurs and analyze beliefs that support the existence of these equilibria. Next, we show that equilibria of type II always fail D1 if $1 - \lambda < \theta$ and that there is an equilibrium outcome that passes D1 if $1 - \lambda > \theta$. The special case $1 - \lambda = \theta$ is then considered. Finally, we characterize the amount of trade.

  a. *Uniqueness of price.* Suppose that trade occurs at more than one price. We prove the result for the case of two different equilibrium prices p' and p'' with $p'' > p'$. The same argument applies to any number of prices higher than one. Clearly, p', p'' could be a pair of equilibrium prices if and only if profits for type l sellers were the same at the two prices, which would imply that $J(p'') < J(p')$. It follows that $J(p'')$ must be lower than 1. But then, any seller announcing p'' would profit from deviating to $p'' - \epsilon$, where ϵ is greater than zero but sufficiently small. From R5(ii), he could sell with probability one at $p'' - \epsilon$ since out of equilibrium beliefs cannot assign him a quality lower than l. Therefore, the only possible equilibrium outcome implies a single price p^*.

  b. *Equilibrium price.* Notice first that in every type II equilibrium p^* necessarily lies in the interval $[c(l), u(l)]$. From (1), $J(p^*) = \theta K/(1-\lambda)$. If $1-\lambda > \theta$, sellers of type l are the long side of the market, and $J(p^*) < 1$. In this case, no price p^* greater than $c(l)$ can emerge in equilibrium; if $p^* > c(l)$ sellers would profit from slightly reducing their price and sell with probability one (R5(ii)). This implies $p^* = c(l)$. (Type h sellers then announce price $\bar{p}$ and do not trade.) Consider now the case $1 - \lambda < \theta$. If $p^* < u(l)$, buyers make positive surplus and, therefore, are all willing to buy at p^*. Given $1 - \lambda < \theta$, K must be less than one. But then (R5(iii)) implies that type l sellers could announce a price $p \in (p^*, u(l))$ and be able to sell with probability one. Hence, $p^* = u(l)$. At this price, type l sellers must have no incentive to cut their prices. Hence, $J(p^*) = 1$ must hold. (Again, type h sellers announce price $\bar{p}$ and do not trade.)

  Finally, in both cases, there must be out of equilibrium beliefs such that no one has any incentive to deviate. These beliefs trivially exist. For instance, beliefs assigning $\sigma(h|p) = 0$ for all out of equilibrium prices induce $J(p) = 0$ for all $p > p^*$ so that no seller wants to deviate. The next step shows that beliefs which sustain equilibria of type II are robust to D1 if and only if $1 - \lambda \geq \theta$.

  c. *Robustness.* It is immediate to show that there are type II equilibria which satisfy D1 when $1 - \lambda > \theta$. Both the RHS and the LHS of (4) are zero for all p, so that (4) never holds. By contrast, type II equilibria fail D1 when $1 - \lambda < \theta$. In equilibrium,

low quality sellers make profits $\pi^*(l) = u(l) - c(l)$ while high quality sellers make zero profits. Condition (4) becomes

$$\frac{u(l) - c(l)}{p - c(l)} > 0. \tag{A.23}$$

This implies that $\sigma(h|p)$ must be one for all $p > c(h)$. Since buyers make zero surplus at p^*, R5 (ii) implies $J(p) = 1$ for all $p \in (c(h), u(h))$. Hence, any deviation $p \in (c(h), u(h))$ would kill the candidate equilibrium.

  d. *Case $1 - \lambda = \theta$.* Notice that also in this case the equilibrium is characterized by a unique price $p^* \in [c(l), u(l)]$. For a deviation $p > c(h)$, condition (4) becomes

$$\frac{p^* - c(l)}{p - c(l)} > 0, \tag{A.24}$$

so that, unless $p^* = c(l)$, $\sigma(h|p) = 1$ for all $p > c(h)$. Hence, unless $p^* = c(l)$, R5(ii) implies $J(p) = 1$ for all p such that

$$u(h) - p > u(l) - p^*. \tag{A.25}$$

In this special case, it is necessary to distinguish between $\gamma \le 0$ and $\gamma > 0$, as in Proposition 1. Consider first the case $\gamma \le 0$ (which implies $u(l) - c(l) \le u(h) - c(h)$). We want to show that only equilibria with $p^* = c(l)$ are robust to D1. Given $\gamma \le 0$, for any $p^* > c(l)$ it is possible to find $p > c(h)$ such that (A.25) is satisfied. Since type h would gain from deviating to p, p^* must be equal to $c(l)$. Consider now the case $\gamma > 0$. Again, $p^* > c(l)$ implies $\sigma(h|p) = 1$ for all $p > c(h)$. However, whether there is any $p > c(h)$ such that (A.25) holds now depends on p^*. A deviation $p > c(h)$ such that (A.25) holds is possible only if $p^* > u(l) - [u(h) - c(h)]$. Hence, robustness to D1 requires $p^* \le u(l) - [u(h) - c(h)]$. In principle, any $p^* \in [c(l), u(l) - [u(h) - c(h)]]$ can be an equilibrium price when $\gamma > 0$. The reason why D1 does not permit to select a price in this case is that, when $\gamma > 0$, a discontinuity arises at $\theta = 1 - \lambda$. To see this, consider the limit of a robust type I equilibrium for $\theta \to 1 - \lambda$. When $\gamma > 0$, the right limit of the expression for p_l given in Proposition 1 selects $p^* = u(l) - [u(h) - c(h)]$. On the other hand, the left limit for $\theta \to 1 - \lambda$ of the price of a type II robust equilibrium selects $p^* = c(l)$.

  e. *Fraction of quality l traded.* It is immediate to check that $K = 1$ whenever $1 - \lambda \ge \theta$. Therefore, each buyer is able to obtain a unit of a quality l good. It follows that the fraction of quality l goods traded is $\theta/(1 - \lambda)$.

$\square$

Proof of Proposition 3

Case 1. $1 - \lambda > \theta$. From Proposition 2, there are D1-robust equilibria of type II. All of these equilibria induce the same unique outcome. Moreover, according to Proposition 1 no equilibrium of type I is possible. Hence, the equilibrium outcome is unique.

Case 2. $1 - \lambda < \theta$. From Proposition 2 there is no D1-robust equilibria of type II. On the other hand, according to Proposition 1 there exist D1-robust equilibria of type I. Again, all these equilibria induce the same unique outcome, so that the equilibrium outcome is unique. The expressions for $f(h)$ and $f(l)$ follow directly from Propositions 1 and 2.

For completeness, we discuss the special case $1 - \lambda = \theta$. The proof of Proposition 2 implies that, if $\gamma > 0$, a discontinuity arises at $1 - \lambda = \theta$. While traded quantities are still uniquely determined, the price p^* experiences a jump from $c(l)$ to $u(l) - [u(h) - c(h)]$.

$\square$

Proof of Lemma 4

Suppose that a pooling occurs at price $p^* \geq c(h)$ and consider a deviation to a higher price $p' > p^*$. A type l would weakly benefit from deviating to p' if

$$J(p', l)[p' - c(l)] \geq J(p^*, l)[p^* - c(l)], \tag{A.26}$$

where the LHS of the expression is the expected profit from the deviation and the RHS is the expected equilibrium payoff for type l. A type h seller would strictly benefit if

$$J(p', h)[p' - c(h)] > J(p^*, h)[p^* - c(h)]. \tag{A.27}$$

Assume now that, for all price-induced beliefs, $J(p, h)/J(p, l)$ is non-decreasing in p. Given (A.26), it is immediate to check that

$$\frac{p' - c(h)}{p^* - c(h)} > \frac{p' - c(l)}{p^* - c(l)} \geq \frac{J(p^*, l)}{J(p', l)} \geq \frac{J(p^*, h)}{J(p', h)}, \tag{A.28}$$

where the first inequality comes from $p' > p^*$ and $c(h) > c(l)$, the second inequality comes from (A.26) and the third comes from the monotonicity condition on $J(p, h)/J(p, l)$. The above result thus implies that (A.27) is satisfied whenever (A.26) is satisfied. In words, type h strictly benefits from the deviation whenever type l weakly benefits. According to D1, buyers should then assign probability zero to type l when observing a deviation to any $p' > p^*$. Notice that, so long as the signal x does not allow to perfectly assess the quality of the good, buyers always attach a strictly positive probability to type l when the price is p^*. As a result, a deviation to a price slightly above p^* would produce a discrete jump in the expected quality. From R5 (ii), the probability to sell at p' must be one. This implies that sellers have incentive to deviate to higher prices when there is a pooling or a hybrid equilibrium. Hence, any pooling or hybrid equilibrium would fail D1.

$\square$

Extension to any finite number of qualities

This section generalizes the results concerning the set of equilibria robust to D1 to the case of a finite number $(N+1)$ of qualities. We show that, as in the case of two qualities, D1 guarantees separation at all prices at which trade occurs. The comparative statics for the general case are also consistent with the results obtained in the two qualities case. When θ is so low that buyers are relatively more numerous than sellers of the lowest quality, no quality other than the lowest is traded. Increases in the value of θ allow higher qualities to be traded until, for θ sufficiently large, all qualities are traded.

Qualities are indexed by $q = 0, ..., N$. Each seller's quality is drawn from a distribution $\lambda : \{0, 1, ..., N\} \to [0, 1]$, where λ_q, $\sum_{q=0}^{N} \lambda_q = 1$, denotes the probability associated with quality q. Buyers' posterior beliefs are denoted with $\sigma(q|p_s^0)$, $\sum_{q=0}^{N} \sigma(q|p_s^0) =$

1. We maintain the convention that agents who choose not to trade announce $\overline{p} > u(N)$ (sellers) or select $p = 0$ (buyers).

Let us concentrate first on pooling and hybrid equilibria in which two or more types of sellers trade at the same price. In order to assess the robustness of these equilibria, we need to state the equivalent of condition (4). In any of these equilibria there always exists a price p^* at which a non-singleton non-empty set of qualities, $M \subseteq \{0, ..., N\}$, is traded. Let q_M be the highest quality in M. Take any quality $q \in M$, $q \neq q_M$. Given pooling at p^*, condition (4) becomes

$$\frac{p^* - c(q)}{p - c(q)} > \frac{p^* - c(q_M)}{p - c(q_M)}, \tag{A.29}$$

which, given $c(q) < c(q_M)$, holds for any $p > p^*$. Thus, robust beliefs should assign probability 0 to a deviation $p > p^*$ by any quality in M except for q_M. As for qualities $q \notin M$, the following applies. Sellers of qualities $q < q_M$ who do not trade at p^* make at least the same profits they would make at p^* by charging a different price (since they could always announce p^*). Since $c(q) < c(q_M)$, they should be assigned probability zero. Sellers of qualities $q > q_M$ should also be assigned probability zero so long as $p < c(q_M + 1)$. Thus, deviations $p^* < p < c(q_M + 1)$ are attributed to sellers of quality q_M with probability 1. As for buyers, the probability to sell at p is one so long as

$$u(q_M) - p > \sum_{q \in M} \sigma(q|p^*)[u(q) - p^*], \tag{A.30}$$

which is always true for p close enough to p^*. Thus deviating to a price slightly higher than p^* would allow sellers of type q_M to reveal their type and induce buyers to buy. Therefore, neither pooling nor hybrid equilibria in which two or more types trade at the same price survive D1.

The set of robust equilibria therefore includes only separating equilibria and hybrid equilibria with the necessary condition that each price at which trade occurs is announced by only one type of seller (i.e. pooling only occurs at $\overline{p}$). We now focus on these equilibria. If $\theta \leq \lambda_0$, the discussion made in the previous sections leads to the immediate conclusion that only quality 0 is traded.

By converse, when $\theta > \lambda_0$, sellers of quality 0 make positive profits and, therefore, higher qualities must be traded. In order to characterize these equilibria, we analyze the properties of the ICC. We start by showing that when the "adjacent upward" ICC is satisfied, all the ICC with respect to all higher qualities are satisfied. The relevant ICC for sellers is

$$J(p_{q-s})[p_{q-s} - c(q-s)] \geq J(p_q)[p_q - c(q-s)], \tag{A.31}$$

for all q and $s = 0, ...q$. It is immediate to check that $J(p_0) = 1$ holds. This, together with (A.31) yields $J(p_q) < J(p_{q-1}) < ... < J(p_1) < 1$, whenever $p_q > p_{q-1} > ... > p_0$. From equation (A.31):

$$J(p_{q-1})[p_{q-1} - c(q-1)] \geq J(p_q)[p_q - c(q-1)], \tag{A.32}$$

and

$$J(p_q)[p_q - c(q)] \geq J(p_{q+1})[p_{q+1} - c(q)]. \tag{A.33}$$

Then, by using $c(q) > c(q-1)$ and $J(p_q) > J(p_{q+1})$, it follows that

$$J(p_{q-1})[p_{q-1} - c(q-1)] \geq J(p_{q+1})[p_{q+1} - c(q-1)], \tag{A.34}$$

always holds. Applying the same reasoning to qualities higher than $q + 1$ shows that when the "adjacent upward" ICC are satisfied, all the ICC with respect to all higher qualities are satisfied.

The next step is to show, by applying D1, that the "adjacent upward" ICC of a given quality must hold with equality whenever the "adjacent upward" quality is traded. It is immediate to check that R5 (ii) ensures that buyers' surplus is constant across all quantities that are traded,

$$u(q) - p_q = k \ \forall q = 0, 1, ..., N, \tag{A.35}$$

where k is a constant.

For $q > 0$, buyers may only be attracted through deviations $p < p_q$. Hence, we restrict attention to deviations $p < p_q$. Notice that D1 requires that type q should be assigned probability zero of deviating to price p, if there exists q' such that

$$\frac{J(p_q)[p_q - c(q)]}{p - c(q)} > \frac{J(p_{q'})[p_{q'} - c(q')]}{p - c(q')}. \tag{A.36}$$

This is the equivalent of condition (4). The next Lemma generalizes Lemma 3.

Lemma 6. *For all $q = 1, ..., N$, in any robust equilibrium in which quality q is traded, the "adjacent upward" ICC of sellers of quality $q - 1$ holds with equality unless $p_q = c(q)$.*

Proof. Consider a deviation $p \in (c(q), p_q)$. Notice that buyers are willing to buy at p if they think that the deviation comes from type q, since $p < p_q$. We argue that whenever it is possible to delete type $q - 1$ from the deviation it is also possible to delete all types $q - s$, $s \geq 2$. To show this point, assume that type $q - 1$ can be eliminated:

$$\frac{J(p_{q-1})[p_{q-1} - c(q - 1)]}{p - c(q - 1)} > \frac{J(p_q)[p_q - c(q)]}{p - c(q)}. \tag{A.37}$$

Consider now type $q - s$. From the incentive compatibility condition:

$$\frac{J(p_{q-s})[p_{q-s} - c(q - s)]}{p - c(q - s)} \geq \frac{J(p_{q-1})[p_{q-1} - c(q - s)]}{p - c(q - s)}. \tag{A.38}$$

But then, for any $p > c(q) > p_{q-1}$, the following relationship,

$$\frac{J(p_{q-s})[p_{q-s} - c(q - s)]}{p - c(q - s)} > \frac{J(p_{q-1})[p_{q-1} - c(q - 1)]}{p - c(q - 1)}, \tag{A.39}$$

holds, which implies that type $q - s$ can be deleted, whenever type $q - 1$ can be deleted. Since $p < p_q < u(q) < c(q+1)$, sellers of type higher than q are never willing to deviate to p. This implies that if type $q - 1$ can be deleted, beliefs should be that the deviation comes from q. As in the two-quality case, we show that whenever $p_q > c(q)$, a viable deviation $p \in (c(q), p_q)$, for which type $q - 1$ can be deleted, exists so long as the incentive compatibility condition of type $q - 1$ holds with inequality. Suppose then that the ICC holds with strict inequality. Assume that the deviation consists in a price $p = p_q - \epsilon$, $\epsilon > 0$ which is a small undercutting of price p_q. We want to show that there exists $\epsilon > 0$ such that

$$\frac{J(p_{q-1})[p_{q-1} - c(q - 1)]}{p_q - \epsilon - c(q - 1)} > \frac{J(p_q)[p_q - c(q)]}{p_q - \epsilon - c(q)}. \tag{A.40}$$

Condition (A.40) can be rewritten as:

$$\frac{J(p_{q-1})[p_{q-1} - c(q-1)]}{J(p_q)[p_q - c(q-1)]} > \frac{p_q - c(q)}{p_q - c(q-1)} \frac{p_q - \epsilon - c(q-1)}{p_q - \epsilon - c(q)}. \tag{A.41}$$

For $p_q > c(q)$, the LHS (which does not depend on ϵ) is strictly greater than 1 whenever the ICC of type $q-1$ holds with strict inequality. On the other hand, the RHS goes to 1 as ϵ becomes small. Thus, there always exists ϵ such that type $q-1$ can be deleted unless either the ICC holds with equality or $p_q = c(q)$. In fact, in the case $p_q = c(q)$, undercutting is never profitable for type q. Hence, either the ICC holds with equality or $p_q = c(q)$.

$\square$

We are now ready to characterize the robust equilibria. We distinguish between the case in which all qualities are traded and the case in which a subset of qualities is traded.

a) *All $N+1$ qualities traded*:
If all qualities are traded, sellers' ICC ensure that $p_q > c(q)$ for all qualities except, possibly, quality N. Thus, for $q < N$, sellers' ICC and Lemma 6 imply:

$$J(p_q) = J(p_{q-1}) \frac{u(q-1) - c(q-1) - k}{u(q) - c(q-1) - k}. \tag{A.42}$$

Unless quality N is the only quality in $\arg\min_{q \in \{1,...,N\}} u(q) - c(q)$, $p_N > c(N)$ must hold and (A.42) holds with equality for quality N as well. In the special case in which N is the only quality in $\arg\min_{q \in \{1,...,N\}} u(q) - c(q)$, $J(p_N)$ is between 0 and the value implied by (A.42).

Using the initial condition $J(p_0) = 1$, equation (A.42) yields

$$J(p_q) = \prod_{i=0}^{q-1} \frac{u(q-i-1) - c(q-i-1) - k}{u(q-i) - c(q-i-1) - k}. \tag{A.43}$$

Since $k \geq 0$, the maximum value of $J(p_q)$ is achieved when $k = 0$. Thus, the probability to trade at p_q, for $q > 0$, is bounded above by

$$\overline{J}_q = \prod_{i=0}^{q-1} \delta_{q-i}, \tag{A.44}$$

where

$$\delta_q \equiv \frac{u(q-1) - c(q-1)}{u(q) - c(q-1)}. \tag{A.45}$$

Notice also that $k = 0$ implies $p_q = u(q) \ \forall q = 0, ..., N$ (i.e. buyers make zero surplus). Assume $k > 0$, so that buyers make positive surplus at all prices. Then, all buyers want to trade. Clearly, $k > 0$ can only emerge if $K = 1$ – otherwise sellers of quality $q = 0$ could always profit from raising their price. Hence, the requirement that the number of goods sold is equal to the number of goods bought (1) can be restated as

$$\sum_{q=0}^{N} \lambda_q J(p_q) = \theta, \tag{A.46}$$

where $J(p_q)$ is given by (A.43) for $q < N$. $J(p_N)$ is also given by (A.43) unless N is the only quality in $\arg\min_{q\in\{1,...,N\}} u(q) - c(q)$, in which case $J(p_N)$ is between zero and the value implied by (A.43). For the case in which all qualities are traded, finding an equilibrium outcome is equivalent to finding a value k^* for which (A.46) holds. From (A.43), the LHS of equation (A.46) is monotonically decreasing in k, for $k \in [0, \hat{k}]$, where $\hat{k} \equiv \min_{q\in\{0,...,N\}} u(q) - c(q)$. We note that simultaneous satisfaction of all the participation constraints requires that k always lie in the interval $[0, \hat{k}]$. Given monotonicity, there always exists at most one value k^* in the above interval. This also implies that if there exists an equilibrium, its outcome must be unique. As for existence, consider the following. From (A.43), the LHS of expression (A.46) reaches its maximum in the relevant interval when $k = 0$ and its minimum when $k = \hat{k}$. Therefore, necessary and sufficient conditions for an interior solution are

$$\theta < \lambda_0 + \sum_{q=1}^{N} \lambda_q \prod_{i=0}^{q-1} \delta_{q-i}, \tag{A.47}$$

and

$$\theta > \lambda_0 + \sum_{q=1}^{N-1} \lambda_q \prod_{i=0}^{q-1} \frac{u(q-i-1) - c(q-i-1) - \hat{k}}{u(q-i) - c(q-i-1) - \hat{k}} + \lambda_N J_N^{min}, \tag{A.48}$$

where J_N^{min} is given by

$$J_N^{min} = \begin{cases} \prod_{i=0}^{N-1} \frac{u(N-i-1) - c(N-i-1) - \hat{k}}{u(N-i) - c(N-i-1) - \hat{k}} & \text{if } \arg\min_{q\in\{1,...,N\}} u(q) - c(q) \neq \{N\} \\ 0 & \text{if } \arg\min_{q\in\{1,...,N\}} u(q) - c(q) = \{N\}. \end{cases} \tag{A.49}$$

Otherwise, if one of the above conditions is not satisfied, the robust equilibrium takes a different form. If condition (A.47) is not satisfied, then k must be equal to zero and the equilibrium is characterized by $p_q = u(q)$ for all $q = 0, 1, ..., N$ and probabilities $\overline{J}_q$ given by (A.44). As $\overline{J}_q > 0 \; \forall q = 0, ..., N$, all qualities are traded also in this case. Thus, a sufficient condition for all qualities being traded is that θ is high enough to ensure that (A.48) holds. Below, we show that this condition is also necessary. In order to gather intuitions on condition (A.48), notice that it is always satisfied when the gains from trade are nondecreasing in the quality since, in this case, $\hat{k} = u(0) - c(0)$ and the RHS is equal to λ_0. Therefore, as long as $\lambda_0 < \theta$, an internal solution in which all qualities are traded must exist.

b) More than one and less than $N + 1$ qualities are traded:
Assume now that θ is relatively small so that condition (A.48) is not satisfied. If the value of the RHS of equation (A.48) for $k = \hat{k}$ is greater than or equal to θ, then some qualities are not traded. To see this, note that k cannot exceed $\hat{k}$. Let $\underline{J}_q$ be the probability to sell at p_q when k equals $\hat{k}$. Let also $\hat{q}$ be the lowest quality in $\arg\min_{q\in\{0,...,N\}} u(q) - c(q)$. Then, by definition, $u(\hat{q}) - c(\hat{q}) - \hat{k} = 0$ so that, from equation (A.43), $\underline{J}_q$ is zero for all $q > \hat{q}$. Thus, when $k = \hat{k}$, all qualities above the quality which provides the lowest gain from trade are not traded. Buyers' surplus k should increase, however it fails to increase because, by increasing, it would violate the participation constraint of sellers of type $\hat{q}$. Hence, the incentive compatibility for type $\hat{q}$ requires that no higher quality is traded. Thus, (A.48) is a necessary condition for all qualities being traded.

What are the characteristics of the equilibrium when (A.48) is violated? Note that if (A.48) is not satisfied we have

$$\sum_{q=0}^{\hat{q}} \lambda_q \underline{J}_q = \sum_{q=0}^{N} \lambda_q \underline{J}_q > \theta, \tag{A.50}$$

where the equality comes from the fact that all qualities higher than $\hat{q}$ are not traded. This suggests that even if only $\hat{q}+1$ qualities are traded out of $N+1$, the quantity sold is still higher than the quantity bought. Then, quality $\hat{q}$ cannot be traded in equilibrium. Let us assume that quality $\hat{q} - 1$ is traded. It follows that its price, $p_{\hat{q}-1}$, must be compatible with D1. In other words, there must be no incentive to deviate for sellers of type $\hat{q}$ or above. Therefore, $p_{\hat{q}-1}$ must be such that $k_1 \equiv u(\hat{q}-1) - p_{\hat{q}-1} \geq u(\hat{q}) - c(\hat{q})$. If so, there is no price sellers of type $\hat{q}$ could possibly announce to attract buyers and still make no loss. Of course, buyers incentive compatibility implies $u(q) - p_q = k_1$ for all qualities that are traded, i.e. $q = 0, ..., \hat{q} - 1$. Now, let $\tilde{q}$ be the lowest quality in $\arg\min_{q \in \{0, ..., \hat{q}-1\}} u(q) - c(q)$, i.e. the lowest quality among those which give minimum gain from trade when attention is restricted to qualities lower than $\hat{q}$. It is clear that k_1 should now satisfy $u(\hat{q}) - c(\hat{q}) \leq k_1 \leq u(\tilde{q}) - c(\tilde{q})$. Therefore, all that remains to be checked is whether there exists k_1^* such that

$$\sum_{q=0}^{\hat{q}-1} \lambda_q \prod_{i=0}^{q-1} \frac{u(q-i-1) - c(q-i-1) - k_1^*}{u(q-i) - c(q-i-1) - k_1^*} = \theta. \tag{A.51}$$

If it does, then the equilibrium is such that qualities $q = 0, ..., \hat{q} - 1$ are traded. If it does not, then all the process starts again by choosing $\tilde{q}$ as the first quality that is not traded. It should be noted that, since we are assuming $\theta > \lambda_0$, the process eventually leads to an equilibrium in which more than one quality is traded. In fact, as long as $\theta > \lambda_0$, qualities 0 and 1 are always traded. By iterating this process, one can show that the number of qualities traded in equilibrium decreases with θ. Thus, price dispersion increases as θ increases.

The extension to $N + 1$ qualities generalizes the result of an inverse relationship between price dispersion and the degree of competition (as measured by θ) derived for the case of two qualities. When competition among sellers is so strong that $\theta \leq \lambda_0$, only the lowest quality is traded and there is no price dispersion. When competition is weak (θ satisfies condition (A.48)) all qualities are traded and price dispersion is maximized. For intermediate values of θ such that $\theta > \lambda_0$ while (A.48) is not satisfied, the number of qualities which are traded and the degree of price dispersion (weakly) increase with θ.

A.2 Tedious part of the proof of Lemma 1 – Not meant for publication

If $p > u(h)$ R5(i) requires $J(p) = 0$ for all values of $\sigma(h|p)$. Consider deviations $p < u(h)$. Given $p > u(l)$, if $K < 1$, then R5(i) and R5(iii) are jointly satisfied for any $J(p) \in [0, 1]$ if and only if beliefs are such that $\sigma(h|p)u(h) + [1 - \sigma(h|p)]u(l) = p$. Moreover, R5(iii) requires probability to sell equal to one at all prices that leave positive surplus to the buyers, so that R5(ii) imposes no restriction (since all prices $p' \neq p$ give higher surplus to the buyer than p and $J(p') = 1$ at all p'). If $K = 1$

then R5(iii) does not apply, so that R5(i) and R5(iii) are jointly satisfied for any $J(p) \in [0, 1]$ if and only if $\sigma(h|p)$ is such that $\sigma(h|p)u(h) + [1 - \sigma(h|p)]u(l) \geq p$. If at all prices p' announced in the candidate equilibrium buyers make lower surplus than p (but the surplus has to be non-negative) and $J(p') > 0$, then pick $\sigma(h|p)$ such that $\sigma(h|p)u(h) + [1 - \sigma(h|p)]u(l) = p$ so that p does not yield higher surplus and any $J(p) \in [0, 1]$ satisfies R5(ii). If there are prices p' which yields higher surplus than p, then R5(ii) requires $J(p) = 0$ for all $\sigma(h|p)$ whenever $J(p') < 1$.

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Figure 1: Equilibrium prices as a function of θ
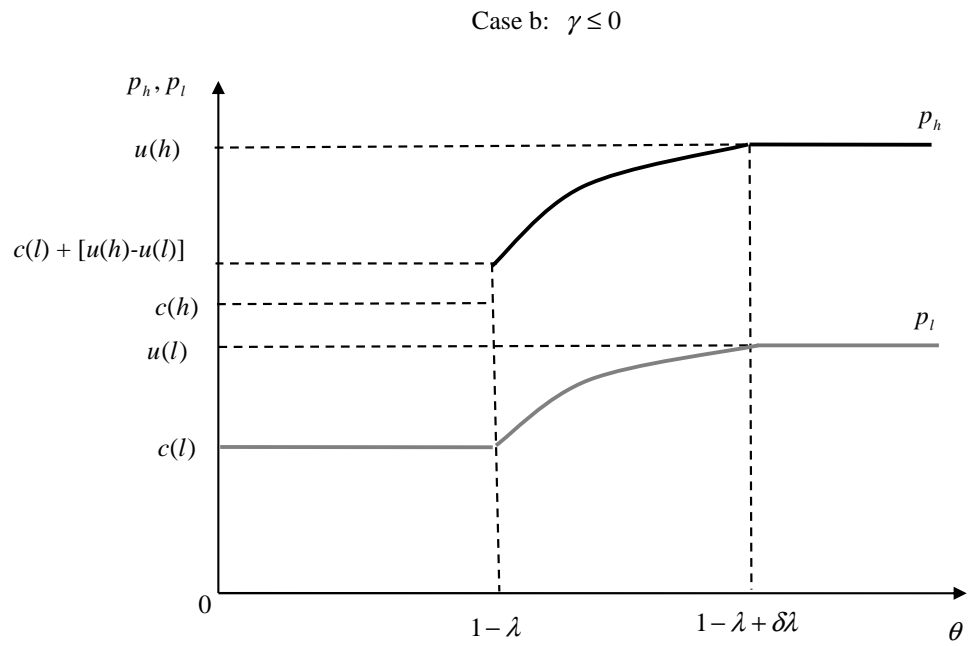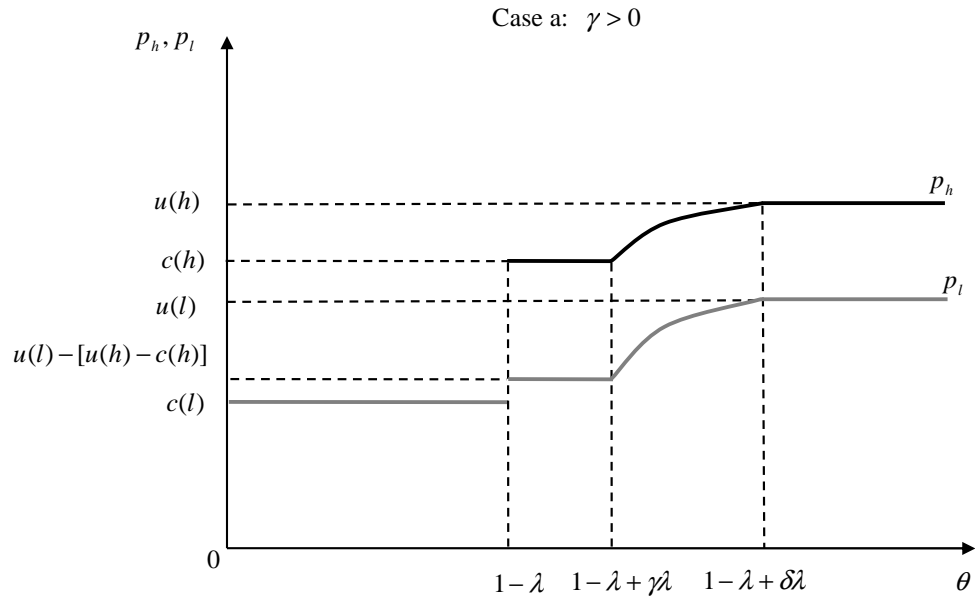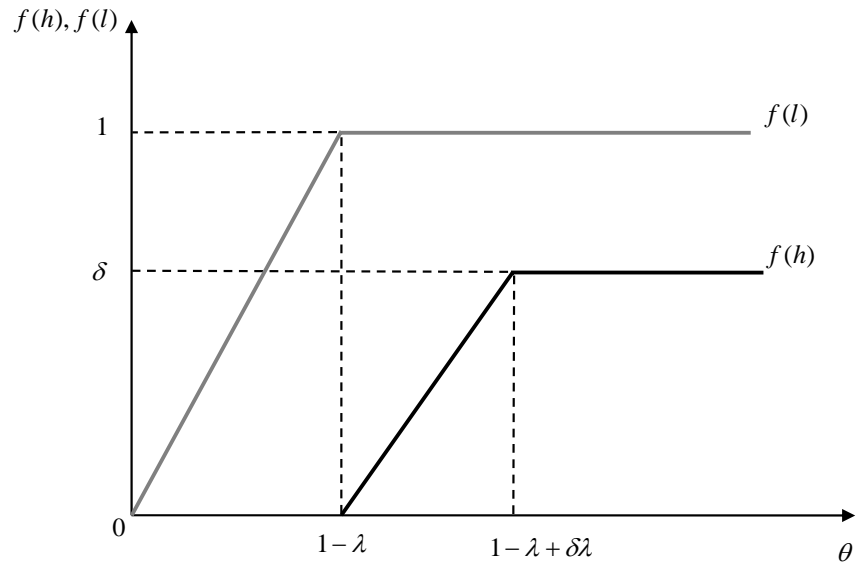
Case a: $\gamma > 0$

Case b: $\gamma \leq 0$

Figure 2: Quality of trade as a function of θ

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